Chapter 2

Literature on atmospheric vortices

Our endeavor to explore latent laws of nature for several millennia is the inherent human characteristic we can be proud of. It looks proper to quote,

"Man lacked the enchanted hands of the unseen forces that created the universe brilliantly but the way he attempted and explored the mysteries with his limited talent and skill is no less splendid. The moment he opened his eyes, he observed: on the one hand was lying stupendous beauty and on the other were standing unyielding principles. He looked out of curiosity into the laws connecting thins in nature and tried to grasp ideas from them. His ventures passed from one generation to the other and finally he created an artificial world of his own."

Sanjay Kumar Pandey (1997)

The efforts made by researchers to investigate atmospheric vortices theoretically and through simulations as well are enormous in magnitude and hence the literature is pretty rich with investigations from genesis point of view and also from view point of analyzing other characteristic aspects. Despite the height we have scaled, the natural phenomena remain underexplored. The comprehensive reporting follows.

2.1 Analytical models

Rankine (1882) vortex model was a pioneer work among the initial efforts to model a vortex type phenomenon. He assumed an ideal steady-state radially symmetric circular solid mass as tornado vortex model possessing only azimuthal velocity and the outer region free from vorticity. That simplest model with the assumptions of negligibly small radial and axial velocities,, i.e., $u = 0$, $w = 0$ respectively, is given by the azimuthal velocity, $v(r) = v_0 r/a$ for $r < a$ and $v = v_0 a/r$ for $r \geq a$, where r, a, v and v_0 are respectively radial coordinate, radius of the vortex and the maximum azimuthal velocity at $r = a$. The part of the motion beyond the core vortex is called free vortex whose velocity, unlike the core vortex, in inversely proportional to the radial distance.

An unsteady axisymmetric columnar vortex model known as Oseen (1912)-Lamb (1932) model incorporated viscous and unsteady aspects of the flow and modified it under the assumption that: radial velocity is negligibly small, i.e., $u = 0$, the axial velocity is negligibly small, i.e., $w = 0$ and the azimuthal velocity is given by $v(r,t) = \frac{\Gamma_0}{2\pi r}\{1 - \exp(-\frac{r^2}{4\nu t})\}$, where $\Gamma(r,t) = 2\pi r v$, $\{\Gamma(0,0) = 0, \Gamma(0,t) = 0, \Gamma(\infty,t) = \Gamma_0\}$, Γ, t and v represent circulation, time and kinematic viscosity respectively.

The velocity profile of Tayler (1918) vortex is given by $u = 0$, $w = 0$, and $v(r,t) = Mr/8\pi v t^2 \exp(-\frac{r^2}{4\nu t})$, where M represents the total angular momentum about the axis given by $M = \int_0^\infty 2\pi r^2 v dr$. This solution has zero total circulation and finite angular momentum M.

Flower (1936) and Williams (1948) noticed that dust devils occur strictly in the afternoon, mainly one to two hours after the noon. Battan (1958) wrote a good review on the present knowledge. Regarding mechanisms of occurrence, Battan

(1958) referred to Ives (1947), who had opined that a super-adiabatic lapse rate is required for dust devil initiation. Battan (1958) further endorsed the hypothesis described by Michelson (1951) that the kinetic energy of a dust devil is extracted from potential and internal energy. This is essentially required due to conservation of energy if the diabatic forcing is absent, indirect speaking that direct radiative heating is not a primary source of energy for dust devils.

Burgers (1940, 1948) and Long (1957)-Rott (1958) independently obtained Burgers-Rott vortex model for steady viscous vortex embedded in a radially inward stagnation point flow over a plane boundary in the form: $u = -ar$, $v(r,t) = \frac{\Gamma_0}{2\pi r}\{1 - \exp(-\frac{ar^2}{2\nu})\}$ and $w = 2az$, where $a = -(\partial u/\partial r)_0$.

The Sullivan (1959) vortex is an exact solution and has some similarity to the Burgers-Rott vortex model. This has a one-celled vortex as well as a two-celled vortex. The two-celled vortex has an inner cell in which air flow descends from above and flows outward to meet a separate air flow that is converging radially. Both flows rise at the point of meeting. The Sullivan vortex is probably the simplest vortex that can describe the flow in an intense tornado with a central downdraft, and it is the simplest vortex that localizes its updraft to a particular place-there is a place for the thunderstorm. The mathematical form of the Sullivan vortex is $u = -ar + \frac{6\nu}{r}(1 - \exp(-\frac{ar^2}{2\nu}))$, $w = 2az(1 - 3\exp(-\frac{ar^2}{2\nu})$ and $v = \frac{\Gamma}{2\Pi r}\frac{H(\frac{ar^2}{2\nu})}{H(\infty)}$, where Γ is the circulation strenght of the vortex, $a = -(\partial u/\partial r)_0$ is the strength of the suction and $H(x)$ is the function defined as $H(x) = \int_0^x \exp(f(t))$, where $f(t) = -t + 3\int_0^t (1 - \exp(-y))dy/y$, where ν is considered to be a constant eddy viscosity which dominates the value of this coefficient, not molecular viscosity. Moreover, the distribution pressure in the atmosphere is given as $p(r,z) = p_0 + \rho\int_0^r \frac{v^2}{r}dr - \frac{\rho r^2}{2}(r^2 + 4z^2) - \frac{18\rho\nu^2}{r^2}(1 - \exp(-\frac{ar^2}{2\nu}))$. The axial pressure gradient is $\partial p/\partial z = -4\rho z a^2$, and increase vertically without bound.

The Batchelor (1964) vortex is an approximate solution to the Navier-Stokes equations obtained using a boundary layer approximation. The physical reasoning behind this approximation is the assumption that the axial gradient of the flow field of interest is of much smaller magnitude than the radial gradient. The solution presented are meant for trailing line vortices from lifting surfaces, jet intake vortices, bath drain vortices, tornadoes etc.

The most simplified form of the Batchelor vortex is q-vortex. It is axisymmetric stretch free columnar vortex. A q-vortex is a model of isolated vortex flow with both axial and azimuthal velocity components. The q-vortex is defined by $u(r) = 0$, $v(r) = \frac{\Gamma_0}{2\pi r}\{1 - \exp(-r^2/a^2)\}$ and $w(r) = \frac{\Gamma_0}{2\pi a q}\{1 - \exp(-r^2/a^2)\}$, where a is the core radius, q is the swirling parameter, the initial parameter Γ_0 is an arbitary parameter.

2.1.1 Dust devils

The field work of Sinclair (1964, 1969) in the Avra Valley and the Tuscon basin, Arizona between 1960 and 1962 is generally observed as the leading attempt of intensive research on dust devils. Sinclair considered the dust devil as a particular type of thermal phenomenon that takes place when a convective updraft forms a vortex. In that model concerning concept, the radial velocity component close to the ground draws warm boundary layer air, which enhances temperature in the core.

Classical vortex models, by considering dust devils and tornadoes as tall and thin columns and neglecting verical diffusion of masses, were designed for homogeneous fluids with constant density (Long, 1958, 1961) and for buoyant fluids which allow for the treatment of the convective forcing in a schematic way (Gutman, 1957; Kuo, 1966, 1967).

Based on observation of dust devils in northwest Libya, McGinnigle (1966) discovered that low background winds were favorable to dust devil formation. It was also noticed that surfaces with low thermal conductivity and high thermal capacity with spatially uneven heating also creates conditions favorable to dust devil. Low thermal conductivity allows the uppermost layers of the soil to heat in the afternoon without considerable loss of energy to the cooler layers below. The tendency to produce a hot ground, however, would be tempered by high thermal capacity. Owing to uneven heating of the surface, large-scale, thermally-driven circulations have some influence on the formation of dust devil. Bretherton and Turner (1968) explained how the presence of preferred direction in the mixing process induces an anisotropic mingling causing a mean radial flux of angular momentum, which make the fluid's angular momentum homogeneous.

Ward (1956, 1972) studied temperature inversion as a factor in the formation of tornadoes. The same author (1972) later explored certain features of tornado dynamics using a laboratory model. Kuo (1966, 1967) studied the dynamics of convective atmospheric vortices. It was followed his (1967) note on the similarity solutions of the vortex equations in an unsteady stratified atmosphere. Bellany-Knights (1970) attempted to get an unsteady two cell solutions of the Navier –Stokes equations while Serrin (1972) modelled a swirling vortex.

Buoyancy created by the solar heating of the surface plays a crucial role in the formation of dust devils. Dust devils are formed within rising spirals of warm air in the convective boundary layer (Battan, 1958; Ryan and Carroll, 1970; Cortese and Balchandar, 1993; Shapiro and Kogan, 1994; Renno et al., 1998; Michaels and Rafkin, 2004). They are also known as swirling rising plumes (Zhao et al., 2004). These swirling rising plumes originate in the regions where the air close to the ground is identified by a super-adiabatic temperature lapse rate (Sinclair, 1976; Oke et al., 2007). Intense dust devils can rise up tot $1 - 2$ km altitude (Hess and Spillane, 1990).

An observation by Sinclair (1969) was that dust frequency is suppressed when the wind exceeds a certain value. Increasing wind speeds increase mechanical mixing reducing the magnitude of the lapse rate near the surface and the strength of convective mixing related to dust devils. Further, wind shear disrupts the coherence of the dynamical structures embedding dust devils.

Ryan and Carrol (1970) improved the results of Sinclair with more accurate measurements of horizontal and vertical wind. A, 500×500 m area of the Mojave desert was scraped to produce a more uniform surface and to provide loose material suitable for lifting. Little mixing was observed between the dust devil core and the surrounding air. Air entered into the dust devil core was only from the friction layer near the surface, which was conformed the concept of Sinclair.

The dust devil is born in an unstable atmospheric environment. To aptly describe the impact of angular momentum radial mixing, partition of the turbulent into "relative vorticity and angular momentum diffusing elements" was proposed by Bretherton and Turner (1969) and McEwan (1973).

Ryan (1972) discovered a relationship between lapse rate and dust devil frequency. A strong correlation was found between dust devil frequency and the lapse rate in the super-adiabatic layer from 0.3 to 10 m above the ground level; however, almost no correlation was found between dust devils and higher level lapse rates in the super-adiabatic layer.

Deissler (1977) presented some models to describe various aspects of atmospheric vertices by taking a single gravity driven vortex and a frictionless adiabatic model. The effects of heat drag, heat transfer and participation induced downdrafts were discussed. It was a generalization of the model proposed by Deissler and Boldman (1974). The azimuthal velocity v was considered as a function of only radial coordinate r whereas the axial velocity w depended on z only. The radial velocity u was then computed and found to be dependent on r and z.

Laboratory experiments with vortex chambers (Ward, 1972; Church et al., 1979) and the supporting numerical simulations (Rotunno, 1977, 1979) suggested that the single most important non-dimensional parameter that governs the tornado-like vortex structure/dynamics is the swirl ratio S. It is the ratio of tangential velocity imposed at the fixed outer radius to the mean vertical velocity of the total flow-through imposed in the vortex chamber (Davies-Jones, 2015). At low S there is a one-cell vortex with updraft flow in its centre but when S exceeds a certain (for fixed geometrical parameters) threshold value, then according to the laboratory and numerical simulation results the vortex should possess a two-cell structure, with updraft flow in the vortex center (Fiedler,1986 and Rotunno, 1986).

Experimental observations by Vatistas (1986) suggest that in the concentrated vortex the azimuthal velocity component does not depend strongly in the axial direction. Therefore, under these assumptions the radial velocity component can be obtained from the θ-momentum equation, the normalized radial velocity function $\bar{u}$ is given by $\bar{u} = -\frac{2(1+n)\bar{r}^{2n-1}}{(1+\bar{r}^{2n})}$, where ν is the kinematic viscosity and the normalized axial velocity function $\bar{w}$ is given by $\bar{w} = \frac{4(1+n)\bar{r}^{2(n-1)}}{(1+\bar{r}^{2n})^2}$.

Snow and McClelland (1990) conducted a field survey in the New Mexico desert in 1986/1987 and found results largely consistent with previous studies. Dust devils were observed to be drawn towards flatter, cleared areas of ground. They assumed that this behaviour was the result of those areas warming faster, producing a series of thermals which drew in unstable air from around the region. However, the location of peak dust devil activity shifted from 1986 to 1987, which could have been due to the higher precipitation that occurred in 1986, potentially changing the surface thermal properties.

Vatistas (1991) proposed the tangential velocity profiles for vortices with continuous distributions of the flow quantities. The normalized tangential-velocity function $\bar{v}$, of the Vatistas model is given by $\bar{v} = \frac{\bar{r}}{(1+\bar{r}^{2n})^{1/n}}$, where $\bar{r} = r/r_c$, r_c is the core radius, and v is the tangential velocity. This approaches Rankine profile as $n \rightarrow \infty$. These equations have singularity on the vortex centre for $n < 1$. Therefore distribution for $n < 1$ should not be permitted. The values of all velocity components are well behaved for any finite value of n. Vatistas (1998) model is a generalization of few well-known vortex tangential-velocity profiles in aerodynamics.

Kanak (2005) opined that the azimuthal velocity profile of Burgers- Rott model matches the high-resolution large eddy simulation results for dust-devil vortices better than the Rankine combined model does.

Balme and Greeley (2006) indicated that the vertical velocity in dust devil is generally about a quarter of the value of the maximum rotational velocity. Using video of a dust devil observed from a close distance in the desert, Ito and Niino (2014) applied particle image velocimetry confirming the indication. Terrestrial dust devils have a warm core with typical temperature perturbations from 4 to 8 K above ambient background values (Sinclair, 1973).

Dust devils can reach several kilometers of height on Mars with convective boundary layer depth approximately three times larger than that on the Earth (Fenton and Lorenz, 2015). According to an existing hypothesis (Willis and Deardorff, 1979; Hess and Spillane, 1990; Kanak et al., 2000) the tallest dust-devil vortices are created in narrow updrafts at the intersections of convective cells within the convective boundary layer.

2.1.2 Tornado

Lewellen (1993) propounded a detailed tornado vortex theory. Davies-Jones (1995) also discussed tornadoes extensively. Larcheveque and Chaskalovic (1994) discussed tornado genesis and found that the basic flow is generated simultaneously by a strong vertical gradient of temperature and by a storm in the troposphere, which is a non-rotating updraft.

Lewellen et al. (2000) introduced the local corner flow swirl ratio S_c, the ratio of a characteristic swirl velocity to a characteristic flow-through velocity in the surface/corner/core portion of the flow in tornado-like vortices. The classic swirl ratio S tends to have smaller value than S_c for similar corner flow behavior, but there is no one-to-one correspondence between the two quantities in general. They measured different properties of the flow, and S_c was affected by other factors (for example, by diverse surface roughness; cf. Davies-Jones ,2015) besides the "outer" swirl ratio S (Lewellen et al., 2000).

Renno et al. (1998) stated, "The classical Rankine vortex model assumes potential vortex flow at the vortex periphery". But field observations showed examples where the tangential wind falls with radius much more slowly that it would follow from the hyperbolic law characteristic for the Rankine vortex. Many observations, including those in tornadoes (Wurman and Gill, 2000) and dust devils (Bluestein et al., 2004), indicate that the tangential wind v grows linearly with radius r up to the radius r_m at which the wind is maximum v_m, in accord with Rankine model. Tratt et al. (2003) showed a well-documented encounter with a devil in Arizona, and found that the tangential wind speed fall-off outside the wall is described as $r^{-1/2}$ rather than r^{-1}, especially close to the ground (e.g. Balme and Greeley, 2006), but it secures the finiteness of the total pressure load-if one takes the analytic description of surface pressure drop and integrates to infinite radius-and of the power demands (integrating dissipations by surface drag) to sustain the vortex.

The flow is very much asymmetric with formation of multiple vortices at large S. In general, this variety of vortex physical appearances is conformed with the Doppler-radar observations of tornadoes and dust devils (e.g., Bluestein, 2005). However, it is hard to relate the swirl ratio S defined for the vortex chambers and their numerical analogs to the swirl ratio in nature, which is most relevant vortices, in particular, tornadoes and dust devils (cf. Bluestein, 2013).

Lorenz (2014) noted that the entire velocity profile in natural dust devils is well-described by the Vatistas et al. (1991) model.

Rotunno (2013) and Davies-Jones (2015) reviewed the fluid dynamics of tornadoes. Those reviews together with those by Lewellen (1976, 1993) describe dust devils to some extent. Tornadoes and dust devils are distinct in several respects and also in terms of altitudinal position of their main buoyant forcing features (Zhao et al., 2004). The buoyant forcing in tornadoes is on account of the release of energy by the water vapours condensing at a great height above the ground; while this forcing is independent of moist processes in dust devils and is located adjacent to the surface. The convergence of radial air flows in dust devils is confined to a surface layer of thickness of a few meters (e.g. Hess and Spillane, 1990). Raasch and Franke (2011) observed that the occurrence of core pressure drop of dust devils and vorticity over their lifetimes shows growth, steady state and waning respectively for about 30%, 50% and 20% the dust devil lifetime.

Researchers (Tanamachi et al., 2006; Yih, 2007); Makarieva and Gorshkov, 2009*a*, 2009*b*, 2011; Bestray et al., 2011, 2012; Makarieva et al., 2011; Arsen'yev, 2011; Rotunno, 2013; Davies-Jones, 1995, 2015 and Ben-Amots, 2016 etc.) have discussed some more aspects particularly of tornado like vortex, used different methods or reviewed the literature extensively in recent reports.

2.1.3 Boundary layer dynamics of mature hurricane

The boundary layer of a mature hurricane is an important feature. It administers the radial distribution of moisture, vertical motion and absolute angular momentum in the eyewall clouds. Attempts were made to isolate the dynamics of the boundary layer of a mature hurricane by Rosenthal (1962), Smith (1968), Carrier (1971) and Eliassen (1971). Those investigations highlighted a feature called secondary circulation and that the circulation is associated with the imbalance of forces in the layer brought about by surface friction (e.g. Greenspan, 1968). Based on scale analysis of the boundary-layer equations, Smith (1968) opined that the vertical gradient of perturbation pressure may be ignored and thus the radial pressure gradient in the boundary layer equals that in the flow above the layer. Surface friction, however, reduces the tangential wind speed and hence the centrifugal and Coriolis forces in the layer, which leaves a net inward residual force for a cyclonic vortex. This net force induces a secondary circulation in the vortex. In a hurricane, the inflow wind collects moisture by evaporation at the surface of the sea and the moist air rises to form clouds in the eyewall.

The fact is that the induced flow in the boundary layer is allied with gradient wind imbalance; but still some considered strict gradient wind balance including a sink of absolute angular momentum at the surface (Ogura, 1964; Ooyama, 1969; Schubert and Hack, 1983; Emanuel, 1986, 1989, 1995, 1997, 2004; Frisius, 2005, 2006; Wirth and Dunkerton, 2006).

Smith et al., 2008 showed that the assumption of balance is not proper in Emanuel's steady-state hurricane model (Emanuel, 1986) and in his theory for potential intensity (Emanuel, 1995; Bister and Emanuel, 1998).

The models of Ooyama (1969) and Sundqvist (1970) were balanced as the tangential or primary circulation was considered in axisymmetric gradient wind balance, and yielded almost realistic simulations of hurricanes. Of course, the axisymmetric gradient balance assumption seems to be more accurate than the free troposphere except outflow layer in the upper level. This is endorsed by a scale analysis for a rapidly spinning vortex whose radial flow component is much less than the tangential flow (Willoughby, 1979).

The early models assumed the boundary layer also in the gradient wind balance. Smith (1968) and Carrier (1971) showed that it is not likely to be an accurate approximation in the inner core. This was endorsed by numerical simulations of hurricanes (Persing and Montgomery, 2003; Kepert and Wang, 2001).

In the cyclonic flow of a hurricane, the radial pressure gradient per unit mass is balanced by the sum of the centripetal force and the Coriolis forces. Cyclostrophic balance is a factor above the hurricane boundary layer where surface friction is negligible and inflow radial wind component is zero (Willoughby, 1979).

Most of the researchers study pressure distribution by using this relation for different profiles of the azimuthal velocity. But the most popular model for the azimuthal velocity is the Rankine combined vortex model (1882), where the azimuthal velocity depends only on the radial coordinate. This has been widely used to provide the best tool for studying and explaining the observed tangential wind and deduced pressure distribution in dust devils (Sinclair, 1973; Cantor et al., 2006), waterspouts (Leverson and Sinclair, 1977) and tornadoes (Hoecker, 1961;

Wakimoto and Wilson, 1989; Winn et al., 1999; Lee and Samaras, 2004; Wurman and Samaras, 2004; Lee and Wurman, 2005; Tanamachi et al., 2013).

Nevertheless many fundamental problems in the science of hurricane continue unanswered. Those include abrupt variation in the wind direction in the lower as well as upper part of the hurricane, radial increase in wind angular momentum in the boundary layer temperature, etc.

Most of the existing theoretical models, idealized some or the other way, are based on the balanced vortex model in association with Sawyer-Eliassen transverse circulation equation (Eliassen, 1951; Charney and Eliassen, 1964; Ooyama, 1969; Shapiro and Willoughby, 1982; Emanuel, 1986; Schubert and Hack, 1982; Emunuel, 1995; Nolan et al., 2007; Wirth and Dunkerton, 2009). Models based on balanced vortex combine the hydrostatic and gradient wind balances with radial momentum and thermodynamic equations. The balanced vortex model conserves the absolute angular momentum and predicts the azimuthal wind to be maximum at the lower levels and decay upward (Emanuel, 1986; Stern and Zhang, 2016).

2.2 Laboratory models

Laboratory experiments in vortex chambers were carried out in the $1970 - 80s$ at Purdue University and at the University of Oklahoma/National Sever Storms Laboratory. Vortex chamber experiments generated the earliest quantitative measurements of vortex characteristics without using radar and computer technology. The inflow direction at the edge of the convection zone, under those conditions, was almost radial deviating merely by $1 - 2$ degree.

This indicated that, at the edge of the updraft, the tangential component was quite small and suffered only a little from frictional losses for contact with the boundaries of the inflow zone. Consequently, no secondary flows took place beyond the core, such as the downward motion reported by Kuo (1969) and Ying and Chang (1970).

Several experimental models for tornado generation exist in the literature. Davies-Jones (1976), who put forth a thorough review of those laboratory models, inferred that the Ward-type Tornado Vortex Chamber (Ward, 1972) demonstrated geometric as well as dynamic similarity to vortices occurring in nature.

Ward (1972), Davies-Jones (1973) indicated through experiments that the core size is primarily a function of swirl ratio. Snow et al. (1980), Pauley et al. (1982), Church and Snow (1985) and Pauley (1989) obtained surface pressure profiles as a function of swirl ratio for the Ward-type tornado vortex chamber. Snow and Lund (1988) amended the Ward-type tornado vortex chamber by substituting the rotating screen by adjustable vanes and adding non-intrusive velocity measurement instrumentation such as Laser Doppler Velocimeter. Lund and Snow (1993) presented primary Laser Doppler Velocimeter results.

Haan (2007) presented a Iowa Laser Doppler Velocimeter with translation ability. The Laser Doppler Velocimeter contained two concentric cylinders forming a duct with a fan in the center. The air circulated between base surface and the duct. The swirl component was added through vanes in the roof of the inner cylinder. The Iowa-Laser Doppler Velocimeter produced one-celled turbulent vortices but didn't produce the multiple vortex flows.

Hashemi-Tari et al. (2007) presented a Laser Doppler Velocimeter similar to the Iowa-Laser Doppler Velocimeter without translation feature and presented an exhaustive set of particle image velocimetry measurements for swirl ratios less than 1. Those were the initial complete set of particle image velocimetry measurements in a tornado vortex chamber characterizing both the mean and the turbulent flow fields.

Those were limited experiments in terms of the size of the apparatus and by the range of possible swirl ratios (Hashemi-Tari et al., 2010).

2.3 Numerical models

Numerical simulations of the squalls on the storm scale, with nested grids used to simulate sub-storms vortices, were also initially carried out when experimental modelling began. It is appropriate to categorize numerical models into two broad classes, viz. (1) thunder storm scales and (2) tornado scales (Nolan and Farrell, 1999).

Meteorological models, created while simulating, regenerated the parent supercell squall with adequate resolution to study tornado genesis and vortex motion. Those are thunderstorm scale simulations (Klemp and Wilhelmson, 1978; Wicker and Wilhelmson, 1995; Grasso and Cotton, 1995). The tornado and the Earth's surface interaction is studied in tornado scale simulation. Those are engineering models and provide the flow pattern and the wind fields, close to the ground, of vortices similar to tornadoes.

The earliest numerical model to simulate vortices resembling tornado in a tornado vortex chamber was constructed by Harlow and Stein (1974). Using a free-slip lower boundary condition, a two-dimensional axisymmetric model was able to generate one-celled and two-celled vortices.

Rotunno (1977) numerically simulated Ward' (1972) laboratory experiment by using constant density, viscosity of an axisymmetric flow and concluded that the numerical and laboratory surface pressures are in agreement and the core radius is not affected by high Reynolds number. He (1979) furthered the study numerically on tornado vortex dynamics of the laboratory experiment of Ward (1972). He establish that the swirl ratio is the single most significant parameter governing tornado vortex structure. For swirl ratio less than 1, a concentrated vortex is created in the upper chamber but not in the corner. However, at moderate swirl ratio, the observation was vortex breakdown, larger amplitude inertial waves and intense swirling motion in the corner. Later, Rotunno (1984) introduced random noise to a three dimensional model of Ward-type tornado vortex chamber and simulated multiple vortices. The new observation was formation of secondary vortices with tangential velocity higher than the mean flow by $20 - 30$ %.

Wilson and Rotunno (1986) simulated a low swirl ratio ($S = 0.28$) laminar vortex and conformed the experimental results of Baker (1981). They propounded the theory of four principal regions. The vortex was observed to be nearly inviscid rotating with a thin viscous region in the core along the central axis and a small viscous sub-layer whose depth reduced towards the central axis.

Fiedler (1994, 1995, 1997, and 1998) used an axisymmetric model. Vortices formed by introducing buoyancy in a rotating cylindrical fluid mass within a domain with rigid boundaries. The results revealed that the vortex touch-down produces wind speeds exceeding the thermodynamic speed limit by 5 folds and produced multiple vortices at higher swirl ratios with introduction of random fluctuations. Simulations disclosed for compressible fluid that, even at subsonic flow, compressibility slightly decreases the wind speeds. Xia et al. (2003) included compressibility and observed similarity with Fiedler (1997). The compressibility effects are not likely to alter the fundamental dynamics of vortex close to the the surface.

Nolan and Farrell (1999) observed that Vortex Reynolds number (ratio of far field circulation to eddy viscosity) is more effective than the conventional swirl ratio when the structure of a vortex is to be predicted. Large-eddy simulations of tornadoes began in the late $1990s$ using grid spacing $1 - 3\ m$ in some places, to represent the turbulence of tornadoes in a more precise way. Those yielded remarkable measurements. Lewellen and Lewellen (1997) and Lewellen et al. (2000) showed the effect of turbulence to generate high wind speeds close to the ground and creation of multiple vortices when the swirl ratio is high. The time averaged maximum velocity was observed to take place in close proximity to the surface within $50\ m$ of the ground.

Hangan and Kim (2008) considered Reynolds Stress Model for Ward-type laboratory scale tornado in order to match the Doppler radar data for real scale tornado. They approximately found a relation in the swirl ratio and the Fujita scale. They concluded that F4 Fujita scale tornado corresponds approximately to a swirl ratio $S = 2.0$.

Kuai (2008) simulated the Iowa-type tornado vortex chamber using Re-Normalisation Group (RNG) $k - \epsilon$ model and compared the results with real scale tornado. Still consistent attempts lacked to simulate vortices over a very large values of swirl ratios. Hangan and Kim (2008) indicated that there is an exploratory relation between the fluid mechanics parameter swirl ratio ($S = 2.0$) and the forensic Fujita scale parameter ($F4$).

The list is quite large, hence it is not possible to disuss all of them.

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