Chapter 1

An introduction to atmospheric vortices and vortex dynamics

1.1 Atmospheric vortices

The non-living invisible air that carries oxygen for the life has probably more mobility than any of the living beings. Its existence seems to be doubtful when it is calm; the next moment it may turn into a pleasant cold breeze shattering all our melancholy or maybe it forms a horrible heat wave blowing to burn the entire habitat; it may mesmerize by its sudden birth as a twinkling twister revolving as a column and tossing dust and debris but may elope once in a while; and drags clouds, carries snow or hails or sprinkles rains the way it likes. It gets violent to form a storm, can uproot trees and shake even a sea. It has an alliance with heat and affinity with moisture, and is able to incarnate itself into a huge hurricane or a terrible tornado.

Vortices are rotating columns of fluid masses. Various forms of vortices exist in nature and have various nomenclature based on the size and strength, the duration of survival and the continent of occurrence. The nomenclature also depends on the language of the region. Amongst them tornado typhoons, cyclones, hurricanes, whirlwinds etc. are popular names. The glossary of meteorology has a common term cyclone for typhoons, hurricanes and tornadoes. It is physically winds circulating around a region of low atmospheric pressure, which is placed centrally and rotates counterclockwise in the northern hemisphere but clockwise in the southern hemisphere. A cyclone originating over the tropical oceans is called tropical cyclone; that in Hongkong, Japan, Philipiens etc. is called typhoon and in the Atlantic and Pacific oceans is known as hurricane

1.1.1 Tornado

Tornado is a fast whirling columnar vortex wind system hanging as a pendent from a cumuliform cloud in contact with the surface of the Earth. It is witnessed as a funnel merging into clouds with circulating dust at the foot. This low pressure visual core vortex rotating with terrific energy also travels along the ground surface. It is most violent and destructive atmospheric vortex on the surface of the Earth. It is observed all over the world such as Japan, Bangladesh, Britain, Australia, but in large number in Tornado Alley in the USA. It is generally believed that that the circulation of a tornado vortex is maintained by the rotating mother cloud and the sense of the vortex rotation coincides with that of the associated mother cloud (Ying and Chang, 1970). Volcanic eruptions are also sometimes reasons behind the birth of tornadoes and waterspouts.

Sometimes it so happens that a tornado fails to touch the ground. However, it is positioned beneath but connected to a cumuliform buoyant convective cloud. When a rotating columnar wind is observed as a condensation funnel which does not touch the ground then it is called a funnel cloud. It is sometimes not known if it has any connection with a mighty circulation at the ground. If the wind is not humid or the pressure is not amply low to form a condensation funnel or to connect to the ground, then it may look like a revolving column of dust and debris; or it may not be visible at all if there is no lofted dust or debris. Even when this is wrapped within rains, then it may disappear from vision.

Most of the tornadoes rotate cyclonically in super-cells, but rarely some anti-cyclonic tornadoes are also observed. Some tornadoes consist of smaller-scale vortices within them which rotate about the tornado-axis. Ted Fujita named such contained vortices as suction, satellite or secondary vortices. Tornadoes with secondary vortices are called multiple-vortices tornadoes. The secondary vortices cause the maximum damage. Ted Fujita first pointed out that in open areas they produce cycloidal damage marks which reveal smaller scale vortices spinning around a common axis.

Tornadoes may last for seconds to an hour or so. Most of them last for about 10 min. The diameter of a tornado lies between 10 m to 2.0 km; but usually it is observed of the order of 200 m. It is distinct and narrower than rotating thunderstorms called meso-cyclones, whose diameter lies between 2.0 $km - 5.0$ km.

Wind speed in tornadoes ranges from 20 $m\ s^{-1}$ to 140 $m\ s^{-1}$. Weaker ones are just dust devils. A tornado may be created throughout the year. Tornadoes are fundamentally an atmospheric phenomena created above ground while hurricanes and typhoons are absolutely an oceanic phenomena. Hurricanes and typhoons finally wane on the land as the source of moisture seizes to exist. The mighty rotation of a tornado causes destruction and damages.

The pressure exerted by the tornadic wind, i.e., the transient responses to wind gusts, causes damage. The pressure varies directly to the square of the wind speed. This produces aerodynamic effect as the airflow interacts with structures. The nature of tornado-caused-damage is investigated under controlled conditions. The debris is lofted by wind currents moving upward or by aerodynamic effects. This is sensed by polarimetric Doppler radar. For detection by radar at long range the debris ought to be lofted above the radar horizon.

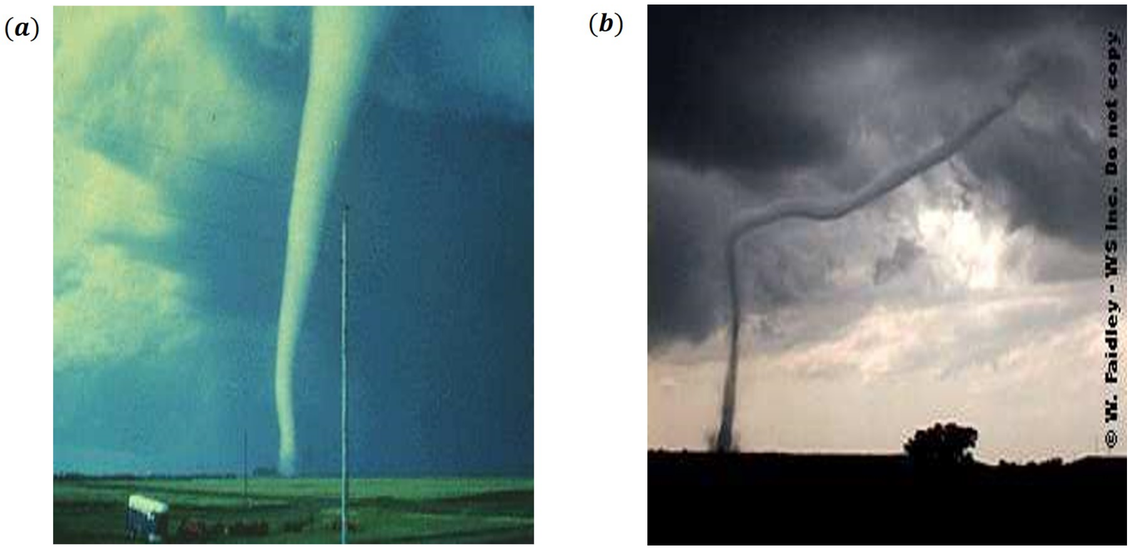

Figure 1.1: (a) A tornado with funnel shape on the plains of North Dakota, USA (http://ww2010.atmos.uiuc.edu/(Gh)/guides/mtr/svr/torn/home.rxml) (Courtesy: National Severe Storms Laboratory (NSSL), USA). (b) A tornado with and unusual shape making right angle bend in the middle. (http://www.weatherstock.com/tornadocat3.html) (Courtesy: The Weather stock, USA).

Theodore Fujita and Alan Pearson in 1971 first found the relation between speeds of tornado rotation with the magnitude of damage and created a scale called Fujita-Pearson scale. This scale is widely used as a standard system to classify the intensities of tornadoes. Fujita-Pearson rates tornadoes by several classes. It ranges from $F0$ $(18 - 32\ m\ s^{-1}; 40 - 72\ mph)$ to $F5$ $(117 - 142\ m\ s^{-1}; 26 - 318\ mph)$, Tornadoes of scale $F0 - F1$ are week, $F2 - F3$ are strong, and above $F4$ are violent.

But the relationship between wind speed and the nature of damage caused is not estimated. Enhanced Fujita (EF) scale replaced Fujita scale in 2007. EF-scale is calibrated to some extent. However, when tornadoes attack meagerly polluted regions with little vegetation, it becomes hard to evaluate wind speeds based solely on the damage or disruption inflicted.

1.1.2 Waterspouts

Tornadoes observed over the surface of water are called "waterspouts". Waterspouts are sometimes responsible for land erosion and while tornadoes sometimes pass over water surfaces. Thus, fundamentally tornadoes and waterspouts are not dynamically dissimilar. Tornadoes are observed even over terrain and mountains. At highly elevated places over terrains of mountainous they are drier than those at lower locations or which are near water sources.

1.1.3 Dust devil

Some atmospheric phenomena similar to tornadoes and waterspouts are dust devils, steam devils, and fire whirls. They are all driven by surface heating, which leads to superadiabatic lapse rates near the ground. Existence of fire whirls is entirely due to intense surface heat sources. While the cause of each of these vortices is somewhat different, they all involve intense vortices making contact with the ground. Steam dust occur when very cold air is advected over a relatively warm water surface.

Dust devil is a tiny whirlwind in comparison to other atmospheric vortices like tornadoes, hurricanes and typhoons etc. Dust devils are observed the world over and occasionally throughout the year. These are formed due to temperature gradients created locally owing to excessive heating in the locality.

Figure 1.2: Naturally occurring atmospheric column vortex or dust devil formed in Arizona. (Source: https://en.wikipedia.org/wiki/Dust devil).

This is formed suddenly and is observed to rise into the sky but, unlike tornadoes, it is short-lived and dies out soon. In summer, the sunlight is quite intense. Whenever a particular area of the Earth's surface is exposed to excessive heating, the water contents are evaporated and move upward; the dry and hot air becomes rarer in density and locally creates a depleted airy zone, forming a horizontal gradient of temperature and air pressure. In order to fill the depleted airy zone, colder winds close to the Earth's surface blow from all the directions and sometimes culminate into the formation of a rotating columnar mass of air mixed with dust, leaves etc., called dust devil. Dust devils are driven by exposure to sunlight at the ground, but also depend on frictionally induced vertical shear near the surface, or cellular convection of clear air. They may be generated baro-clinically by horizontal vorticity. Pictures of naturally occurring dust devils are shown in Fig. 1.2. Renno et al. (1998) put forward a thermo-dynamical theory for dust devils explaining how vertical and horizontal temperature gradients are created. Many more detailed descriptions are given by Balme and Greeley (2006).

Dust devils typically are formed in an unsaturated atmosphere. In the absence of dust or debris, it may not be visible. It often happens that while whirling about the vertical axis, it translates in the surroundings and whenever it is placed in an area with no dust or dirt, it disappears; but soon after some time it is again visible. This off and on visibility continues until it dies out.

Although, since it is a weak whirlwind, it is generally harmless and entertaining but sometimes it may get mightier and topple huts, break tree branches and even loft anything including animals and humans.

1.1.4 Hurricane

Hurricanes are quasi-symmetric and extraordinary fascinating weather phenomena that have huge societal impacts. These are low pressure systems that originate over tropical oceans and are tropical cyclones with winds exceeding 33 ms^{-1} in the Atlantic and the Eastern North Pacific, typhoon in the western North Pacific which is an identical phenomenon as hurricane and severe tropical cyclones in other ocean basins. A hurricane is a natural occurrence beheld as a solitary vertical aero-vortex spinning in the cyclonic direction around the axis of rotation with additional radial and vertical winds close to the bottom.

The central part a hurricane is a vertical column of low pressure calm zone of about 20 km called the eye. It is surrounded by a thicker layer of $30 - 50$ km called the eye-wall experiences intense updraft. Above the hurricane boundary layer of thickness, the radius of external eye-wall changes with height. The air within the eye-wall along with radial inflow performs intense rotation and the maximum is achieved at the inner eye-wall. The vertical velocity is considered weak in comparison to the radial and azimuthal wind velocities and is mostly contained within the eye-wall region whereas outside the eye-wall vertical velocity is negligibly small. This is stationed on the oceanic surface. The eye-wall is encased in a very strongly rotating external wind region which produces water vapour within the boundary layer required for the updraft. This may be $400-600$ km thick with strong azimuthal wind but almost no updraft. Heavy showers are later witnessed in the eye-wall region after the water vapour condenses. The blowing wind along the radial direction is inward at the bottom and outward at the top of the hurricane. The entire vortex could be vertically layered into the bottom hurricane boundary layer, and upper adiabatic layer with the total hurricane height up to $20 - 30$ km. The coastal areas experience heavy rains and usually culminates into heavy loss of lives and property. Fig. 1.3 gives a top view in part (a) and the vertical structure in part (b).

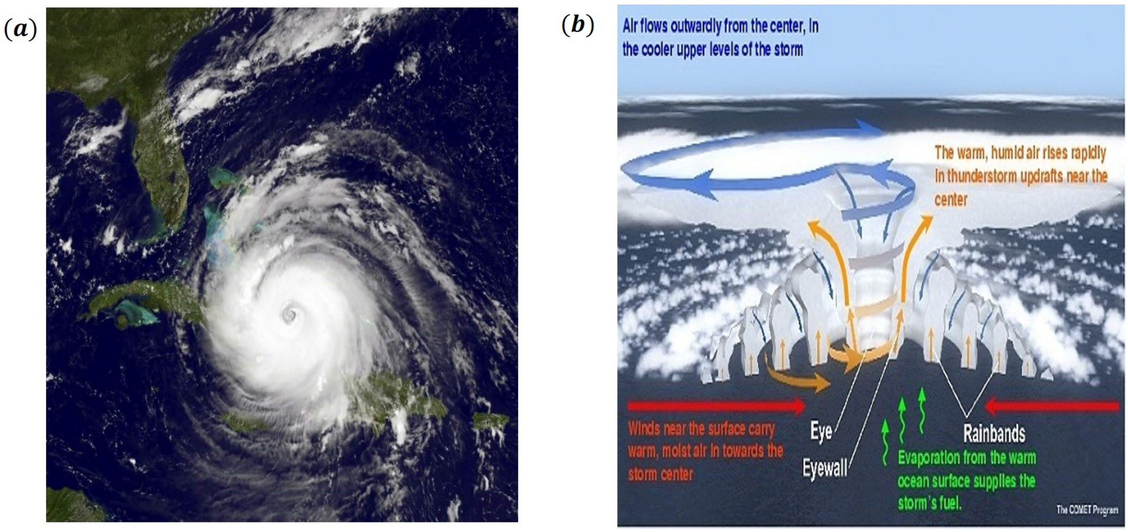

Figure 1.3: (*a*) Top view of Hurricane Irma. (Source: NOAA via the New York Times), (http://www.tampabay.com/hurricane-guide/Watch-out-Florida-Forecasters-predict-active-2018-hurricane season-166989625). (*b*) Vertical structure of hurricane. (Source: https://www.monolitonimbus.com.br/wp-content/uploads/2014/03/hurricane-structure-300x172.jpg).

Hurricane is the most attractive atmospheric phenomenon. It is a complex phenomenon. New insight have been obtained from experiments using numerical simulations based on ideal assumptions, supported by aircraft exploration data. These insights augmented understanding hurricane intensification and structure change. Several aspects of this weather system has been described by Emanuel (2005) in his book entitled *Divine wind: The history and science of hurricanes.*

The flow in mature tropical cyclone may be classified into two different circulations. On the primary circulation, which is a horizontal quasi-symmetric circulation, the secondary circulation, which is a thermally-direct vertical circulation, is superposed. Ooyama (1982) used the terms for the first time. Those two circulations together culminate in a spiraling flow, the characteristic of tropical cyclones. Surface friction has a strong influence on the dynamics of tropical cyclones.

Hurricane is characterized by a warm core. Though the combination of axisymmetric and waning interior friction are approximation for hurricanes, these assumption, nevertheless, are a first step for a clear understanding of the hurricane dynamics (Emanuel, 1986; Wirth, 1998; Hsu and Plumb, 2000).

1.1.5 Supercell

A supercell is a terrible convective squall that mainly comprises a single, quasi-steady spinning updraft continuing for a longer duration (much more than 10–20 min) than it takes an air parcel to rise from the base to the peak of the updraft.

The spinning updrafts are identified by cyclonic vorticity. The supercell structure is internally organized and hence safeguards continuous propagation. It can survive for many hours and form strong vertical wind shear. Propagation of a supercell is unidirectional with a speed different from the mean wind in the environment. Such storms sometimes are born through a split into a pair of supercells with a cyclonic and an anti-cyclonic supercell. Severe weather is accompanied with supercells, which produce strong winds, large hailstorms and mighty long-living tornadoes.

1.1.6 Mesocyclone

A mesocyclone is a smaller version of cyclonically rotating vortex of diameter 2–10 km in a convective storm. The vorticity of a mesocyclone is of the order of 10^{-2} s^{-1} or more. It refers to cyclones within convective storms. Mesocyclones are found often together with updrafts in supercells. Tornadoes occasionally grow in mesocyclones. Stubborn mesocyclones having significant vertical range are spotted by

Doppler radar. Once a mesocyclone signature is noticed, warnings may be issued of the occurrence of a tornado.

1.1.7 Thunderstorm

Thunderstorm is a local phenomenon usually of short duration, lasting for an hour or less, conventionally produced by a cumulonimbus cloud together with lightning and thunder, usually with strong wind squalls, heavy rains and hails off and on.

A thunderstorm is a result of changing atmosphere due to a toppling of layers of air for attaining a stable stratified density. A strong convective updraft subsequently followed by a strong downdraft in a column of precipitation characterizes it. Thunderstorms are often formed at an altitude of $40000 - 50000\ ft$ in mid-latitudes and to even higher altitudes in the tropical regions. A thunderstorm is identified by striking electrical activities and is characterized also by lightning and very complex phenomena of electric charge separation and distribution in the dominion of thunderstorm.

1.2 Vortex dynamics

Some concepts and terms associated with vortex dynamics and frequently used in various chapters of the thesis are briefly explained in the subsequent paragraphs.

1.2.1 General analysis of fluid motion

Let us considering $P(x, y, z)$ and $P^{'}(x + \delta x, y + \delta y, z + \delta z)$ as points representing two neighboring fluid particles moving along with the fluid at time t, with $\boldsymbol{OP} = \boldsymbol{r}$,

$\boldsymbol{OP}^{'} = \boldsymbol{r} + \delta\boldsymbol{r}$. If $\boldsymbol{q} = [u, v, w]$, $\boldsymbol{q} + \delta\boldsymbol{q} = [u + \delta u, v + \delta v, w + \delta w]$ be the velocities of P, $P^{'}$ at instant t, then

$$\left.\begin{aligned} \delta u &= \frac{\partial u}{\partial x}\delta x + \frac{\partial u}{\partial y}\delta y + \frac{\partial u}{\partial z}\delta z \\ \delta v &= \frac{\partial v}{\partial x}\delta x + \frac{\partial v}{\partial y}\delta y + \frac{\partial v}{\partial z}\delta z \\ \delta w &= \frac{\partial w}{\partial x}\delta x + \frac{\partial w}{\partial y}\delta y + \frac{\partial w}{\partial z}\delta z, \end{aligned}\right\} \tag{1.1}$$

which in the matrix form may be given by

$$\begin{bmatrix} \delta u \\ \delta v \\ \delta w \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} \tag{1.2}$$

Symbolizing them as given below

$$a = \frac{\partial u}{\partial x},\ b = \frac{\partial v}{\partial y}, c = \frac{\partial w}{\partial z}$$

$$\left.\begin{aligned} a = \frac{\partial u}{\partial x},\ b = \frac{\partial v}{\partial y},\ c = \frac{\partial w}{\partial z}; \\ f = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right),\ g = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right),\ h = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right); \\ \xi = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right),\ \eta = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right),\ \zeta = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \end{aligned}\right\}$$

the derivative matrix may be split into a symmetric and a skew matrix as

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix} = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} + \begin{bmatrix} 0 & -\zeta & \eta \\ \zeta & 0 & -\xi \\ -\eta & \xi & 0 \end{bmatrix}.$$

i.e.,

$$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} = \begin{bmatrix} a\delta x + h\delta y + g\delta z \\ h\delta x + b\delta y + f\delta z \\ g\delta x + f\delta y + \delta z \end{bmatrix}, \tag{1.3}$$

We now consider the quadric surface

$$a(\delta x)^2 + b(\delta y)^2 + c(\delta z)^2 + 2f\delta y\delta z + 2g\delta z\delta x + 2h\delta x\delta y = const., \tag{1.4}$$

in which the coefficients a, b,..., f, ... are constants with respect to the coordinates $(\delta x, \delta y, \delta z)$, so that Eq. (1.4) is quadric with centre at P. Then partial differentiation of the left side of Eq. (1.4) with respect to δx, δy, δz indicate that the column vector on the right side of Eq. (1.3) is normal to the quadric surface (1.4). Writing $\delta \boldsymbol{q}_1 = u_1\boldsymbol{i} + v_1\boldsymbol{j} + w_1\boldsymbol{k}$, where u_1, v_1 and w_1 are the components of Eq. (1.3), this shows that $\delta\boldsymbol{q}_1$ is everywhere normal to the quadric surface (1.4) having P as centre. Such a velocity $\delta\boldsymbol{q}_1$ is ascribable to a motion which is called a pure strain. The quadric is called the rate of strain quadric. By a suitable rotation of coordinate axes, the equation of the quadric surface (1.4) may be reduced to the form

$$a^{'}(\delta x^{'})^2 + b^{'}(\delta y^{'})^2 + c^{'}(\delta z^{'})^2 = const.,$$

in which the coordinate axes coincide with the principle axes of the quadric. The velocity components due to pure strain parallel to the axes of the quadric are respectively $[a^{'}\delta x^{'}, b^{'}\delta y^{'}, c^{'}\delta z^{'}]$. Lines drawn parallel to the principle axes are undergoing elongations at uniform rates.

Next, we observe that

$$\begin{bmatrix} 0 & -\zeta & \eta \\ \zeta & 0 & -\xi \\ -\eta & \xi & 0 \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} = \begin{bmatrix} \eta\delta z - \zeta\delta y \\ \zeta\delta x - \xi\delta z \\ \xi\delta y - \eta\delta z \end{bmatrix}, \tag{1.5}$$

Writing $\delta\boldsymbol{q}_2 = u_2\boldsymbol{i} + v_2\boldsymbol{j} + w_1\boldsymbol{k}$, where u_2, v_2 and w_2 are the components of the matrix on the R.H.S. of Eq. (1.5), and also $\boldsymbol{\omega} = \xi\boldsymbol{i} + \eta\boldsymbol{j} + \zeta\boldsymbol{k}$, we see that

$$\delta\boldsymbol{q}_2 = \boldsymbol{\omega} \times \delta\boldsymbol{r}. \tag{1.6}$$

This shows $\delta\boldsymbol{q}_2$ is ascribable to a rigid-body rotation with angular velocity $\boldsymbol{\omega}$. Further substitution for ξ, η, ζ shows that

$$\boldsymbol{\omega} = \frac{1}{2}curl\boldsymbol{q} = \frac{1}{2}\boldsymbol{\zeta}. \tag{1.7}$$

where $\boldsymbol{\zeta}$ is the vorticity vector. Thus the vortex line through the fluid element is its instantaneous axis of rotation. This analysis justifies the terminology 'irrotational' for those flows for which $curl\boldsymbol{q} = 0$, leading as we have seen to $\boldsymbol{q} = -\nabla\phi$.

We can conclude the following results from the foregoing analysis of the general motion of a fluid particle in the following way (Chorlton, 1985):

- There is a translation velocity $\boldsymbol{q}$ of any point P of it.
- Relative to P there is a velocity component $\delta\boldsymbol{q}_1$ of P' which is ascribable to pure strain in the sense that for all other points P' of the particle the component $\delta\boldsymbol{q}_1$ is normal to a certain quadric surface centered on P. This type of motion requires that the fluid is deformable.

- Relative to P there is a velocity component $\delta \boldsymbol{q}_2$ of $P^{'}$ which is ascribable to a rigid-body rotation of the element. This means that if the fluid in the particle were suddenly frozen, then the solid would be instantaneously rotating with vector angular velocity $\frac{1}{2}\boldsymbol{\zeta}$.

- Unlike potential flow for which $curl\boldsymbol{q} = 0$, a rotational motion of vortices is characterized by $curl\boldsymbol{q} \neq 0$ and hence the vorticity vector is defined as $\boldsymbol{\zeta} = curl\boldsymbol{q}$. In order to conceive the physical meaning of vorticity vector, general motion of flow is analyzed in the succeeding section.

1.2.2 Vorticity vector

Vorticity of the fluid particle is simply defined as twice the angular velocity of the fluid particle. It is related to the velocity field of the fluid flow by a curl operator i.e., if we denote the vorticity field by $\boldsymbol{\zeta}$, the velocity field by $\boldsymbol{q}$ then vorticity is defined by $\boldsymbol{\zeta} = curl\boldsymbol{q}$. In general, vorticity is the preferable fundamental quantity for the analysis of incompressible fluid dynamics in terms of vortex flow. Vorticity represents the local rotation rate of fluid particles (Chorlton, 1985).

1.2.3 Circulation

The circulation around any closed contour C is defined as $\oint_C \boldsymbol{q} \cdot d\boldsymbol{s}$. By Stokes' Theorem, $\oint_C \boldsymbol{q} \cdot d\boldsymbol{s} = \int\limits_S \boldsymbol{n} \cdot \zeta \, dS$, where S is an arbitrary surface bounded by C. Thus, the circulation may also be interpreted as the flux of vorticity through a cross section of the tube (Wu et al., 2006).

1.2.4 Conservation of angular momentum

The absolute angular momentum M per unit mass is defined as

$$M = rv + f\frac{r^2}{2}, \tag{1.8}$$

where r is the radius, f is the Coriolis parameters and v is the azimuthal velocity. In the absence of frictional effects with axial symmetry, the azimuthal momentum equation physically means that the quantity M is materially conserved as air parcels moves around the axis of symmetry in a vertical plane (Smith and Montgomery, 2016).

1.2.5 Hydrostatic and gradient wind balance

In case of pure circular motion, where radial velocity $u = 0$ and the motion is indepedent of θ, the radial momentum equation reduces to

$$\frac{\partial p}{\partial r} = \rho\left(\frac{v^2}{r} + fv\right). \tag{1.9}$$

where p is the pressure, r is the radius, f is the Coriolis parameters, v is the azimuthal velocity and ρ is the density. This relation is known as gradient wind equation. This relation represent the balance among, centripetal force, Coriolis forces and force with radial pressure gradient.

Also the scale analysis of the axisymmetric vertical momentum equation upto a first order approximation shows that the, there is hydrostatic balance in the vertical direction, i.e.,

$$\frac{\partial p}{\partial z} = -\rho g. \tag{1.10}$$

where z is the vertical coordinate and g is the acceleration due to gravity. For slowly evolving flow, these equations (Eq. 1.9 and Eq. 1.10) lead to a constraint on the secondary circulation forced by latent heat release in deep convective clouds and friction (Smith and Montgomery, 2016).

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