Chapter 9

Mathematical Model

9.1. Introduction

Friction coefficient of fibre reinforced composites varies with the orientation of fibres with respect to the counter surface [187-191]. In unidirectional composites, friction coefficient for perpendicular orientation of fibres is different from the friction coefficient for parallel orientation. Friction coefficient also varies with the counterface material [192, 193]. Several researchers have used uniphase material as counterface whereas only few have used multiphase material as counterface [151, 194]. Friction coefficient of fibre reinforced composites also depends on the volume fraction of fibres [188, 195, 196] as well as on the relative interaction area of fibres and matrix with the counterface. Relative interaction area is the relative area proportion of fibres and matrix which are in direct contact with the counterface. When fibres are arranged perpendicular to the counterface in a friction couple, relative interaction area doesn’t change with time if the material wear out during in-service conditions. But when fibres are arranged parallel to the counterface, relative interaction area changes with time because both fibres and matrix wear out gradually.

When the counterface material is also a fibre reinforced composite, then the friction coefficient also depends upon the interaction of the material with the counterface matrix, fibres, and their conformity. Change in conformity results in change of stress conditions on the surface.

Friction coefficient of materials also depends upon the hardness [193]. The hardness of the uniphase material does not change with the position on the surface. However, in case of fibre reinforced composites, effective hardness of the surface is generally considered. The superficial hardness of fibre reinforced composites changes with the position on the surface because the hardness of fibres and matrix is usually different. Thus, considering effective hardness to develop mathematical model for friction coefficient of fibre reinforced composites generally yield results which do not fit well with the experimental results.

Friction coefficient of fibre reinforced composites also depends upon the friction coefficient of individual components of composite against same counterface material. It is observed that the composites containing same matrix but reinforced with different fibres show different friction coefficient with the same counter surface. Thus friction coefficients of individual components play a significant role in deciding the friction coefficient of a composite. However, only a few researchers have worked to find out the friction coefficient of individual fibres against a specific material [197, 198]. Thus, there is lack of data available on the friction coefficient of individual fibres against a specific material.

Friction coefficient of an individual fibre against a specific material can be found by capstan apparatus. Roselman and Tabor [197] used the capstan apparatus to find out the friction coefficient of carbon fibre against some materials and related the friction coefficient with the tension in the fibre as

$$T = T_o e^{\mu\theta} \quad (1)$$

where, T and T_o are the tensions at the ends of the fibre. T_o is the tension along applied weight side.

μ is the friction coefficient.

θ is the angle subtended by the length of fibre in contact with the cylinder.

However, friction coefficient of one fibre against another can't be find out by using eq. (1). To find out friction coefficient of one fibre against another, Olga smerdova [199] suggested a method. Olga smerdova found friction coefficient of carbon fibres against carbon fibres in parallel as well as anti-parallel orientation. Friction coefficient of fibres rubbed against another fibres changes with orientation.

In the light of the above, equations are formulated for the friction coefficient of fibre reinforced composites.

9.2. Formulation of equations

Friction coefficient depends upon the orientation of fibres with respect to counterface material. Equation were formulated for the friction coefficient of fibre reinforced composites against uniphase material, for which following assumptions were made.

i. Every single phase of composite material is isotropic in nature and follows coulomb's law of friction.
ii. Overall composite material follows coulomb's law of friction.

9.2.1. Composite having longitudinal fibres aligned perpendicular to the uniphase counterface

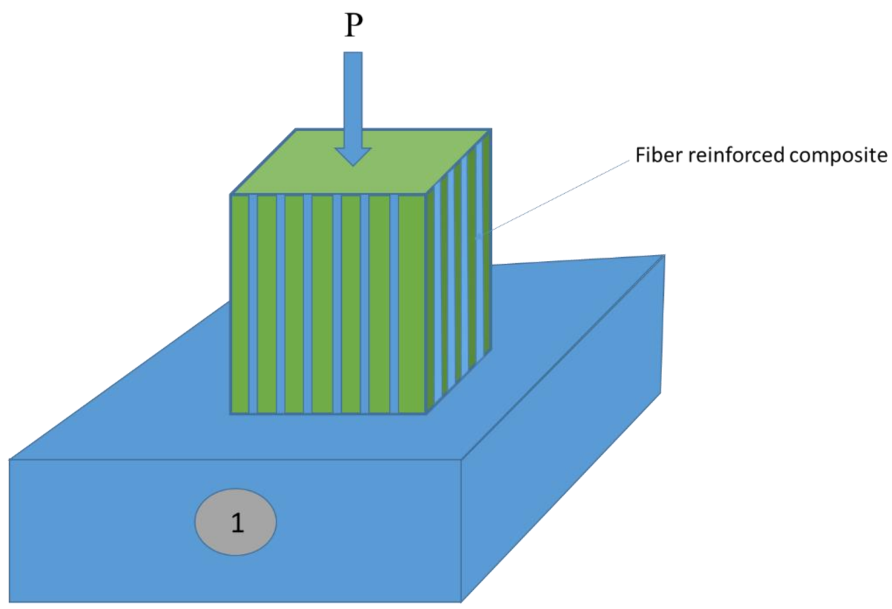


Fig. 9.1. Longitudinal fibre composite in contact with uniphase material 1.

Friction coefficient of composite having longitudinal fibres aligned perpendicular to the uniphase counterface (as shown in Fig. 9.1) is given by

$$\mu_1 = \frac{F}{P} \qquad (2)$$

.

where, μ_l is friction coefficient against counterface designated as 1.

F is the friction force

P is the load applied on the composite.

Since the friction force acting on the composite is the sum of friction forces due to fibres and matrix i.e. sum of friction forces due to all individual components.

Thus,

$$F = F_f + F_m$$

$$= \mu_{f1} P_f + \mu_{m1} P_m$$

$$= \mu_{f1} \sigma_f A_f N_f + \mu_{m1} \sigma_m A_m$$

$$= \mu_{f1} E_f \varepsilon_f A_f N_f + \mu_{m1} E_m \varepsilon_m (A - A_f N_f)$$

$$F = \mu_{f1} E_f \varepsilon_f A_f N_f + \mu_{m1} E_m \varepsilon_m A - \mu_{m1} E_m \varepsilon_m A_f N_f \qquad (3)$$

where, F_f is friction force due to fibres,

F_m is friction force due to matrix,

μ_{f1} and μ_{m1} are the respective friction coefficient of fibres and matrix against counterface 1,

σ_f and σ_m are the stresses in the fibre and matrix,

A_f and A_m are the area of one fibre bundle and the matrix,

A is the total area of contact,

E_f and E_m are the elastic moduli of fibres and matrix,

ε_f and ε_m are strains in fibre and matrix.

Since $\varepsilon_f = \varepsilon_m$ as the composite is loaded longitudinally.

Thus from eq. (3)

$$F = \varepsilon_f \{[A_f N_f (\mu_{f1} E_f - \mu_{m1} E_m)] + \mu_{m1} E_m A\} \qquad (4)$$

Now, $P = P_f + P_m$

$$P = E_f \varepsilon_f A_f N_f + E_m \varepsilon_m (A - A_f N_f)$$

$$P = \varepsilon_f\,[E_f A_f N_f + E_m A - E_m A_f N_f] \quad (5)$$

Thus from eq. (4) and (5), we get

$$\mu_1 = \frac{A_f N_f\,(\mu_{f1} E_f - \mu_{m1} E_m) + \mu_{m1} E_m A}{E_f A_f N_f - E_m A_f N_f + E_m A}$$

$$\mu_1 = \frac{A_f N_f\,(\mu_{f1} E_f - \mu_{m1} E_m) + \mu_{m1} E_m A}{A_f N_f\,(E_f - E_m) + E_m A} \quad (6)$$

Divide and multiply equation (6) by A, we get

$$\mu_1 = \frac{\frac{A_f N_f}{A}(\mu_{f1} E_f - \mu_{m1} E_m) + \mu_{m1} E_m}{\frac{A_f N_f}{A}(E_f - E_m) + E_m} \quad (7)$$

In case of longitudinal fibres arranged perpendicular to the counterface

$$\frac{A_f N_f}{A} = V_f \quad (8)$$

Thus from eq. (7) and (8), we get

$$\mu_1 = \frac{V_f\,(\mu_{f1} E_f - \mu_{m1} E_m) + \mu_{m1} E_m}{V_f (E_f - E_m) + E_m} \quad (9)$$

9.2.2. Fibres aligned parallel to the uniphase counterface

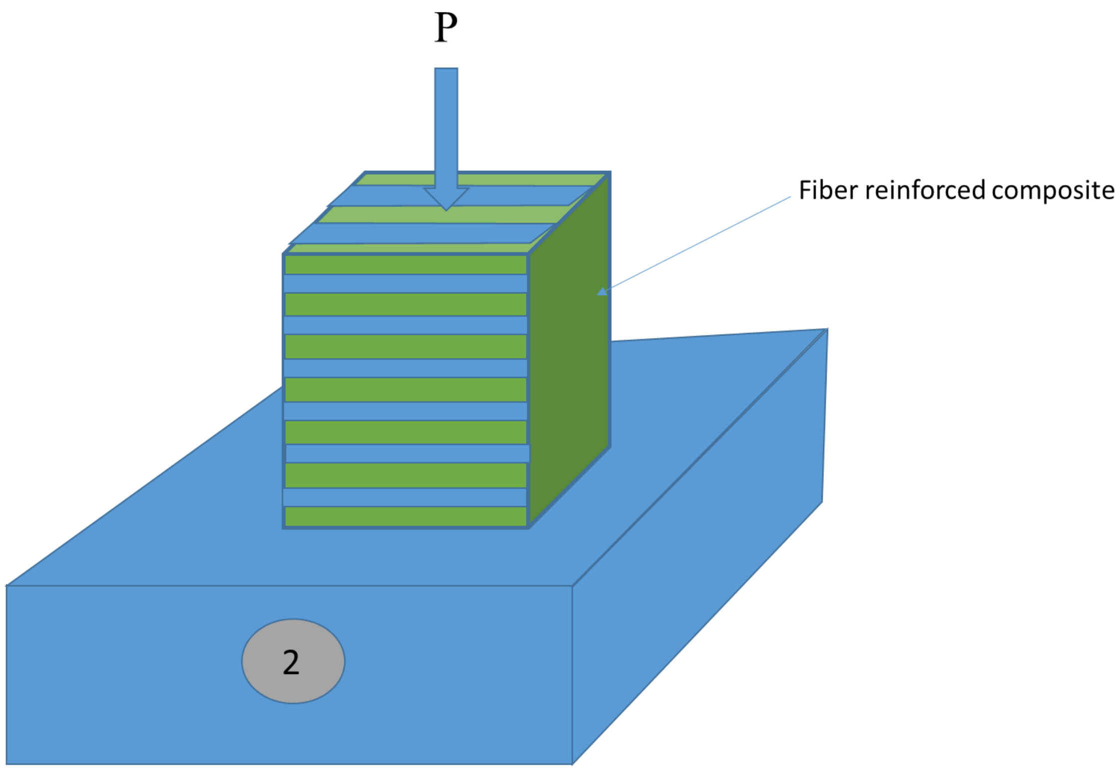


Fig. 9.2. Parallel fibre composite in contact with uniphase material 2.

Friction coefficient of composite having longitudinal fibres aligned parallel to the uniphase counterface (as shown in Fig. 9.2) is given by

$$\mu_2 = \frac{F}{P} \tag{10}$$

where, μ_2 is friction coefficient,

F is the friction force,

P is the load applied on the composite.

The friction force acting on the composite is the sum of friction forces due to fibres and matrix.

Thus,

$$F = F_f + F_m$$

$$= \mu_{f2}\sigma_f \sum_{f=1}^{f=n}(A_{fo}\chi_f) + \mu_{m1}\sigma_m \left[A - \sum_{f=1}^{f=n}(A_{fo}\chi_f)\right]$$

$$= \mu_{f2}\sigma_f \sum_{f=1}^{f=n} A_{fo}\chi_f + \mu_{m1}\sigma_m A - \mu_{m1}\sigma_m \sum_{f=1}^{f=n} A_{fo}\chi_f \qquad (11)$$

where, A_{fo} is the maximum area of fibre that can come in contact with the counterface.

χ_f is the interaction area parameter i.e. the ratio of area of fibre in contact to the maximum area of fibre that can come in contact with the counterface. It varies from 0 to 1.

n is the total number of fibres in contact with the counterface.

Since $\sigma_m = \sigma_f = \sigma_T$ (considering isostress state as there is isostress state in most of the composites loaded in transverse direction)

σ_T is the stress in the composite in transverse direction.

Thus

$$F = \mu_{f2}\sigma_T \sum_{f=1}^{f=n}(A_{fo}\chi_f) + \mu_{m1}\sigma_T A - \mu_{m1}\sigma_T \sum_{f=1}^{f=n}(A_{fo}\chi_f)$$

$$F = \sigma_T \left(\mu_{f2} \sum_{f=1}^{f=n}(A_{fo}\chi_f) + \mu_{m1} A - \mu_{m1} \sum_{f=1}^{f=n}(A_{fo}\chi_f)\right)$$

$$F = \sigma_T \left\{\left[(\mu_{f2} - \mu_{m2}) \sum_{f=1}^{f=n}(A_{fo}\chi_f)\right] + \mu_{m2}A\right\} \qquad (12)$$

Now, $P = \sigma_T A$

Thus, $$\mu_2 = \frac{(\mu_{f2} - \mu_{m2}) \sum_{f=1}^{f=n}(A_{fo}\chi_f) + \mu_{m2}A}{A}$$

$$\mu_2 = \frac{(\mu_{f2} - \mu_{m2}) \sum_{f=1}^{f=n} (A_{fo}\chi_f)}{A} + \mu_{m2}$$

(13)

Since $\frac{\sum_{f=1}^{f=n}(A_{fo}\chi_f)}{A} \;\alpha\; V_f$

Thus,

$$\frac{\sum_{f=1}^{f=n}(A_{fo}\chi_f)}{A} = kV_f \qquad (14)$$

However k is the function of χ_f.

That is $k = f(\chi_f)$

Thus,

$$\mu_2 = kV_f\,(\mu_{f2} - \mu_{m2}) + \mu_{m2} \qquad (15)$$

If fibre bundles are arranged in circular cross section and there are n number of fibres in contact with the counterface, then in equation (13)

$$\sum_{f=1}^{f=n}(A_{fo}\chi_f) = (d_f l_f \chi_f)_1 + (d_f l_f \chi_f)_3 + (d_f l_f \chi_f)_3 + \cdots \ldots \ldots \ldots \ldots .. (d_f l_f \chi_f)_n \qquad (16)$$

If diameter of all fibre bundles are same and portion of all fibres which come in contact with the counterface at the same time, is same, then

$$\sum_{f=1}^{f=n}(A_{fo}\chi_f) = d_f \chi_f\,[(l_f)_1 + (l_f)_3 + (l_f)_3 + \cdots \ldots \ldots \ldots \ldots .. (l_f)_n]$$

$$\sum_{f=1}^{f=n}(A_{fo}\chi_f) = d_f L_f \chi_f \qquad (17)$$

L_f is the total length of fibres in contact.

Thus,

$$\mu_2 = \frac{d_f L_f \chi_f (\mu_{f2} - \mu_{m2}) + \mu_{m2} A}{A} \quad (18)$$

9.3. Validation and discussion

There is lack of data regarding friction coefficient of several fibres against uniphase materials and fibre reinforced composites having different orientation of fibres. However to validate the derived equations, interaction area parameter can be taken as 1 to know the upper limit of friction coefficient of composite if friction coefficient of fibre is more than friction coefficient of matrix. If friction coefficient of fibre is less than friction coefficient of matrix, the interaction area parameter can be taken as 0 to know the upper limit of friction coefficient of composite.

Due to lack of data of various variables in derived equations, only equation (9) is selected for validation. Anyone can check the equation 18 by considering the other parameters (e.g. diameter of fibres, total contact area, and fibre/counterface friction) before starting the friction experiment.

A MATLAB program was used to validate the equation (9). The data was taken from the reported work [199-201] to qualify the validity of present analysis. The variation of friction coefficients (present analysis) of carbon fibre reinforced epoxy (CFRE) composite against epoxy and steel with volume fraction of fibres is shown in Fig. 9.3 and 9.4. The experimental results of friction coefficients of carbon fibre reinforced epoxy composite against epoxy and steel with varying volume fraction of fibres is shown in Fig. 9.5.

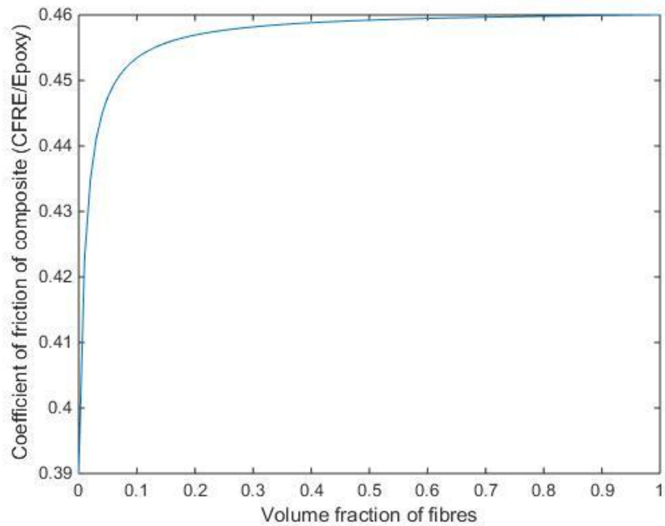


Fig. 9.3. Friction coefficient of CFRE composite against epoxy with varying volume fraction of fibres (Present analysis)

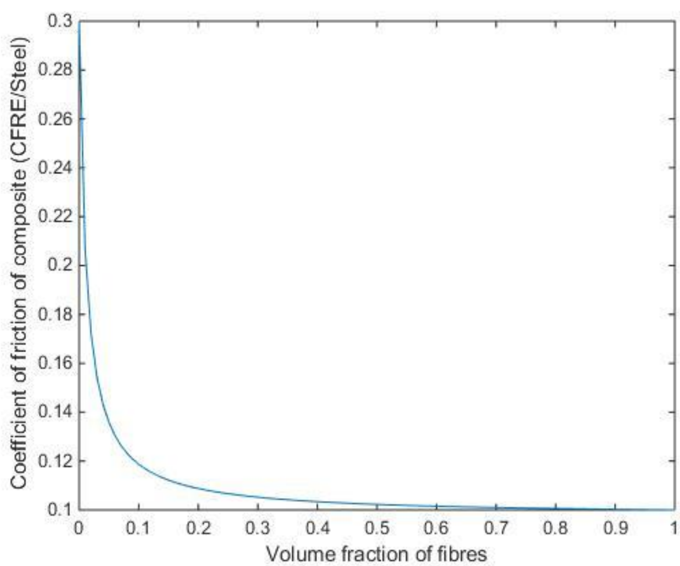
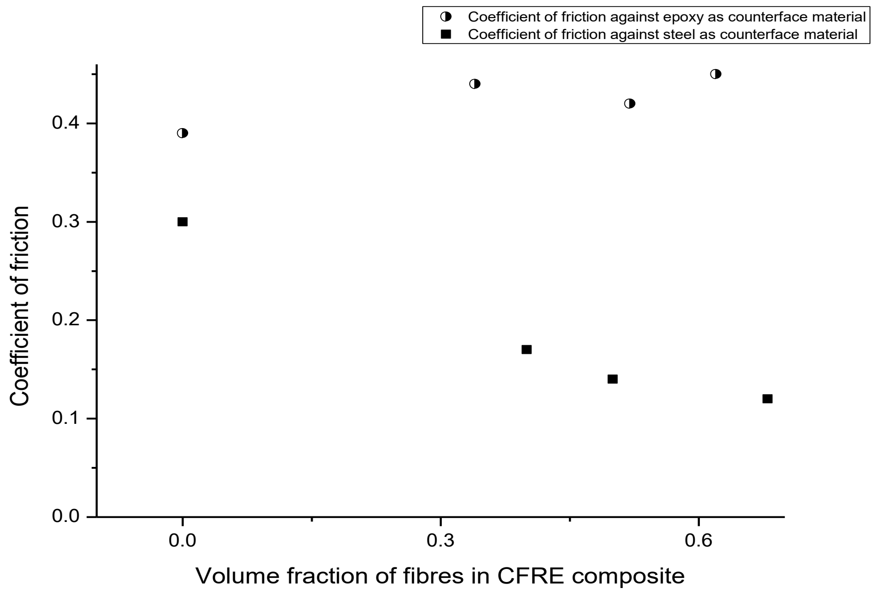


Fig. 9.4. Friction coefficient of carbon fibre reinforced epoxy composite against steel with varying volume fraction of fibres (Present analysis)



Fig, 9.5. Experimental results of friction coefficients of carbon fibre reinforced epoxy composite against epoxy and steel with varying volume fraction of fibres [199].

The deviation in friction coefficient from present analysis and the experimental results may be due to the fact that in the present analysis, wear rate of the surface in contact is assumed to be uniform which is not the case with most of the composites. Fibre peeling is not considered in the present analysis which happens in most of the cases. Thus in future, the present analysis can be modified for variable wear rate of the surfaces in contact.

It is observed that equations fits well the experimental results. However, if effective hardness of the surface is considered, a straight line is observed [155] which shows that friction coefficient varies linearly with the volume fraction of fibres which is not the case always. Thus considering different surface properties of fibres and matrix yields results which shows that friction coefficient doesn't vary linearly with volume fraction of fibres.

9.4. Conclusion

Mathematical model for the calculation of the friction coefficient of fibre reinforced composites has been introduced. Equations were formulated for parallel and perpendicular orientation of fibres in a composite. To validate the model, equation for longitudinal fibre composite against uniphase material is selected. This equation was validated for carbon fibre reinforced epoxy composite against epoxy and steel as counterface materials. The results showed that the behaviour of friction coefficient is not linear as the volume fraction of fibres in composite increases which is also observed experimentally by several researchers.