Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 53-1 (2020) 13–18

Design of Robust PID Controller using Static Output Feedback framework

Jitendra Kumar Goyal * Shubham Aggarwal * Pradosh Ranjan Sahoo ** Sandip Ghosh * Shyam Kamal *

* *Department of Electrical Engineering, Indian Institute of Technology (BHU), Varanasi, U.P-221005, India (e-mail: jitendrakg.rs.eee16@iitbhu.ac.in, shubham.aggarwal.eee15@iitbhu.ac.in, sghosh.eee@iitbhu.ac.in, shyamkamal.eee@iitbhu.ac.in).*
** *Department of Electrical and Electronics Engineering, MITS Madanpalle, Chittoor, A.P-517325, India (e-mail: sahoo.pradosh@gmail.com)*

Abstract: This paper considers the design of a robust PID controller for higher order MIMO plants. The design problem is first recast into a Static Output Feedback (SOF) controller design problem and then the transformed SOF problem is solved within the framework of Linear Matrix Inequalities (LMIs) through a decomposition of the Lyapunov matrix variable. Sufficient LMI criteria are derived that ensure H_∞ performance of the underlying system. By means of a numerical example, it is shown that the designed controller yields less conservative results. Also, a comparative study is done with the existing techniques to demonstrate the efficacy of the proposed method.

Keywords: Static output feedback controllers, H_∞ controller, Proportional Integral Derivative controller, Linear matrix inequalities, Multi Input Multi Output plants

1. INTRODUCTION

Industrial processes commonly use PID controllers due to the inherent ease in implementation and tuning. Potential areas of application of such controllers include chemical process industries, food processing industries, aerospace industry, robotic industry and numerous other engineering domains. Due to their popularity, a host of tuning methods have been proposed in literature such as by Åström and Hägglund (1995), Tan et al. (2012) which consider several modeling and autotuning techniques to determine the gains of the PID controllers associated with the proportional, integral and derivative inputs. One of the main challenges in their design process is the appropriate gain tuning of these controllers. Among the approaches existing in literature for tuning the P, I and D gains are those of root locus, bode plots, Ziegler-Nichols method, etc. Though there exist a spectrum of tools that give different types of performances [1], unfortunately, a single tuning method usually does not satisfy a variety of practical issues such as load disturbances, sensitivity of the system to measurement noise and model uncertainties. Also, with the growing need for improved process control, we need to design controllers in a robust way to extract satisfactory performance even in uncertain environments. A good robust controller should ensure stability of the overall closed-loop system and performance over the entire uncertainty domain (Åström and Hägglund (1995); Goodwin et al. (2001)).

Therefore, to overcome the above-mentioned shortcomings, LMI based controller design (Boyd et al., 1994) is one of the widely used approach for designing a robust controller since it provides solution to a large set of convex problems effectively without any restriction on the selection of certain parameters. Additionally, it provides simplicity and flexibility in tuning the controller gain parameters rather than other techniques such as Ziegler-Nichols, Model Reference Adaptive Control (MRAC), Particle Swarm Optimization (PSO), Adaptive Particle Swarm Optimization (APSO) and the like.

Static Output Feedback (SOF) controller design is a fundamental problem in control engineering and it finds use in a raft of design problems such as finding constant feedback gain matrices for centralized control and design of PI/PID controllers. Motivated by the above problems and scope of solution in the design of controllers in the framework of LMIs, the problem of PID controller design is transformed into design of robust SOF controller using the transformations given in Zheng et al. (2002). Various iterative algorithms have been developed by Cao et al. (1998a,c); El Ghaoui et al. (1997); He and Wang (2006) in the LMI framework, which are widely used in the design of PID controllers. But such iterative algorithms are complex and require large computation time.

To avoid this, an approach for solving the SOF problem was addressed in Rubió-Massegú et al. (2013), which is based on the decomposition of the Lyapunov matrix variable. But this approach omits the off-diagonal entries of the Lyapunov matrix and considers only the diagonal elements. This, however results in restrictions in the design criteria. To this end, we recently presented new results on the design of SOF controller based on the decomposition of the Lyapunov matrix variable which considers the diagonal

10.1016/j.ifacol.2020.06.003

as well as the off-diagonal terms and the method was shown to provide less conservative results in Sahoo et al. (2019). This paper extends the same approach of SOF design to the PID controller design problem. The nonlinear terms ($BKC\mathcal{X}$) arising due to the coupling of controller gain (K) and system matrices as defined later in (1) are handled through the decomposition of the Lyapunov matrix. Based on this method, a robust PID controller is then designed.

The remaining portion of this technical note is given as follows. Section 2 provides necessary preliminaries useful in deriving the main results. The main idea behind designing the SOF controller involves decomposition of Lyapunov matrix variable which is developed in section 3. Section 4 provides the approach for the robust H_∞ controller design. Robust PID controller based on SOF design is given in section 5. A numerical example is provided in section 6 to illustrate the efficacy of the proposed method. Finally, the paper is concluded in section 7 with major highlights of this manuscript.

Notations: The symbol $*$ is used to represent the symmetrical off diagonal terms. $(\cdot)^{-1}$ denotes inverse of a matrix and $(\cdot)^T$ denotes transpose operation. I denotes an identity matrix with suitable dimensions. $He\{Z\}$ denotes symmetric matrices, i.e., $He\{Z\} = Z + Z^T$.

2. PRELIMINARIES

Consider the continuous-time (CT) LTI system described by the following state space equations as:

$$\begin{aligned} \dot{x}(t) &= \mathcal{A}x(t) + \mathcal{B}_w w(t) + \mathcal{B}u(t) \\ z(t) &= \mathcal{C}_z x(t) + \mathcal{D}_{zu} u(t) + \mathcal{D}_{zw} w(t) \\ y(t) &= \mathcal{C}x(t) + \mathcal{D}_{yw} w(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $z(t) \in \mathbb{R}^{p_1}$ denote the vector of states, control input and the controlled output of the system, respectively. Also, $w(t) \in \mathbb{R}^{m_2}$ and $y(t) \in \mathbb{R}^{n_y}$ are exogenous disturbance and measured output respectively. $\mathcal{A}, \mathcal{B}, \mathcal{B}_w, \mathcal{C}_z, \mathcal{D}_{zu}, \mathcal{D}_{zw}, \mathcal{C}, \mathcal{D}_{yw}$ are constant matrices having compatible dimensions.

Let the SOF control law for system (1) be given as:

$$u(t) = Ky(t) \tag{2}$$

where K is the feedback gain matrix. Then the closed-loop system with the above control law (2) can be expressed by the state equations as

$$\begin{bmatrix} \dot{x}_l(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_l & \mathcal{B}_l \\ \mathcal{C}_l & \mathcal{D}_l \end{bmatrix} \begin{bmatrix} x_l(t) \\ w(t) \end{bmatrix} \tag{3}$$

where $x_l(t) = x(t), \mathcal{A}_l = \mathcal{A} + \mathcal{B}K\mathcal{C}, B_l = \mathcal{B}_w + \mathcal{B}K\mathcal{D}_{yw}, \mathcal{C}_l = \mathcal{C}_z + \mathcal{D}_{zu}K\mathcal{C}$ and $\mathcal{D}_l = \mathcal{D}_{zw} + \mathcal{D}_{zu}K\mathcal{D}_{yw}$ are closed-loop system matrices with suitable dimensions. Also, the transfer function matrix for this overall system from $w(t)$ to $z(t)$,i.e., $\mathcal{T}_{wz}(s)$ can be defined as

$$\mathcal{T}_{zw}(s) = \mathcal{C}_l(sI - \mathcal{A}_l)^{-1}\mathcal{B}_l + \mathcal{D}_l \tag{4}$$

The following lemmas are used for deriving the main results of this work.

First, we use the below lemma for evaluating the H_∞ performance of (4).

Lemma 1. (Bounded Real Lemma Boyd et al. (1994)). The ensuing statements hold equivalence for $\gamma > 0$.

(1) $\|\mathcal{T}_{wz}(s)\| < \gamma$ and $\mathcal{A}_l$ is Hurwitz
(2) There exists $\mathcal{X} = \mathcal{X}^T > 0$ satisfying

$$\begin{bmatrix} He\{\mathcal{A}_l\mathcal{X}\} & * & * \\ \mathcal{B}_l^T & -\gamma^2 I & * \\ \mathcal{C}_l\mathcal{X} & D_l & -I \end{bmatrix} < 0 \tag{5}$$

Lemma 2. (Chang et al. (2015)). For the matrices $\mathcal{Q}$, $\mathcal{S}$, $\mathscr{L}$ and $\mathscr{N}$ of appropriate dimensions and a scalar ϱ, the inequality

$$\mathcal{Q} + \mathcal{S}\mathscr{N} + \mathscr{N}^T\mathcal{S}^T < 0$$

implies

$$\begin{bmatrix} \mathcal{Q} & * \\ \varrho\mathcal{S}^T + \mathscr{L}\mathscr{N} & -He\{\varrho\mathscr{L}\} \end{bmatrix} < 0 \tag{6}$$

3. DECOMPOSITION OF LYAPUNOV MATRIX

Let the matrix $\bar{Q} \in \mathcal{N}(\mathcal{C})$ represent the null space of the output matrix $\mathcal{C}$. Similarly the range space of $\mathcal{C}$ is denoted by the matrix $\bar{R} \in \mathcal{R}(\mathcal{C})$. Based on the following facts, we derive the main results.

Fact 1. A symmetric matrix $\mathcal{X} > 0$ can be decomposed as follows:

$$\begin{cases} \mathcal{X} = \begin{bmatrix} \bar{Q}^T \\ \bar{R}^T \end{bmatrix}^T \begin{bmatrix} \mathcal{X}_Q & \mathcal{X}_S \\ \mathcal{X}_S^T & \mathcal{X}_R \end{bmatrix} \begin{bmatrix} \bar{Q}^T \\ \bar{R}^T \end{bmatrix}, \\ \begin{bmatrix} \mathcal{X}_Q & \mathcal{X}_S \\ \mathcal{X}_S^T & \mathcal{X}_R \end{bmatrix} > 0, \end{cases} \tag{7}$$

where $\mathcal{X}_Q \in \mathbb{R}^{(n-n_y)\times(n-n_y)}$, $\mathcal{X}_S \in \mathbb{R}^{(n-n_y)\times n_y}$ and $\mathcal{X}_R \in \mathbb{R}^{n_y\times n_y}$.

Considering the fact that $\mathcal{C}\bar{Q} = 0$, the matrix $\bar{R}$ is selected such that the equality $\mathcal{C}\bar{R} = I$ is satisfied. One can then easily obtain

$$\mathcal{C}\mathcal{X} = \mathcal{X}_R\bar{R}^T + \mathcal{X}_S^T\bar{Q}^T. \tag{8}$$

Fact 2. There exists a matrix $Y_R \in \mathbb{R}^{m\times n_y}$ and an invertible matrix $\mathcal{X}_R \in \mathbb{R}^{n_y\times n_y}$ such that the following decomposition holds valid for all K and $\mathcal{X}$

$$K\mathcal{C}\mathcal{X} = Y_R\bar{R}^T + Y_R\mathcal{X}_R^{-1}\mathcal{X}_S^T\bar{Q}^T \tag{9}$$

with

$$K\mathcal{X}_R = Y_R. \tag{10}$$

4. H_∞ CONTROLLER DESIGN

The following result is the reproduction of the result in Sahoo et al. (2019).

Theorem 1. Given a CT system described by (1) along with SOF controller (2), the former is stable and a performance $\|\mathcal{T}_{wz}(s)\| < \gamma$ is guaranteed if, there exist matrices $\mathcal{X} = \mathcal{X}_Q^T$, $\mathcal{X}_R = \mathcal{X}_R^T$, $\mathcal{X}_S$, Y_R and scalars α and β such that the below LMI conditions are satisfied.

$$\begin{bmatrix} \Phi_1 & * & * & * \\ \Phi_2 & -\gamma^2 I & * & * \\ \Phi_3 & \mathcal{D}_{zw} & -I & * \\ \Phi_4 & \mathcal{D}_{yw} & \alpha Y_R^T\mathcal{D}_{zu}^T & -\Phi_5 \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} \mathcal{X}_Q & \mathcal{X}_S \\ \mathcal{X}_S^T & \mathcal{X}_R \end{bmatrix} > 0 \tag{12}$$

where

$$\Psi = \mathcal{A}\bar{Q}\mathcal{X}_Q\bar{Q}^T + \mathcal{A}\bar{R}\mathcal{X}_S^T\bar{Q}^T + \mathcal{A}\bar{Q}\mathcal{X}_S\bar{R}^T + \mathcal{A}\bar{R}\mathcal{X}_R\bar{R}^T$$
$$\Phi_1 = He\{\Psi\} + He\{\Xi\},$$
$$\Phi_2 = \mathcal{B}_w^T - \beta\mathcal{D}_{yw}^T\bar{R}^T$$
$$\Phi_3 = \mathcal{C}_z\mathcal{X} + \mathcal{D}_{zu}Y_R\bar{R}^T$$
$$\Phi_4 = \alpha\Gamma^T + \mathcal{X}_S^T\bar{Q}^T$$
$$\Phi_5 = He\{\alpha\mathcal{X}_R\}$$
$$\Xi = \mathcal{B}Y_R\bar{R}^T - \beta\bar{R}\mathcal{X}_S^T\bar{Q}^T$$
$$\Gamma = \mathcal{B}Y_R + \beta\bar{R}\mathcal{X}_R$$

The feedback controller gain can be computed as $K = Y_R\mathcal{X}_R^{-1}$.

Proof: Given $\begin{bmatrix}\bar{Q} & \bar{R}\end{bmatrix}$ is full rank and from (7) and (12), it is clear that $\mathcal{X} > 0$. Then it remains to show that (11) is sufficient for (5). Replacing (9) in (5), one can rewrite a sufficient criterion of (5) as

$$\begin{bmatrix}\Phi_1 & * & * \\ \Phi_2 & -\gamma^2 I & * \\ \Phi_3 & \mathcal{D}_{zw} & -I\end{bmatrix} + \begin{bmatrix}He\{\Gamma\mathcal{X}_R^{-1}\mathcal{X}_S^T\bar{Q}^T\} & * & * \\ \mathcal{D}_{yw}^T\mathcal{X}_R^{-1}\Gamma^T & 0 & * \\ \mathcal{D}_{zu}Y_R\mathcal{X}_R^{-1}\mathcal{X}_S^T\bar{Q}^T & \mathcal{D}_{zu}Y_R\mathcal{X}_R^{-1}\mathcal{D}_{yw} & 0\end{bmatrix} < 0 \quad (13)$$

The above equation (13) can be rewritten as

$$\begin{bmatrix}\Phi_1 & * & * \\ \Phi_2 & -\gamma^2 I & * \\ \Phi_3 & \mathcal{D}_{zw} & -I\end{bmatrix} + \begin{bmatrix}\Gamma \\ 0 \\ \mathcal{D}_{zu}Y_R\end{bmatrix}\mathcal{X}_R^{-1}\begin{bmatrix}\bar{Q}\mathcal{X}_S \\ \mathcal{D}_{yw}^T \\ 0\end{bmatrix}^T + \begin{bmatrix}\bar{Q}X_S \\ \mathcal{D}_{yw}^T \\ 0\end{bmatrix}\mathcal{X}_R^{-1}\begin{bmatrix}\Gamma \\ 0 \\ \mathcal{D}_{zu}Y_R\end{bmatrix}^T < 0 \quad (14)$$

Finally, applying Lemma 2 and substituting $\mathscr{L} = \mathcal{X}_R$, one obtains (11). □

In the upcoming section, we discuss the method of PID controller design by appropriately transforming it into SOF design problem.

5. PID CONTROLLER DESIGN

The schematic diagram of the overall closed-loop system is shown in Figure 1. The figure clearly demonstrates that the PID controller undergoes a transformation using the $T(\cdot)$ function block to SOF controller design which is inverse transformed using the $T^{-1}(\cdot)$ function block to compute the original gains of the PID controller. The appropriate control signal is then generated using these recovered gains and is fed back into the plant.

5.1 Transformation from PID to SOF Controller Design Problem

Consider the nominal CT LTI system with $w(t) = 0$ in (1), given by

$$\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u(t), \quad y(t) = \mathcal{C}x(t), \quad (15)$$

with the PID controller in the form as

$$u(t) = K_1 y(t) + K_2\int_0^t y(\tau)d\tau + K_3\frac{dy(t)}{dt} \quad (16)$$

where $K_1, K_2, K_3 \in \mathbb{R}^{m\times n_y}$ represent the proportional, integral and derivative gains, respectively which are to be designed. Here, our objective is to transform the PID controller design problem into an SOF design problem. In order to achieve our goal, we consider the coordinate transformation variable defined by Zheng et al. (2002) as $\nu_1(t) = x(t)$, $\nu_2(t) = \int_0^t y(\tau)d\tau$ to incorporate the states to be tracked for incorporating the integral control terms. Let us define $\nu(t) = \begin{bmatrix}\nu_1^T(t) & \nu_2^T(t)\end{bmatrix}^T$, where $\nu(t)$ can be seen as the new state vector of the transformed system, whose dynamics is given by

$$\begin{cases}\dot{\nu}_1(t) = \mathcal{A}\nu_1(t) + \mathcal{B}u(t), \\ \dot{\nu}_2(t) = \mathcal{C}\nu_1(t).\end{cases} \quad (17)$$

Rewriting the equation (17), we get,

$$\begin{cases}\dot{\nu}(t) = \bar{\mathcal{A}}\nu(t) + \bar{\mathcal{B}}u(t), \\ \bar{y}(t) = \bar{\mathcal{C}}\nu(t),\end{cases} \quad (18)$$

where $\bar{\mathcal{A}} = \begin{bmatrix}\mathcal{A} & 0 \\ \mathcal{C} & 0\end{bmatrix}$, $\bar{\mathcal{B}} = \begin{bmatrix}\mathcal{B} \\ 0\end{bmatrix}$, $\bar{\mathcal{C}} = \begin{bmatrix}\mathcal{C}_1 \\ \mathcal{C}_2 \\ \mathcal{C}_3\end{bmatrix}$, $\bar{y}(t) = \begin{bmatrix}y_1 \\ y_2 \\ y_3\end{bmatrix}$, $\mathcal{C}_1 = [\mathcal{C}\ 0]$, $\mathcal{C}_2 = [0\ I]$, $\mathcal{C}_3 = [\mathcal{C}\mathcal{A}\ 0]$.

Assuming that the matrix $(I - K_3\mathcal{C}\mathcal{B})$ is invertible and using above matrix definitions, the system (18) and controller (16) reduce to an SOF controller as

$$u(t) = \bar{K}\bar{y}(t) \quad (19)$$

where, $\bar{K} = \begin{bmatrix}\bar{K}_1 & \bar{K}_2 & \bar{K}_3\end{bmatrix}$, $\bar{K}_1 = (I - K_3\mathcal{C}\mathcal{B})^{-1}K_1$, $\bar{K}_2 = (I - K_3\mathcal{C}\mathcal{B})^{-1}K_2$, $\bar{K}_3 = (I - K_3\mathcal{C}\mathcal{B})^{-1}K_3$.

Once the controller gain matrix $\bar{K} = \begin{bmatrix}\bar{K}_1 & \bar{K}_2 & \bar{K}_3\end{bmatrix}$ is computed, the original PID controller gains can be recalculated as

$$\begin{cases}K_3 &= \bar{K}_3(I + \mathcal{C}\mathcal{B}\bar{K}_3)^{-1} \\ K_2 &= (I - K_3\mathcal{C}\mathcal{B})\bar{K}_2 \\ K_1 &= (I - K_3\mathcal{C}\mathcal{B})\bar{K}_1.\end{cases}$$

The following lemma guarantees the existence and invertibility of the matrix $(I + \mathcal{C}\mathcal{B}\bar{K}_3)$.

Lemma 3. (He and Wang (2006)). The matrix $(I+\mathcal{C}\mathcal{B}\bar{K}_3)$ is always invertible if and only if the matrix $(I - K_3\mathcal{C}\mathcal{B})$ is invertible, where K_3 and $\bar{K}_3$ are related to each other as $\bar{K}_3 = (I - K_3\mathcal{C}\mathcal{B})^{-1}K_3$, or equivalently $K_3 = \bar{K}_3(I + \mathcal{C}\mathcal{B}\bar{K}_3)^{-1}$

5.2 H_∞ Based Robust PID controller

This section focusses on the design of PID controllers with the underlying performance criteria chosen as H_∞ performance (γ). Consider the system (1) and PID controller (16). Under the assumption that the matrix $(I - K_3\mathcal{C}\mathcal{B})$ is invertible and using the transformation discussed in the earlier section to change the PID controller design problem to SOF design problem, the augmented system dynamics are as follows

$$\begin{cases}\dot{\tilde{x}}(t) &= \tilde{\mathcal{A}}\tilde{x}(t) + \tilde{\mathcal{B}}\tilde{u}(t) + \tilde{\mathcal{B}}_w w(t) \\ \tilde{z}(t) &= \tilde{\mathcal{C}}_z\tilde{x}(t) + \tilde{\mathcal{D}}_{zu}\tilde{u}(t) + \tilde{\mathcal{D}}_{zw}w(t) \\ \tilde{y}(t) &= \tilde{\mathcal{C}}\tilde{x}(t) + \tilde{\mathcal{D}}_{yw}w(t) \\ \tilde{u}(t) &= \tilde{K}\tilde{y}(t)\end{cases} \quad (20)$$

where $\tilde{\mathcal{A}} = \begin{bmatrix}\mathcal{A} & 0 \\ \mathcal{C} & 0\end{bmatrix}$, $\tilde{\mathcal{B}} = \begin{bmatrix}\mathcal{B} \\ 0\end{bmatrix}$, $\tilde{\mathcal{B}}_w = \begin{bmatrix}\mathcal{B}_w \\ 0\end{bmatrix}$, $\tilde{\mathcal{C}} = \begin{bmatrix}\mathcal{C}_1 \\ \mathcal{C}_2 \\ \mathcal{C}_3\end{bmatrix}$, $\mathcal{C}_1 = [\mathcal{C}\ 0]$, $\mathcal{C}_2 = [0\ I]$, $\mathcal{C}_3 = [\mathcal{C}\mathcal{A}\ 0]$, $\tilde{\mathcal{C}}_z = [\mathcal{C}_z\ 0]$,

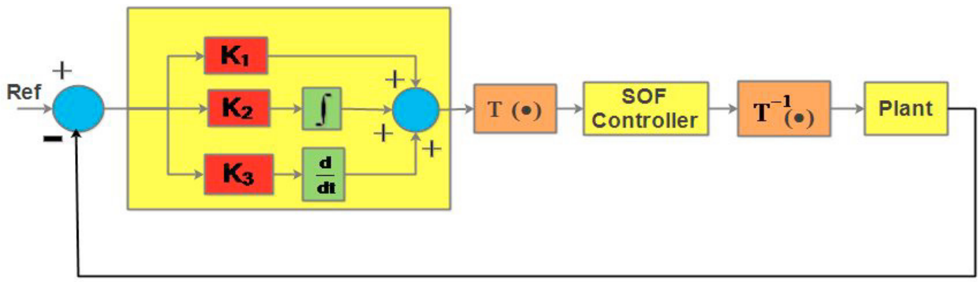

Fig. 1. Closed-loop system block diagram

$\tilde{\mathcal{D}}_{zu} = \mathcal{D}_{zu}$, $\tilde{\mathcal{D}}_{zw} = \mathcal{D}_{zw}$, $\tilde{\mathcal{D}}_{yw} = \mathcal{D}_{yw}$. Thus, the composite feedback controller gain matrices $\bar{K} = \begin{bmatrix}\bar{K}_1 & \bar{K}_2 & \bar{K}_3\end{bmatrix}$ can be obtained by applying Theorem 1 to system (20).

6. NUMERICAL EXAMPLE

A numerical example is considered in this section to demonstrate the efficacy of the proposed design. Note that the scalar parameters α, β in Theorem 1 are obtained using linear search algorithm of *fminsearch*, which is a function of the optimization toolbox in Matlab as in Grace (1993).

Example 1. Consider the state space linearized model of an aircraft system given by Zheng et al. (2002) where only H_∞ output feedback optimization problem is considered with the given parameters.

$$\mathcal{A} = \begin{bmatrix} -0.0266 & -36.6170 & -18.8970 & -32.0900 & 3.2509 & -0.7626 \\ 0.0001 & -1.8997 & 0.9831 & -0.0007 & -0.1708 & -0.0050 \\ 0.0123 & 11.7200 & -2.6316 & 0.0009 & -31.6040 & 22.3960 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -30 & 0 \\ 0 & 0 & 0 & 0 & 0 & -30 \end{bmatrix} \quad (21)$$

$$\mathcal{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 30 & 0 \\ 0 & 30 \end{bmatrix}, \mathcal{B}_w = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 30 \\ 0 \end{bmatrix}, \mathcal{C} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \mathcal{C}_z = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T,$$

$$\mathcal{D}zu = [1\ 1],\ \mathcal{D}zw = 0,\ \mathcal{D}yw = [0\ \ 0]^T.$$

The results computed using Theorem 1 are listed in Table 1. These are further compared with the existing results given by Zheng et al. (2002) and Cao et al. (1998b) for two different cases. The first is that of an SOF stabilization problem and the second considers SOF design with H_∞ performance measure (γ). Note that smaller the value of the performance measure, better is the disturbance rejection capability of the controller. Also, the gains in the Table 1 correspond to the original gains of the PID controller. It is clearly seen that the proposed results give improved results over existing designs.

7. ACKNOWLEDGEMENT

This work was supported by the Science and Research Board, SERB project, India (CRG/2018/004481).

8. CONCLUSION

This paper addresses the problem of robust PID SOF controller design for CT LTI systems. Sufficient conditions are developed in the framework of LMIs for designing the controller ensuring H_∞ performance. The development is based on decomposition of the Lyapunov matrix which provides ease in handling the nonlinear terms arising due to the coupling of system matrices and controller gain. Invertible transformations convert the PID design problem into an SOF one which allows for easy recovery of the original PID controller gains. The proposed theory is supported through a numerical example and from the obtained results, it is seen that the proposed gives less conservative results.

REFERENCES

Åström, K.J. and Hägglund, T. (1995). *PID controllers: theory, design, and tuning*, volume 2. Instrument society of America Research Triangle Park, NC.

Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory*, volume 15. Philadelphia: SIAM.

Cao, Y.Y., Lam, J., and Sun, Y.X. (1998a). Static output feedback stabilization: an ILMI approach. *Automatica*, 34(12), 1641–1645.

Cao, Y.Y., Lam, J., and Sun, Y.X. (1998b). Static output feedback stabilization: an ILMI approach. *Automatica*, 34(12), 1641–1645.

Cao, Y.Y., Sun, Y.X., and Mao, W.J. (1998c). Output feedback decentralized stabilization: ILMI approach. *Systems & Control Letters*, 35(3), 183–194.

Chang, X.H., Park, J.H., and Zhou, J. (2015). Robust static output feedback H_∞ control design for linear systems with polytopic uncertainties. *Systems & Control Letters*, 85, 23–32.

El Ghaoui, L., Oustry, F., and AitRami, M. (1997). A cone complementarity linearization algorithm for static output-feedback and related problems. *IEEE Transactions on Automatic Control*, 42(8), 1171–1176.

Goodwin, G.C., Graebe, S.F., Salgado, M.E., et al. (2001). *Control system design*, volume 240. Prentice Hall New Jersey.

Grace, A. (1993). Optimization toolbox for use with MATLAB, the MathWorks. *Inc., South Natick, MA.*

Table 1. SOF and PID Controllers and their respective performances

Design Approach	Problem Type	Feedback Gains	Closed-loop Poles	Performance	Parameters
Zheng et al. (2002), Cao et al. (1998b)	SOF stabilization	$K=\begin{bmatrix}7.0158 & -4.3414\\ 2.1396 & -4.4660\end{bmatrix}$	$-0.0475 \pm j0.0853$, $-0.7576 \pm j0.7543$, $-29.2613, -33.6825$	Stable	α = -1.9 $\times 10^{-4}$
He and Wang (2006)	SOF stabilization	$K=\begin{bmatrix}0.6828 & 0.2729\\ -0.1024 & -0.0348\end{bmatrix}$	$-1.3274 \pm j4.6317$, $-0.7735, -0.0665$, $-30.6841, -30.0117$	Stable	$\alpha = -0.1330$
Proposed	SOF stabilization	$K=\begin{bmatrix}0.4099 & 0.1906\\ -0.1504 & 0.0610\end{bmatrix}$	$-1.4831 \pm j3.0245$, $-0.7875, -0.1083$, $-31.0626, -30.0006$	Stable	$\alpha = 77.6023$, $\beta = 0.4403$
Zheng et al. (2002)	PID stabilization	$K_1=\begin{bmatrix}10.1359 & -1.7947\\ 6.9912 & -9.4140\end{bmatrix}$, $K_2=\begin{bmatrix}0.3817 & -0.6939\\ 0.6528 & -1.1978\end{bmatrix}$, $K_3=\begin{bmatrix}2.6162 & -1.4722\\ 0.8212 & -1.6284\end{bmatrix}$	$-17.38 \pm j34.73$, $-3.07 \pm j0.46$, $-0.0003, -0.0243$, $-0.0774, -37.0751$	Stable	α =-4.4 $\times 10^{-4}$
He and Wang (2006)	PID stabilization	$K_1=\begin{bmatrix}13.9831 & 2.0458\\ 0.8758 & -1.3309\end{bmatrix}$, $K_2=\begin{bmatrix}0.1710 & -0.3188\\ 0.1550 & -0.2891\end{bmatrix}$, $K_3=\begin{bmatrix}5.7954 & -4.5855\\ 1.1572 & -1.9783\end{bmatrix}$	$-19.4227 \pm j31.7602$, $-49.8699, -4.9964$, $-0.6490, -0.0275$, $-0.0388, -1.3296 \times 10^{-5}$	Stable	α = -4.4 $\times 10^{-4}$
Proposed	PID stabilization	$K_1=\begin{bmatrix}-0.0798 & 0.6675\\ -7.6582 & -0.7689\end{bmatrix}$, $K_2=\begin{bmatrix}0.1291 & -0.1357\\ 0.1152 & -0.1466\end{bmatrix}$, $K_3=\begin{bmatrix}-0.6425 & 1.2466\\ -3.6122 & 3.2701\end{bmatrix}$	$-19.9208 \pm j22.2367$, $-15.2952, -4.8236$, $-0.6799, -0.0043$, $-0.0468, -0.0326$	Stable	$\alpha = 17.2488$, $\beta = 0.4327$
Zheng et al. (2002), Cao et al. (1998b)	SOF (H_∞)	$K=\begin{bmatrix}0.2838 & 0.0313\\ -0.8725 & -0.0289\end{bmatrix}$	$-0.1042 \pm j0.1536$, $-1.6525 \pm j4.3864$, $-30.0027, -31.0378$	$\gamma = 0.863$	α =-4.5 $\times 10^{-2}$
He and Wang (2006)	SOF (H_∞)	$K=\begin{bmatrix}2.3982 & 0.2302\\ -3.3982 & -0.2302\end{bmatrix}$	$-0.0010 \pm j11.6634$, $-0.0547, -0.1260$, $-34.3691, -30.0061$	$\gamma = 0.323$	$\alpha = -0.0020$
Proposed	SOF (H_∞)	$K=\begin{bmatrix}0.1892 & 0.0438\\ -1.1895 & -0.0438\end{bmatrix}$	$-1.5557 \pm j4.8905$, $-0.1145 \pm j0.1284$, $-31.2124, -30.0052$	$\gamma = 0.0870$	α =1.4500 $\times 10^3$, β = -0.0001 $\times 10^3$
Zheng et al. (2002)	PID (H_∞)	$K_1=\begin{bmatrix}442.17 & 221.64\\ -188.84 & -104.44\end{bmatrix}$, $K_2=\begin{bmatrix}0.3845 & -0.5019\\ 0.1068 & -0.3373\end{bmatrix}$, $K_3=\begin{bmatrix}48.03 & -3.85\\ -19.31 & -8.80\end{bmatrix}$	$-0.0001, -0.0020$, $-0.0050, -0.7200$, $-19.9100, -191.000$, $-48.02 \pm j77.79$	$\gamma = 1.003$	α =-4.9 $\times 10^{-4}$
He and Wang (2006)	PID (H_∞)	$K_1=\begin{bmatrix}22.5780 & 5.9056\\ -15.3968 & -4.9824\end{bmatrix}$, $K_2=\begin{bmatrix}39.0276 & 20.4624\\ -24.1171 & -13.3738\end{bmatrix}$, $K_3=\begin{bmatrix}6.2065 & -4.6857\\ -4.1822 & 2.9566\end{bmatrix}$	$-22.9441 \pm j32.1154$, $-35.1083, -9.1967$, $-4.7860, -0.0249$, $-0.0637, -0.6649$	$\gamma = 1.001$	α =-1.3142 $\times 10^{-4}$
Proposed	PID (H_∞)	$K_1=\begin{bmatrix}26.3361 & 6.4134\\ -27.2839 & -6.4000\end{bmatrix}$, $K_2=\begin{bmatrix}-0.0013 & 0.0361\\ 0.0013 & -0.0360\end{bmatrix}$, $K_3=\begin{bmatrix}5.4216 & -2.3951\\ -5.4274 & 2.3991\end{bmatrix}$	$-25.7949 \pm j62.4829$, $-29.8858, -9.5803$, $-0.4392, -0.0223$, $-0.0069, -0.0000$	$\gamma = 1.000$	$\alpha = 69.2078$, $\beta = -0.0132$

He, Y. and Wang, Q.G. (2006). An improved ILMI method for static output feedback control with application to multivariable PID control. *IEEE Transactions on Automatic Control*, 51(10), 1678–1683.

Rubió-Massegú, J., Rossell, J.M., Karimi, H.R., and Palacios-Quinonero, F. (2013). Static output-feedback control under information structure constraints. *Automatica*, 49(1), 313–316.

Sahoo, P.R., Goyal, J.K., Ghosh, S., and Naskar, A.K. (2019). New results on restricted static output feedback H_∞ controller design with regional pole placement. *IET Control Theory & Applications*, 13(8), 1095–1104.

Tan, K.K., Wang, Q.G., and Hang, C.C. (2012). *Advances in PID control*. Springer Science & Business Media.

Zheng, F., Wang, Q.G., and Lee, T.H. (2002). On the design of multivariable PID controllers via LMI approach. *Automatica*, 38(3), 517–526.