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New Decentralised Event-Triggered Consensus Strategy for Single and Double Integrator Multi-Agent Systems

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ABSTRACT This paper proposes new decentralised event-triggering conditions for single- and double integrator multi-agent systems. The developed conditions are based on the relative ratio of the state measurement error and norm of a state function for actuating the controller updates. With higher limits on the maximum tolerable state measurement error, the controller is shown to reduce the actuation updates and hence, the use of available resources. The network topology is assumed to be undirected and connected. The inter-event intervals are shown to be strictly positive for all agents to eliminate the zeno phenomenon. The theoretical concepts are further demonstrated through numerical comparisons and illustrative simulations.

INDEX TERMS Decentralised event-triggered control, inter-event interval, multi-agent systems, zeno phenomenon.

I. INTRODUCTION

Cooperative technologies for consensus control of multi-agent systems (MASs) have been widely explored in the past decades and have been a trending research area due to their diverse applications in areas, for instance, power engineering, artificial intelligence, defence, robotics, medical imaging, agricultural applications, etc (see references [1]–[3]). Such technologies focus on information sharing between component subsystems to bring about agreement or consensus on a state value. A host of literature has been explored for consensus control of such MASs (see [4]–[7] and references therein).

A typical classification of the aforementioned control algorithms, namely centralised and decentralised, is based on the manner of communication between individual agents. Although, basic consensus algorithms with continuous feedback [8] allow agents to achieve average consensus, however, limited computational availability necessitates effective use of the available resources. Decentralised control [9] has thus, recently advanced to a great extent given its advantages of scalability and relatively less information requirement. It involves information collection from neighbouring agents only, without any significant knowledge of global parameters. Implementation-wise, two important aspects with respect to decentralised algorithms include development of communication and controller actuation strategies. The underlying aim of this paper is to lessen the tally of controller actuator updates of an individual agent by development of new decentralised event-driven strategies.

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Recently, event-triggered control has gained popularity and a lot of event-driven strategies have been presented in literature [10]–[12] to better utilize the capacity of embedded microprocessors. The difference between the traditionally employed time-executed control and the event-executed control technique for scheduling tasks is that the former allows only periodic actuation updates whereas the update in the latter strategy occurs when a set of rules are violated, which are given by an event-triggering law. The law usually specifies a correlation between the state measurement error and the maximum bound on that error. Assuming that the system is Input to State (ISS) stable with respect to these errors, the method eliminates the problem of synchronously exchanging the information with other agents which may cause sheer traffic in the communication channel, once the agents become large in number.

In the multi-agent domain, the event-driven protocol has been studied for application in different control strategies such as consensus based control [14], output feedback control [15], spacecraft formation attitude synchronization [16] and robust cooperative stabilization in nonlinearly interconnected multi-agent systems [17]. Other such developments can be seen in [18], [19].

The major contribution of this paper lies in the new approach to derive improved event-triggered conditions that we propose, to achieve average consensus in single and double integrator homogeneous undirected and connected multi-agent systems. The proposed conditions entail a higher maximum bound on threshold error than in [14], [20], which inturn provide a longer inter-event interval, thereby, reducing the number of controller actuator updates significantly. To demonstrate that the proposed strategy is effective enough in reducing the controller computations, a numerical comparison with the results obtained [14] is given for the first-order case. Additionally, each agent's inter-execution time is shown to be stricly greater than zero under the proposed conditions, which is a stronger criterion as against lower bounding atleast one agent's inter-execution time [14], [18]. This is necessary for eliminating zeno's behaviour [21] in continuously evolving systems, i.e., occurrence of infinite events within a finite interval of time, which is an impractical situation.

The remaining paper is organised as follows. Section II presents a discussion on the algebraic graph theory and related results used in this note. Sections III and IV give the event-triggering conditions to achieve average consensus in first- and second-order systems, respectively. Section V presents numerical simulations to showcase the efficacy of the proposed conditions and the note is concluded in Section VI with the major highlights of the paper.

Notations: $vec(x_i) = [x_1, x_2, \cdots, x_N]^T$, $i = 1, \cdots, N$. $\otimes$ denotes the kronecker product operation. For a symmetric matrix X, $X \geq 0$ (resp. > 0) means that X is positive semi-definite (resp. positive definite). I_n and $0_{n\times n}$ are identity and null matrices of dimensions $n \times n$. $\mathbf{1_n}$ is vector consisting of n ones. $\|.\|$ represents the 2-norm for vectors or the induced 2-norm for matrices, unless stated otherwise.

II. PRELIMINARIES

A. GRAPH THEORY

For a MAS of N agents, if $\mathcal{G} = (\mathcal{V}, \varepsilon)$ denotes a communication graph, where $\mathcal{V} = \{1,\ldots, N\}$ is a finite non-empty set of vertices, then $\varepsilon \subset \mathcal{V} \times \mathcal{V}$ is a set of ordered pairs of vertices, called edges of the graph [22]. An agent j is said to constitute the neighbourhood ($\mathcal{V}_i$) of agent i, if there exists an edge between them. An edge $(i, j) \in \varepsilon$ in an undirected graph represents bidirectional information flow. The in-degree Laplacian matrix associated with the graph $\mathcal{G}$ is defined as $\mathscr{L} = \Delta(\mathcal{G}) - A(\mathcal{G})$, where $\Delta(\mathcal{G})$ is a diagonal matrix with diagonal entries representing the number of agents which communicate their state information to the i^{th} agent. The adjacency matrix $A(\mathcal{G})$ is defined as

$$A(\mathcal{G}) = \begin{cases} w_{ij} & (j, i) \in \varepsilon \\ 0, & otherwise \end{cases} \tag{1}$$

where w_{ij} denotes the weights of the edges. Then, Laplacian $\mathscr{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ such that $l_{ii} = \sum_{j\in\mathcal{V}_i} w_{ij}$ and $l_{ij} = -w_{ij}$, $i \neq j$. For an undirected graph, the adjacency and Laplacian matrix are symmetric. Also, we let $w_{ij} = 1$ in this work for simplicity. A graph is connected if a path exists between any of its two vertices. For such a graph, the Laplacian has one eigenvalue as zero with the corresponding right eigenvector $\mathbf{1_n}$.

We now state the following Lemmas that will be useful in deriving the main results.

Lemma 1 [23]: The below conditions are equivalent

1)

$$\begin{bmatrix} \mathbb{U} & \mathbb{T} \\ \mathbb{T}^T & W \end{bmatrix} \geq 0,$$

2) $W > 0, \mathbb{U} - \mathbb{T}W^{-1}\mathbb{T}^T \geq 0.$

Lemma 2 [24]: For any a,b $\in \mathbb{R}$ and $\alpha > 0$, the following property holds

$$ab \leq \frac{\alpha}{2}a^2 + \frac{1}{2\alpha}b^2. \tag{2}$$

III. SINGLE INTEGRATOR SYSTEMS

This section presents an event-triggered law for state agreement of a system with single integrator agents. The control model is given for a decentralised strategy and is shown to drive the agents towards average consensus. Further, an interval analysis is done to show that the inter-event times for all agents are strictly positive and thus, the controller executions are free of zeno's behaviour.

A. PROBLEM DEFINITION

Consider a MAS with N agents, each having the following single integrator dynamics

$$\dot{x}_i(t) = u_i(t), \quad i \in \mathcal{V}, \tag{3}$$

where $x_i(t), u_i(t) \in \mathbb{R}$ are the state and control input of the i^{th} agent respectively. Given any initial condition, the average consensus problem is solved, if $\lim_{t\to\infty} |x_i(t) - x_j(t)| = 0$, $\forall i, j \in \mathcal{V}$. Given the decentralised nature of control, all the agents are allowed information sharing with their neighbors only, to reach a common state value.

The following subsection presents a decentralised event-based control law for consensus in single integrator systems.

B. DECENTRALIZED CONTROLLER DESIGN

In the decentralised approach for event-triggered control, the control input is upgraded for each agent at event times depending on its self measurement error and information from its neighbors. Let $t_0^i, t_1^i, t_2^i, \ldots,$ denote the event instants for agent $i \in \mathcal{V}$. Then, its state measurement error is given by:

$$e_i(t) = x_i(t_m^i) - x_i(t), \quad t \in [t_m^i, t_{m+1}^i) \tag{4}$$

where t_m^i represents the m^{th} event time for agent i. The distributed control update for i^{th} agent is now given by

$$u_i(t) = -\sum_{j \in \mathcal{V}_i} (x_i(t_m^i) - x_j(t_{m'(t)}^j)), \tag{5}$$

where $m'(t) = arg \min_{k \in \mathbb{N}: t \geq t_k^j} \{t - t_k^j\}$ is the latest event time of agent j for $t \in [t_m^i, t_{m+1}^i)$. The control input for agent i is thus updated both at its own defined event instants $t_0^i, t_1^i, t_2^i, \ldots$ as well as those of its neighbors $t_0^j, t_1^j, t_2^j, \ldots, j \in \mathcal{V}_i$.

Now, from (4) and (5), the agent dynamics can be written as:

$$\dot{x}_i(t) = -\sum_{j \in \mathcal{V}_i} (x_i(t) - x_j(t)) - \sum_{j \in \mathcal{V}_i} (e_i(t) - e_j(t)), \tag{6}$$

with the overall system dynamics given as

$$\dot{x}(t) = -\mathscr{L}(x(t) + e(t)), \tag{7}$$

where $x(t) = vec(x_i(t))$ and $e(t) = vec(e_i(t))$.

Theorem 1: Consider a homogeneous first-order MAS given by (3) with the control law (5). Then, for some $\gamma_i \in (0, 1)$ and any initial condition in $\mathbb{R}^N$, the system achieves average consensus if the following inequality on the norm of state measurement error is satisfied

$$|e_i(t)| \leq \gamma_i |y_i(t)| \left(-\alpha + \sqrt{\frac{2\alpha}{|\mathcal{V}_i|}}\right), \ \alpha \in \left(0, \frac{2}{|\mathcal{V}_i|}\right). \tag{8}$$

where $y_i(t)$ denotes relative state information that is accessible to agent i from its neighbours and its own sensor measurements.

Proof 1: Let $y(t) = vec(y_i(t)) \triangleq \mathscr{L}x(t)$. Consider now, the Lyapunov function,

$$V = \frac{1}{2} x^T(t) \mathscr{L} x(t) \tag{9}$$

Taking the derivative of (9), we get

$$\begin{aligned} \dot{V} &= -x^T(t)\mathscr{L}(\mathscr{L}(x(t) + e(t))) \\ &= -y^T(t)y(t) - y^T(t)\mathscr{L}e(t). \end{aligned} \tag{10}$$

The above equation (10) can be further expressed as

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^N y_i^2(t) - \sum_{i=1}^N \sum_{j \in \mathcal{V}_i} y_i(t)(e_i(t) - e_j(t)) \\ &= -\sum_{i=1}^N y_i^2(t) - \sum_{i=1}^N |\mathcal{V}_i| y_i(t) e_i(t) + \sum_{i=1}^N \sum_{j \in \mathcal{V}_i} y_i(t) e_j(t). \end{aligned}$$

Using Lemma 2, we get

$$\begin{aligned} \dot{V} \leq& \sum_{i=1}^N \left(-|y_i(t)|^2 + |\mathcal{V}_i||y_i(t)||e_i(t)|\right) \\ &+ \sum_{i=1}^N \sum_{j \in \mathcal{V}_i} \left(\frac{\alpha}{2}|y_i(t)|^2 + \frac{1}{2\alpha}|e_j(t)|^2\right). \end{aligned} \tag{11}$$

Since the graph is symmetric, we have

$$\sum_{i=1}^N \sum_{j \in \mathcal{V}_i} \frac{1}{2\alpha}|e_j(t)|^2 = \sum_{i=1}^N \sum_{j \in \mathcal{V}_i} \frac{1}{2\alpha}|e_i(t)|^2. \tag{12}$$

Substituting (12) in (11), one gets

$$\begin{aligned} \dot{V} \leq& \sum_{i=1}^N \Bigg(-|y_i(t)|^2 + |\mathcal{V}_i||y_i(t)||e_i(t)| \\ &+ \frac{\alpha}{2}|\mathcal{V}_i||y_i(t)|^2 + \frac{1}{2\alpha}|\mathcal{V}_i||e_i(t)|^2\Bigg). \end{aligned} \tag{13}$$

From (13), consider the following quadratic equation by denoting corresponding value of $e_i(t)$ as $e_i^*(t)$:

$$\begin{aligned} &\left(-\frac{1}{2\alpha}|\mathcal{V}_i|\right)|e_i^*(t)|^2 - (|\mathcal{V}_i||y_i(t)|)|e_i^*(t)| \\ &+ |y_i(t)|^2\left(1 - \frac{\alpha}{2}|\mathcal{V}_i|\right) = 0. \end{aligned} \tag{14}$$

Solving for the roots of quadratic equation (14), we get

$$|e_i^*(t)| = |y_i(t)|\left(-\alpha \mp \sqrt{\frac{2\alpha}{|\mathcal{V}_i|}}\right), \tag{15}$$

The objective is now to determine the sign of RHS of (13) when the error is perturbed from the solution obtained in (15). As can be easily seen, $\dot{V} \leq 0$ for $|e_i(t)| \leq |y_i(t)|\left(-\alpha + \sqrt{\frac{2\alpha}{|\mathcal{V}_i|}}\right)$. Further, equality ($\dot{V} = 0$) occurs if and only if consensus has been achieved. Thus, the upper threshold limit for error e_i(t) is defined as follows,

$$|e_i(t)| \leq |y_i(t)|\left(-\alpha + \sqrt{\frac{2\alpha}{|\mathcal{V}_i|}}\right), \ \alpha \in \left(0, \frac{2}{|\mathcal{V}_i|}\right). \tag{16}$$

The proof is complete.

Thus, for $\gamma_i \in (0, 1), \ i \in \mathcal{V}$, the event-triggering function can be stated below as

$$f(t) = |e_i(t)| - \gamma_i|y_i(t)|\left(-\alpha + \sqrt{\frac{2\alpha}{|\mathcal{V}_i|}}\right). \tag{17}$$

Denote now, the average state of all agents as x_{av}. Then, $x_{av}(t) = \frac{1}{N}\sum_i^N x_i(t)$. Then, the evolution of $x_{av}(t)$ is given using (7), as $\dot{x}_{av}(t) = \frac{1}{N}\sum_i^N \dot{x}_i(t) = -\frac{1}{N}\sum_i^N \sum_{j \in \mathcal{V}_i}\{(x_i(t) - x_j(t)) + (e_i(t) - e_j(t))\} = 0$. Thus, $x_{av}(t) = \frac{1}{N}\sum_i^N x_i(0)$ is a constant of motion.

C. EVENT-TIME ANALYSIS

This section gives a lower bound on the interval between two event instants to eliminate the zeno phenomenon.

Theorem 2: Consider the system given by (3), control (5) and update rule (16). Under the assumption of an undirected connected network with $\gamma_i \in (0, 1)$, $\forall i \in \mathcal{V}$, for any initial condition in $\mathbb{R}^N$, there exists a minimum of one agent $n \in \mathcal{V}$, such that its inter-event time τ_d is strictly greater than zero and is given by

$$\tau_d = \frac{\mu}{N + \sigma\|\mathscr{L}\|}, \ \sigma = \gamma_n\left(-\alpha + \sqrt{\frac{2\alpha}{|\mathcal{V}_n|}}\right). \tag{18}$$

Proof 2: Suppose that (17) holds $\forall i \in \mathcal{V}$ at time $t \leq 0$ and all errors drop to zero at the same time. It is shown that for at least one node in $\mathcal{V}$, its next inter-event time is lower bounded by $\tau_d > 0$. For this, denote $n = arg \max_{i\in\mathcal{V}} |y_i|$ and given the fact that $|e_i| \leq \|e\|, \forall i$, we then have $\frac{|e_n|}{N|y_n|} \leq \frac{\|e\|}{\|y\|}$. Similar to [13], differentiating $\frac{\|e\|}{\|y\|}$, we get

$$\frac{d}{dt}\frac{\|e\|}{\|y\|} = \frac{d}{dt}\frac{\|e\|}{\|\mathscr{L}x\|} = -\frac{e^T\dot{x}}{\|e\|\,\|\mathscr{L}x\|} - \frac{(\mathscr{L}x)^T\mathscr{L}\dot{x}}{\|\mathscr{L}x\|^2}\frac{\|e\|}{\|\mathscr{L}x\|}$$
$$\leq \frac{\|e\|\,\|\dot{x}\|}{\|e\|\,\|\mathscr{L}x\|} + \frac{\|\dot{x}\|\,\|\mathscr{L}\|\,\|e\|}{\|\mathscr{L}x\|^2} \leq \left(1 + \frac{\|\mathscr{L}\|\,\|e\|}{\|\mathscr{L}x\|}\right)^2$$

If we denote $\phi = \frac{\|e\|}{\|\mathscr{L}x\|}$, we have $\dot{\phi} \leq (1 + \|\mathscr{L}\|\phi)^2$ so that ϕ is bounded by $\phi(t) \leq \theta(t, \theta_0)$ where $\theta(t, \theta_0)$ is obtained on solving the below differential equation $\dot{\theta} = (1 + \|\mathscr{L}\|\theta)^2, \theta(0, \theta_0) = \theta_0$. On solving, we get $\theta(\tau, 0) = \frac{\tau}{1-\tau\|\mathscr{L}\|}$. For an agent i, the lower bound (τ_d) on the next inter event interval, satisfies $\frac{N\tau_d}{1-\tau_d\|\mathscr{L}\|} = \sigma$. On performing simple manipulations, it is easy to show (18).

Remark 1: Assuming the system is ISS, threshold limit represents the maximum allowed error before the control input is updated. With a higher value of upper threshold limit, number of input updates to reach consensus are reduced, which has been illustrated via numerical examples in Section V. Note in [14], the upper threshold limit for error e_i(t) and τ_d are defined as follows,

$$e_i^2(t) \leq (\frac{\alpha}{|\mathcal{V}_i|} - \alpha^2)y_i^2(t), \ \alpha \in \left(0, \frac{1}{|\mathcal{V}_i|}\right), \tag{19}$$

$$\tau_d = \gamma_n\alpha\frac{1 - \alpha|\mathcal{V}_n|}{N|\mathcal{V}_n| + \|\mathscr{L}\|\gamma_n\alpha(1 - \alpha|\mathcal{V}_n|)}. \tag{20}$$

Now, we show that for all agents, the inter-event time [25] is strictly greater than zero. Consider that the event for the i^{th} agent triggers at instant t_m^i and thus, $e_i(t_m^i) = 0$. Then error evolution in the same interval is given as $\dot{e}_i(t) = -\dot{x}_i(t)$. Then,

$$\frac{d|e_i(t)|}{dt} \leq |\dot{x}_i(t)| = \left|\sum_{j\in\mathcal{V}_i}\{x_i(t_m^i) - x_j(t_{m'(t)}^j)\}\right|.$$

Let $\dot{\zeta}_i(t) = \left|\sum_{j\in\mathcal{V}_i}\{x_i(t_m^i) - x_j(t_{m'(t)}^j)\}\right| = f_e(t)$. Then we have $e_i(t) \leq \zeta_i(t) = \int_{t_m^i}^t f_e(\tau)d\tau$. Now, until the consensus has been reached, the next inter-event interval (τ_m^i) is lower bounded by the interval it takes $\zeta_i(t)$ to progress from 0 to $\gamma_i|y_i(t)|\left(-\alpha + \sqrt{\frac{2\alpha}{|\mathcal{V}_i|}}\right)$. This proves that τ_m^i is a strictly positive value and hence, zeno's behaviour is eliminated.

IV. DOUBLE INTEGRATOR SYSTEMS

The second-order case cannot simply be dealt by extending the first order case. Thus, it has been taken up separately in this section for analysis.

A. PROBLEM DESCRIPTION

Consider an undirected connected topology with each agent having second-order dynamics defined as follows:

$$\begin{aligned}\dot{x}_{1_i}(t) &= x_{2_i}(t)\\ \dot{x}_{2_i}(t) &= u_i(t), \quad \forall i \in \mathcal{V}\end{aligned} \tag{21}$$

where $x_{1_i}(t),\ x_{2_i}(t),\ u_i(t) \in \mathbb{R}$ can be considered as the i^{th} agent's respective position, velocity and control input. The consensus problem for a system with dynamics (21) is solved, if for any given set of initial conditions

$$\lim_{t\to\infty} |x_{1_i}(t) - x_{1_j}(t)| = 0,$$
$$\lim_{t\to\infty} |x_{2_i}(t) - x_{2_j}(t)| = 0, \quad \forall i, j \in \mathcal{V}.$$

Following similar steps as in the first-order case, initial average can be proven to be a constant of motion for the double integrator case also, hence omitted here.

B. DECENTRALIZED EVENT-TRIGGERED CONDITION

The control signal for the distributed approach depends on both the states and is given as

$$u_i = -\sum_{j\in\mathcal{V}_i} x_{1_i}(t_m^i) - x_{1_j}(t_{m'(t)}^i)$$
$$- \mu\sum_{j\in\mathcal{V}_i} x_{2_i}(t_m^i) - x_{2_j}(t_{m'(t)}^i), \ t \in [t_m^i, t_{m+1}^i) \tag{22}$$

where $\mu > 0$, t_m^i is the m^{th} event time for agent i, $t_{m'(t)}^i$ is the latest event time for j^{th} agent, $x_{1_i}(t_m^i)$ and $x_{2_i}(t_m^i)$ are the i^{th} agent's position and velocity at the last event, respectively. The same follows for $x_{1_j}(t_{m'(t)}^i)$ and $x_{2_j}(t_{m'(t)}^i)$. Define $e_{x_1}(t) = vec(e_{x_1}^i(t))$, where $e_{x_1}^i(t) = x_{1_i}(t_m^i) - x_{1_i}(t)$. Define again $e_{x_2}(t) = vec(e_{x_2}^i(t))$, where $e_{x_2}^i(t) = x_{2_i}(t_m^i) - x_{2_i}(t),\ \forall i \in \mathcal{V}$.

The closed-loop system formed using (21) and (22) can now be expressed as

$$\begin{aligned}\dot{x}_1(t) &= x_2(t)\\ \dot{x}_2(t) &= -\mathscr{L}(x_1(t) + e_{x_1}(t)) - \mu\mathscr{L}(x_2(t) + e_{x_2}(t))\end{aligned} \tag{23}$$

We now drop the argument of vectors for simplicity. Define two vectors $y = [x_1^T, x_2^T]^T$ and $e = [e_{x_1}^T, e_{x_2}^T]^T$. Then, the system dynamics is given as follows

$$\dot{y}(t) = \begin{bmatrix} 0_{N\times N} & I_N \\ -\mathscr{L} & -\mu\mathscr{L}\end{bmatrix}y + \begin{bmatrix} 0_{N\times N} & 0_{N\times N} \\ -\mathscr{L} & -\mu\mathscr{L}\end{bmatrix}e. \tag{24}$$

Let $\hat{x}_1 = \mathscr{L}x_1$, $\hat{x}_2 = \mathscr{L}x_2$ and $\hat{y} = (I_2 \otimes \mathscr{L})y$. Similar to [18], define the Lyapunov function

$$V = \frac{1}{2}y^T \begin{bmatrix} k_1\mathscr{L}^T\mathscr{L} & k_2\mathscr{L}^T \\ k_2\mathscr{L} & k_3\mathscr{L} \end{bmatrix} y \tag{25}$$

where k_1, k_2, k_3 are positive constants.

Lemma 3: The function (25) is a valid candidate for the system (21), if there exists constant $k_4 > 0$ such that $k_1 \geq \frac{k_2^2}{k_3k_4}$.

Proof 3: Consider the case where consensus has yet not been attained. From (25), it follows that

$$\begin{aligned} V &= \frac{1}{2}(k_1x_1^T\mathscr{L}^T\mathscr{L}x_1 + 2k_2x_1^T\mathscr{L}^Tx_2 + k_3x_2^T\mathscr{L}x_2) \\ &= \frac{1}{2}(k_1\hat{x}_1^T\hat{x}_1 + 2k_2\hat{x}_1x_2 + k_3x_2^T\mathscr{L}x_2). \end{aligned} \tag{26}$$

It is easy to see that $k_1\hat{x}_1^T\hat{x}_1 = k_1\sum_{i=1}^{N}\|\hat{x}_{1i}\|^2 \geq 0$, with equality occuring only for the case when the consensus is reached. We have $V \geq \frac{1}{2}y^T \begin{bmatrix} k_1\mathscr{L}^T\mathscr{L} & k_2\mathscr{L}^T \\ k_2\mathscr{L} & k_3k_4 \end{bmatrix} y$ where $k_4 \triangleq \min_{\mathscr{L}x_2\neq 0, x_2\neq 0} \frac{x_2^T\mathscr{L}x_2}{x_2^Tx_2}$. From Lemma 1, $\begin{bmatrix} k_1\mathscr{L}^T\mathscr{L} & k_2\mathscr{L}^T \\ k_2\mathscr{L} & k_3k_4 \end{bmatrix} \geq 0$, if $k_3k_4 > 0$ and $\mathscr{L}^Tk_1\mathscr{L} - \frac{\mathscr{L}^Tk_2{}^2\mathscr{L}}{(k_3k_4)} \geq 0$, which implies that, if the condition (28) holds, the Lyapunov function (25) is positive semi-definite and equals zero only when the consensus is achieved.

Theorem 3: Consider the MAS (21) with control input (22). Suppose $k_5 > 0$. Then for any given initial condition in $\mathbb{R}^{2N}$, average consensus is achieved, if the decentralised event-triggering condition $f(t) \leq 0$ is enforced, where $f(t)$ is given by

$$\begin{aligned} f(t) = \|e^i_{x_1}\|^2 + \|e^i_{x_2}\|^2 - \frac{\|\hat{x}^i_1\|^2}{\|\mathscr{L}\|^2}\left(\sqrt{\frac{2\gamma_ikk_2}{k_3}} - \frac{kk_2}{k_3}\right)^2 \\ - \frac{1}{\|\mathscr{L}\|^2}(\sqrt{2\gamma_ikk_3|k_2-k_5|} - k_3k)^2\|\hat{x}^i_2\|^2 \end{aligned} \tag{27}$$

and the following conditions hold

$$\mu k_2 - k_3 \geq \frac{k_2^2}{k_3k_4},\ k_5\mathscr{L}^T\mathscr{L} - k_2\mathscr{L} \geq 0, \tag{28}$$

$$k_3\mu = k_2, \tag{29}$$

$$1 > \gamma_i > \max\left(\frac{kk_2}{2k_3}, \frac{1}{2(k_2-k_5)}\right). \tag{30}$$

Proof 4: Consider the Lyapunov function in (25). Its derivative along the trajectory (24) is given by

$$\begin{aligned} \dot{V} &= y^T \begin{bmatrix} k_1\mathscr{L}^T\mathscr{L} & k_2\mathscr{L}^T \\ k_2\mathscr{L} & k_3\mathscr{L} \end{bmatrix} \dot{y} \\ &= y^T \begin{bmatrix} -k_2\mathscr{L}^T\mathscr{L} & k_1\mathscr{L}^T\mathscr{L} - \mu k_2\mathscr{L}^T\mathscr{L} \\ -k_3\mathscr{L}^T\mathscr{L} & k_2\mathscr{L} - \mu k_3\mathscr{L}^T\mathscr{L} \end{bmatrix} y \\ &\quad + y^T \begin{bmatrix} -k_2\mathscr{L}^T\mathscr{L} & -\mu k_2\mathscr{L}^T\mathscr{L} \\ -k_3\mathscr{L}^T\mathscr{L} & -\mu k_3\mathscr{L}^T\mathscr{L} \end{bmatrix} e. \end{aligned} \tag{31}$$

Consider now the former part of (31) which can be written as

$$\begin{aligned} & y^T \begin{bmatrix} -k_2\mathscr{L}^T\mathscr{L} & k_1\mathscr{L}^T\mathscr{L} - \mu k_2\mathscr{L}^T\mathscr{L} \\ -k_3\mathscr{L}^T\mathscr{L} & k_2\mathscr{L} - \mu k_3\mathscr{L}^T\mathscr{L} \end{bmatrix} y \\ &= -k_2\hat{x}_1^T\hat{x}_1 + \hat{x}_1^T(k_1 - \mu k_2 - k_3)\hat{x}_2 \\ &\quad + k_2x_2^T\hat{x}_2 - \mu k_3\hat{x}_2^T\hat{x}_2. \end{aligned} \tag{32}$$

If we choose $\mu k_2 - k_3 \geq \frac{k_2^2}{k_3k_4}$, $k_5\mathscr{L}^T\mathscr{L} - k_2\mathscr{L} \geq 0$, the condition (32) is less than or equal to

$$-k_2\hat{x}_1^T\hat{x}_1 - (\mu k_3 - k_5)\hat{x}_2^T\hat{x}_2. \tag{33}$$

Now, let us consider the second part of (31)

$$\begin{aligned} & y^T \begin{bmatrix} -k_2\mathscr{L}^T\mathscr{L} & -\mu k_2\mathscr{L}^T\mathscr{L} \\ -k_3\mathscr{L}^T\mathscr{L} & -\mu k_3\mathscr{L}^T\mathscr{L} \end{bmatrix} e \\ &\leq k_2\|\hat{x}_1^T\mathscr{L}e_{x1}\| + \mu k_2\|\hat{x}_1^T\mathscr{L}e_{x2}\| \\ &\quad + k_3\|\hat{x}_2^T\mathscr{L}e_{x1}\| + \mu k_3\|\hat{x}_2^T\mathscr{L}e_{x2}\|. \end{aligned} \tag{34}$$

Using Lemma 2, (34) is less than or equal to

$$\begin{aligned} & k_2\|\hat{x}_1\|\ \|\mathscr{L}\|\ \|e_{x1}\| + \frac{\mu k_2k\|\hat{x}_1\|^2}{2} + \frac{\mu k_2\ \|\mathscr{L}\|^2\ \|e_{x2}\|^2}{2k} \\ &+ \frac{k_3k\|\hat{x}_2^2\|}{2} + \frac{k_3\|\mathscr{L}\|^2\ \|e_{x1}\|^2}{2k} + \mu k_3\|\hat{x}_2\|\ \|\mathscr{L}\|\ \|e_{x2}\|. \end{aligned} \tag{35}$$

On rearranging the above equation (35), the terms can be split into two quadratic polynomials as follows

$$V_1 = \frac{k_3\|\mathscr{L}\|^2\ \|e_{x1}\|^2}{2k} + k_2\|\hat{x}_1\|\ \|\mathscr{L}\|\ \|e_{x1}\| + \frac{\mu k_2k\|\hat{x}_1\|^2}{2}, \tag{36}$$

$$V_2 = \frac{\mu k_2\ \|\mathscr{L}\|^2\ \|e_{x2}\|^2}{2k} + \mu k_3\|\hat{x}_2\|\|\mathscr{L}\|\|e_{x2}\| + \frac{k_3k\|\hat{x}_2^2\|}{2}. \tag{37}$$

From (36) and (37), we get

$$\begin{aligned} V_1 &= \left(\sqrt{\frac{k_3}{2k}}\|\mathscr{L}\|\ \|e_{x1}\| + \sqrt{\frac{k_2^2k}{2k_3}}\|\hat{x}_1\|\right)^2 \\ &\quad + \left(\frac{\mu k_2k}{2} - \frac{k_2^2k}{2k_3}\right)\|\hat{x}_1\|^2, \end{aligned} \tag{38}$$

$$\begin{aligned} V_2 &= \left(\sqrt{\frac{\mu k_2}{2k}}\|\mathscr{L}\|\|e_{x2}\| + k_3\sqrt{\frac{k\mu}{2k_2}}\|\hat{x}_2\|\right)^2 \\ &\quad + \left(\frac{k_3k}{2} - \frac{k_3^2k\mu}{2k_2}\right)\|\hat{x}_2\|^2 \end{aligned} \tag{39}$$

If we choose (29), then from (31), (33), (38) and (39), we get

$$\begin{aligned} \dot{V} &\leq \left(\sqrt{\frac{k_3}{2k}}\|\mathscr{L}\|\|e_{x1}\| + \sqrt{\frac{k_2^2k}{2k_3}}\|\hat{x}_1\|\right)^2 \\ &\quad + \left(\sqrt{\frac{1}{2kk_3}}k_2\|\mathscr{L}\|\|e_{x2}|| + k_3\sqrt{\frac{kk_3}{2}}\|\hat{x}_2\|\right)^2 \\ &\quad - k_2\|\hat{x}_1\|^2 - (\mu k_3 - k_5)\|\hat{x}_2\|^2. \end{aligned} \tag{40}$$

Now consider $\|e_{x1}\|\|\mathcal{L}\| \leq \left(\sqrt{\frac{2\gamma kk_2}{k_3}} - \frac{kk_2}{k_3}\right)\|\hat{x}_1\|$ and $\|e_{x2}\|\ \|\mathcal{L}\| \leq \left(\sqrt{2\gamma kk_3|k_2-k_5|} - k_3k\right)\|\hat{x}_2\|$ such that (30) holds, then, the event-triggering condition is given by (27). Thus, if (27) is forced to be no greater than zero, then from (40), $\dot{V} \leq (\gamma-1)(k_2\|\hat{x}_1\|^2 + (k_2-k_5)\|\hat{x}_2\|^2) \leq 0$ and equality occurs only if consensus has been achieved.

C. EVENT-TIME ANALYSIS

Calculating the derivative of $\frac{\|e\|}{\|\hat{y}\|}$ on the same lines as in [18], $\frac{d}{dt}\frac{\|e\|}{\|\hat{y}\|} \leq \frac{\|\dot{e}\|}{\|\hat{y}\|} + \frac{\|e\|\|\dot{\hat{y}}\|}{\|\hat{y}\|^2}$. Now,

$$\dot{\hat{y}} = \left\| \begin{bmatrix} \hat{x}_2 \\ -\mathcal{L}^T\mathcal{L}(x_1+e_{x1}) - \mu\mathcal{L}^T\mathcal{L}(x_2+e_{x2}) \end{bmatrix} \right\| \leq (1+\sqrt{1+\mu^2}\|\mathcal{L}\|)(\|\hat{y}\| + \|\mathcal{L}\|\ \|e\|). \tag{41}$$

Similarly, $\|\dot{e}\| = \|\dot{y}\| = \left\| \begin{bmatrix} x_2 \\ \mathcal{L}(x_1+e_{x1}) + \mu\mathcal{L}(x_2+e_{x2}) \end{bmatrix} \right\|$

$$\leq (1+\sqrt{1+\mu^2})(\|\hat{y}\| + \|\mathcal{L}\|\ \|e\|). \tag{42}$$

From (41) and (42), we have

$$\begin{aligned} \frac{d}{dt}\frac{\|e\|}{\|\hat{y}\|} &\leq (1+\sqrt{1+\mu^2})\frac{\|\hat{y}\| + \|\mathcal{L}\|\ \|e\|}{\|\hat{y}\|} \\ &\quad + \|e\|(1+\sqrt{1+\mu^2}\|\mathcal{L}\|)\frac{\|\hat{y}\| + \|\mathcal{L}\|\ \|e\|}{\|\hat{y}\|^2} \\ &\leq (\Delta+\sqrt{1+\mu^2})\left(1+\frac{\|\mathcal{L}\|\ \|e\|}{\|\hat{y}\|}\right)^2, \end{aligned} \tag{43}$$

where $\Delta = \max(1, \frac{1}{\|\mathcal{L}\|})$. If we represent $(\Delta + \sqrt{1+\mu^2})$ by Γ and $\frac{\|e\|}{\|\hat{y}\|}$ by z, so that z is bounded by $z(t) \leq \theta(t, \theta_0)$, where $\theta(t, \theta_0)$ is obtained on solving the below differential equation $\dot{\theta} = (1+\|\mathcal{L}\|\theta)^2\Gamma$, $\theta(0,\theta_0) = \theta_0$. Solving, we get

$$\theta(\tau, 0) = \frac{\Gamma\tau}{1-\Gamma\tau\|\mathcal{L}\|}. \tag{44}$$

From the event-triggered law (27), we see that the event interval is the time $\|e_{x1}\|^2 + \|e_{x2}\|^2$ takes to advance from zero to $\delta_1\|\hat{x}_1\|^2 + \delta_2\|\hat{x}_2\|^2$, where $\delta_1 = \frac{1}{\|\mathcal{L}\|^2}\left(\sqrt{\frac{2\gamma kk_2}{k_3}} - \frac{kk_2}{k_3}\right)^2$ and $\delta_2 = \frac{1}{\|\mathcal{L}\|^2}\left(\sqrt{2\gamma kk_3(k_2-k_5)} - k_3k\right)^2$. If we denote this period with τ, then τ is certainly longer than the period in which $\|e_{x1}\|^2 + \|e_{x2}\|^2$ grows from zero to $\zeta(\delta_1\|\hat{x}_1\|^2 + \delta_2\|\hat{x}_2\|^2)$ and $\zeta = \min(\delta_1, \delta_2)$. Suppose it takes τ_d for $\frac{\|e_{x1}\|^2+\|e_{x2}\|^2}{\|\hat{x}_1\|^2+\|\hat{x}_2\|^2}$ to reach ζ. Then $(k+1)^{th}$ event occurs after time $t_k^i + \tau_d$.

Theorem 4: Consider the system in (21) and control law (22), which is triggered when the condition (27) is satisfied. Then, for any given initial condition in $\mathbb{R}^{2N}$, if (28), (29) and (30) hold, then there exist at least one agent r $\in \mathcal{V}$, having strictly positive interval $t_{k+1}^r - t_k^r$, lower bounded by $\tau_d > 0$.

Proof 5: Define $r = arg\max_{i\in\mathcal{V}}(\|\hat{x}_1^i\|^2 + \|\hat{x}_2^i\|^2)$. Then,

$$\frac{\|e_{x1}^r\|^2 + \|e_{x2}^r\|^2}{\|\hat{x}_1^r\|^2 + \|\hat{x}_2^r\|^2} \leq \frac{\|e_{x1}\|^2 + \|e_{x2}\|^2}{\|\hat{x}_1\|^2 + \|\hat{x}_2\|^2} \leq N^2\frac{\|e\|^2}{\|\hat{y}\|^2} \tag{45}$$

From (44), $\frac{\theta(\tau,0)}{1+\|\mathcal{L}\|\theta(\tau,0)} = \tau_d\Gamma$. Using $\theta(\tau,0) = \zeta$, $\tau_d = \frac{\zeta}{(N+\|\mathcal{L}\|\zeta)\Gamma}$. It thus follows that $\tau_d > 0$ and the proof is complete.

For a stronger condition such that the inter-execution times are strictly positive for all agents, assume the event triggers for the i^{th} agent at instant t_m^i and thus, $e_{x_1}^i(t_m^i) = e_{x_2}^i(t_m^i) = 0$. The error evolution in the interval $[t_m^i, t_{m+1}^i)$ is given as $\dot{e}_{x_1}^i(t_m^i) = -\dot{x}_{1_i}$, $\dot{e}_{x_2}^i(t_m^i) = -\dot{x}_{2_i}$. Let $x_i(t) = [x_{1_i}(t)\ x_{2_i}(t)]^T$, $e_i(t) = [e_{x_1}^i(t)\ e_{x_2}^i(t)]$, $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = [0\ \ 1]^T$. Then,

$$\begin{aligned} \frac{d|e_i(t)|}{dt} &\leq |\dot{x}_i(t)| = |Ax_i(t) + Bu_i(t)| \\ &\leq |Ae_i(t)| + \left| B\left(\sum_{j\in\mathcal{V}_i}\{x_{1_i}(t_m^i) - x_{1_j}(t_{m'(t)}^j)\}\right.\right. \\ &\quad \left.\left. + \mu\sum_{j\in\mathcal{V}_i}\{x_{2_i}(t_m^i) - x_{2_j}(t_{m'(t)}^j)\}\right) + Ax_i(t_m^i)\right|. \end{aligned}$$

Let $\dot{\xi}_i(t) = |A|\xi_i(t) + \left|B\left(\sum_{j\in\mathcal{V}_i}\{x_{1_i}(t_m^i) - x_{1_j}(t_{m'(t)}^j)\}\right.\right.$ $\left.\left.+\mu\sum_{j\in\mathcal{V}_i}\{x_{2_i}(t_m^i) - x_{2_j}(t_{m'(t)}^j)\}\right) + Ax_i(t_m^i)\right| = |A|\xi_i(t) + f_e'(t)$.
Then we have

$$e_i(t) \leq \xi_i(t) = \int_{t_m^i}^{t} f_e'(\tau)e^{|A|(t-\tau)}d\tau.$$

Now, until the consensus has been reached, the next inter-event interval $\bar{\tau}_m^i$ is lower bounded by the interval it takes $\xi_i(t)$ to advance from 0 to $\frac{1}{\|\mathcal{L}\|^2}\left(\sqrt{\frac{2\gamma_i kk_2}{k_3}} - \frac{kk_2}{k_3}\right)^2\|\hat{x}_1^i\|^2 + \frac{1}{\|\mathcal{L}\|^2}(\sqrt{2\gamma_i kk_3|k_2-k_5|} - k_3k)^2\|\hat{x}_2^i\|^2$. This proves that $\bar{\tau}_m^i$ is a strictly positive value and hence, the proposed conditions are free of zeno's behaviour.

V. ILLUSTRATIVE SIMULATIONS

This section compares the performance of the single-integrator system (3) with the proposed condition (8) and inter-event time (18) to the one proposed in [14] given by (19) and (20) with a numerical example. Another example is given for the double integrator system (21) to achieve consensus under the proposed strategy in Theorem 3.

A. SINGLE INTEGRATOR SYSTEM

Consider a system consisting of four agents with $\mathcal{L} = [2, -1, -1, 0; -1, 2, -1, 0; -1, -1, 3, -1; 0, 0, -1, 1]$. The initial states were chosen arbitrarily to be $x(0) = [0.3, 0, 0.5, 0.2]^T$ with the initial average equal to $\bar{x} = 0.25$. The necessary constants for the triggering law (17) fulfilling

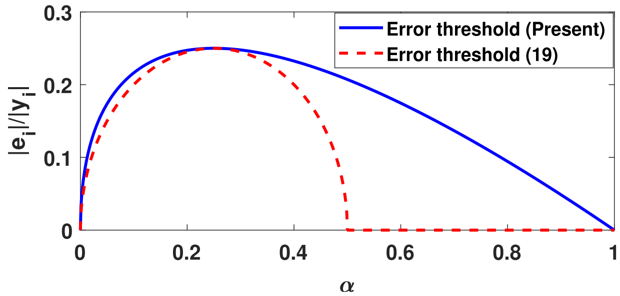

FIGURE 1. Variation of threshold error limit with α ($|\mathcal{V}_2| = 2$).

TABLE 1. Comparison of number of actuation updates.

Case	[14]	Present	% reduction
Agent 2	333	276	17.12%
Agent 3	314	250	20.38%

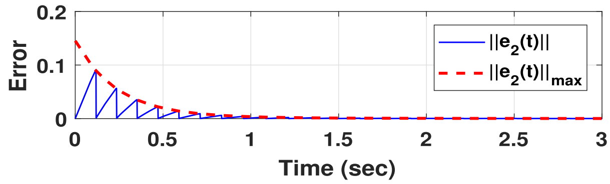

FIGURE 2. Evolution of state error and threshold limit (19) for agent 2 with $\mathcal{V}_2 = 2$.

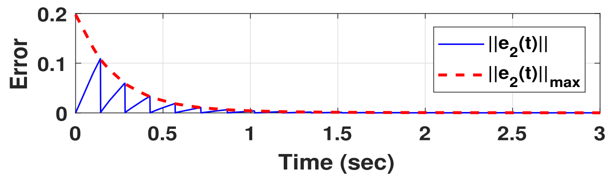

FIGURE 3. Evolution of state error and threshold limit (proposed)(8) for agent 2 with $\mathcal{V}_2 = 2$.

condition (8) are chosen to be $\gamma_1 = 0.55, \gamma_2 = 0.55, \gamma_3 = 0.65, \gamma_4 = 0.65$ and $\alpha = 0.2$. Variation of $\frac{|e_i(t)|}{|y_i(t)|}$ with α for agent 2 with $|\mathcal{V}_2| = 2$ is shown in Figure. 1. It can be clearly observed that the proposed threshold error limit (8) is higher than threshold error limit (19) given in [14]. At $\alpha = \alpha_{max}$, the limit is same for both the results. Figures 2 and 3 demonstrate the event triggers for (19) and (8) for agent 2, respectively. Similarly, Figures 4 and 5 demonstrate the event triggers for (19) and (8) for agent 3, respectively. It can be easily seen from the figures that the number of actuation updates are lesser for the proposed triggering condition. This is further verified by a numerical comparison in Table 1 which compares the number of actuation updates for 2 agents. It can be seen that there is a significant reduction of more than 17% and 20% wrt to [14] in number of event triggers for agent 2 and agent 3, respectively. In addition, the inter-event time as a function of α for agent 2 is also greater for (18) than in (20), as seen from Figure. 6.

B. DOUBLE INTEGRATOR SYSTEM

Consider an undirected connected topology with four agents with $\mathscr{L} = [3, -1, -1, -1; -1, 2, -1, 0; -1, -1, 3, -1; 0, 0, -1, 1]$. The initial states were chosen arbitrarily to be $x_1(0) = [3, 1, 0, -2]^T$ and $x_2(0) = [0, 0, 0, 0]^T$. The necessary constants for the triggering law (27) fulfilling conditions (28), (29) and (30) are chosen to be $k_1 = 99, k_2 = 10, \mu = 10, k_3 = 1, k_4 = 5, k_5 = 8, k = 0.01, \gamma_1 = 0.8, \gamma_2 = 0.7, \gamma_3 = 0.9$ and $\gamma_4 = 0.9$. Figures 7 and 8 respectively demonstrate the achievement of consensus for both the position and velocity states under the triggering condition (27).

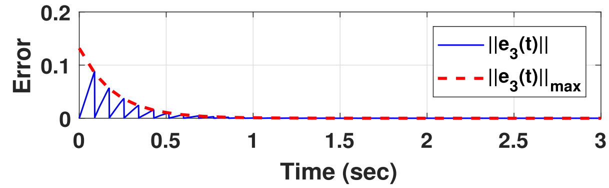

FIGURE 4. Evolution of state error and threshold limit (19) for agent 3 with $\mathcal{V}_3 = 3$.

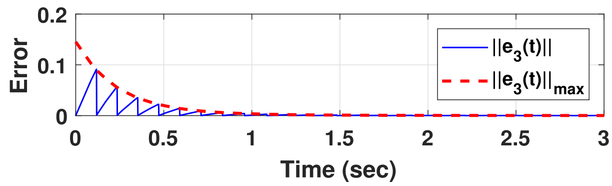

FIGURE 5. Evolution of state error and threshold limit (proposed) (8) for agent 3 with $\mathcal{V}_3 = 3$.

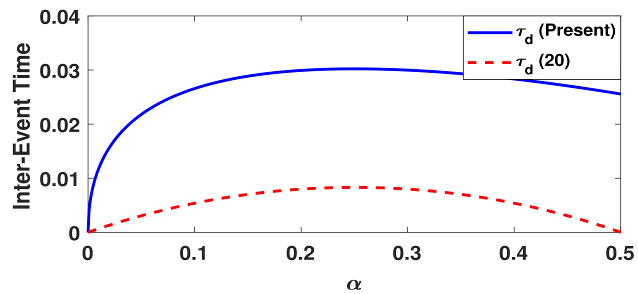

FIGURE 6. Variation of inter-event time with α for $|\mathcal{V}_2| = 2$ and $\gamma_2 = 0.55$.

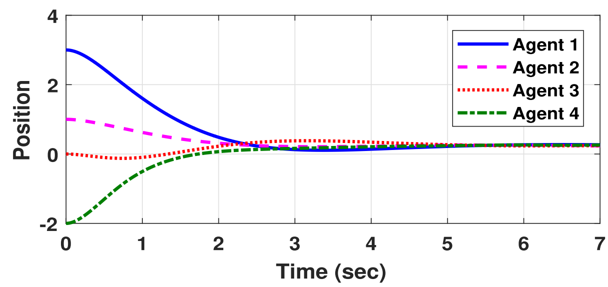

FIGURE 7. Position (x_1) trajectory under triggering law (27).

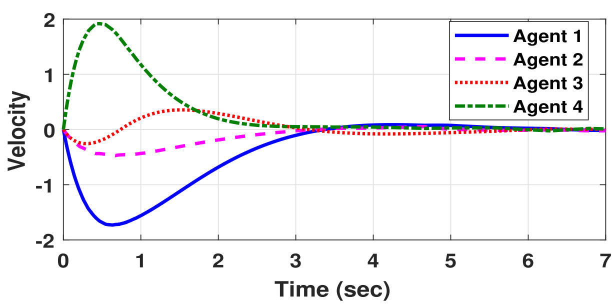

FIGURE 8. Velocity (x_2) trajectory under triggering law (27).

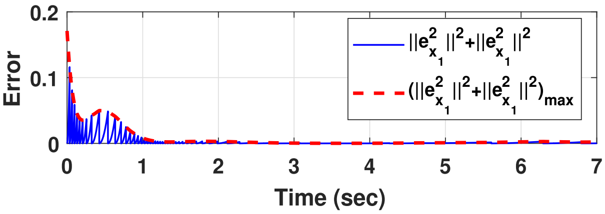

FIGURE 9. Evolution of norm of error for agent 3, along with the maximum bound on error norm (dotted).

Further, Figure. 9 shows the square of error norm for agent 3 in solid line and the maximum bound in dotted. It shows that after the triggering of control law for agent 3 following an event instant, the measurement error resets to zero since the states get updated, and the process repeats.

VI. CONCLUSION

In this paper, new distributed event-triggered conditions have been proposed to achieve average consensus in single- and double-integrator systems. With the assumption of an undirected connected topology, the triggering condition for the first order case associates with itself, a higher limit for maximum error, which reduces number of system input updates, thereby increasing the inter-event time and allowing better use of communication bandwidth. Further, a triggering law for second-order system has also been proposed, along with a minimum bound on the inter-event interval for all agents. The presented results are supported through numerical comparisons and simulations done in Simulink environment.

A disadvantage of the proposed conditions is that the event-triggers occur at both an agent and its neighbours' event times. Thus, future directions involve deriving event conditions based on the same approach for triggers occurring only at an agent's own event time, which further reduces the number of actuations. In additions, similar approach will be employed to directed graphs.

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