PREFACE

Moving boundary problems involving heat or diffusion processes appear in many important areas of science, engineering and industry. Some instances are production of ice, solidification of steel, freezing and thawing foods, casting of metals, crystal growth. All these phenomena involve either a moving freezing, moving melting or moving reaction interface which is unknown. The moving boundary problem consists of determing a temperatue profile and phase change boundary in a material. The whole state of a medium is subject to vary due to heat conduction on account of the action of external and internal heat sources. The transfer of energy in different phases of the material is described by the heat equation and the movement of phase change interface, termed as a moving boundary, governed by the Stefan condition. The Stefan condition represents the energy balance at the moving boundary of the medium from one state to other.

In this thesis, the author discusses some moving boundary problems and their solutions. This thesis contains six chapters. Chapter 1 is introduction which describes moving boundary problems, the historical background of the problem. this chapter includes definition and some basic properties of shifted Chebyshev polynomials, Legendre polynomials and fundamental descriptions of the second kind Chebyshev wavelets. The basic idea of homotopy analysis method and homotopy perturbation method are also demonstrated in this chapter. Definition of Caputo fractional derivative and its properies, utilized in this thesis, are given in the last section of the chapter.

Chapter 2 describes a one-phase Stefan problem in a semi-infinite domain that involves temperature-dependent thermal coefficients and moving phase change material with a speed in the direction of the positive x-axis. The convective boundary condition at the fixed boundary is considered in the problem. An approximate approach to the problem is discussed to solve the problem with the aid of spectral tau method. The existence and uniqueness of the exact solution to the problem are also established for a particular case and the obtained approximate solution is compared with this exact solution which shows that the approximate results are sufficiently accurate. The impact of the parameters on the moving interface is also analysed.

Chapter 3 explores a phase change problem in a one dimensional semi-infinite domain including the time-dependent speed of a phase change material. In this consideration, the Dirichlet type of boundary condition is taken into account, the thermal conductivity and specific heat are assumed as linear functions of temperature. In case of $\alpha = \beta$, the exact similarity solution to the problem is established and its existence and uniqueness are also deliberated. For all α and β, we also present an approximate approach based on spectral shifted Legendre collocation method to solve the problem. The approximate results thus obtained are likened with our exact solution for different parameters and it is shown through tables. From this study, it can be seen that the approximate results are adequately accurate. The impact of different parameters appearing in the considered model on temperature profile and tracking of moving phase-front is also studied.

In chapter 4, we consider a mathematical model of a Stefan problem involving variable latent heat term which is function of position with positive exponent and a numerical solution of this model is discussed. We first convert the governing partial differential equation and its boundary conditions into a boundary value problem involving ordinary differential equations by using similarity transformation. Then an approximate approach based on operational matrix of differentiation of shifted second kind Chebyshev wavelets is used to find a numerical solution of the obtained boundary value problem. For accuracy of the scheme, we compare our results with available analytical solutions.

In chapter 5, a mathematical model of a Stefan problem with fractional order derivatives is discussed and an approximate solution of the problem is obtained by homotopy analysis method (HAM). This model includes a nonlinear boundary condition and the surface heat flux as a power function of time. In order to get numerical results, a suitable value for the auxiliary parameter is determined optimally and the obtained results are presented through figures. For accuracy of our results, comparisons are made between the existing analytic solution and the approximate solution obtained by homotopy analysis method for integer order derivatives. It is found that the homotopy analysis method is valid and feasible technique to study the Stefan problems.

In chapter 6, we study a moving boundary problem with time fractional derivative in Caputo sense. This problem contains time-varying heat flux and variable latent heat term. The solution of the problem is found with the aid of a well-known approximate technique, i.e. homotopy perturbation method. The comparison is made with the solutions of the existing integer order and fractional order problems for some particular cases which show that the obtained numerical results are adequately precise for practical application in the most real-world problems.