# Abstract

The thesis consists of six chapters. Chapter 1 is the collection of definitions, notations, literature review and generalized fractional calculus which is used in subsequence chapters.

In chapter 2, we presented two approximate methods such as Quadratic and Cubic approximations for the Riemann-Liouville fractional integral and Caputo fractional derivatives. The approximations error estimates are also obtained. Numerical simulations for these approximation schemes are performed with the test examples from literature and obtained numerical results are also compared. To establish the application of the presented schemes, the problem of Abel's inversion is considered. Numerical inversion of Abel's equation is obtained using Quadratic and Cubic approximations of the Caputo derivative. Test examples from literature are considered to validate the effectiveness of the presented schemes. It is observed that the Quadratic and Cubic approximations schemes produce the convergence of orders $h^3$ and $h^4$ respectively.

In chapter 3, we studied two numerical approximations for the generalized Abel's integral equations (GAIEs). Two numerical schemes such as linear scheme and quadratic scheme are proposed to solve GAIEs numerically. The error convergence of the presented schemes is also established where it is observed that the quadratic scheme achieves the convergence order up to 3. Some examples of GAIEs from literature are considered to perform the numerical investigations and the obtained numerical results are shown in tabular form. We analyze that the presented schemes work well and provide good numerical results. It is also observed that the accuracy in the numerical solutions can be achieved with smaller value of

the step size. As the GAIEs reduces to the Abel's integral equation of the first kind in special case, therefore a similar scheme could be developed to solve such equations.

In chapter 4, we presented a comparative study three numerical schemes such as Linear, Quadratic and Quadratic-Linear scheme for the fractional integro-differential equations defined in terms of the Caputo fractional derivatives. The error estimates of the respective approximations are also established. Numerical tests of the discussed schemes show that all schemes work well, and when the number of terms approximating the solution are increased, the desired solution is achieved. The accuracy of the numerical schemes with respect to the step size $h$ is analyzed and illustrated through various tables. Finally, comparative performances of the schemes are discussed.

In chapter 5, we have discussed two numerical schemes namely *P1* and *P2* for Generalized Fractional Integro-Differential Equations (GFIDEs) using approximations of Generalized Fractional Derivative (GFD). Two approximation schemes namely linear approximation and quadratic approximation are presented for GFD which are later used in solving GFIDEs defined using GFD. For the special choice of scale and weight functions, GFD reduces to Caputo and Riemann–Liouville derivatives. Error estimate and convergence order (CO) of linear and quadratic approximations are also investigated. Further, an illustrative example is considered to validate the effectiveness of proposed numerical schemes.

In chapter 6, a numerical scheme is formulated and analysed for solving a Generalized Time-Fractional Telegraph-Type Equation (GTFTTE) defined using Generalized Time Fractional Derivative (GTFD) proposed recently. The GTFD involves the scale and weight functions and it reduces to the traditional Caputo derivative for a particular choice of weight and scale functions. Weight and scale functions play an important role in describing the behaviour of real-life physical systems and thus we study the solution behaviour of the GTFTTE by varying weight and scale functions in GTFD. Some cases of the solution profiles of the GTFTTE under these choices are investigated and obtained results are discussed. The convergence and stability of the presented numerical scheme for GTFTTE are also given. We consider three examples to perform numerical simulations. Furthermore, we describe the effects of scale function and weight functions on the numerical solution of GTFTTE.