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Acceleration waves in non-ideal magnetogasdynamics

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Abstract The problem of propagation of acceleration waves in an unsteady inviscid non-ideal gas under the influence of magnetic field is investigated. The characteristic solution to the problem in the neighbourhood of leading characteristics has been determined. An evolution equation governing the behaviour of acceleration waves has been derived. It is shown that a linear solution in the characteristic plane exhibits non-linear behaviour in physical plane. The effect of magnetic field on the formation of shock in non-ideal gas flow with planar and cylindrical symmetry is analysed. It is noticed that all compressive waves terminate into a shock wave. Further, we also compare/contrast the nature of solution in ideal and non-ideal magnetogasdynamic regime.

1. Introduction

In the unsteady flow of a compressible fluid, a surface may exist, on which the acceleration of the fluid particle has a discontinuity, this is known as acceleration wave. The analysis of these waves has been the subject of great interest both from mathematical and physical point of view due to its applications in a variety of fields such as astrophysics, nuclear science, geophysics, plasma physics and interstellar gas masses where the temperature of the gas is very high and the density is too low. The study of jump discontinuities along characteristic curves has been extensively done during the past decades. Generally we do not have the luxury of complete exact solution for nonlinear system of hyperbolic partial differential equations, for analytical work we have to rely on some approximate analytical or numerical techniques which may be useful to set the view and provide useful information towards our understanding of the complete physical phenomena involved. Since at high temperatures and low density the assumption that the gas is ideal is no longer valid, the popular alternative to the ideal gas is a simplified Van der Waals model. Under these assumptions the electromagnetic effects may also be significant. A complete analysis of such a problem should therefore consist of study of the non-ideal magnetogasdynamics flow. The formulation of the problem in non-ideal magnetogasdynamics involves greater complexities; several investigations yielding qualitative description of the flow field have been performed by people with aeronautical interest, using a number of simplifications concerning the gas properties and the boundary walls-producing wave like disturbances. A number of problems relating to weak nonlinear wave propagation in the non-ideal magnetogasdynamics regime within the context of nonlinear breaking of wave-fronts and its analysis have been investigated by Singh et al. [1–3] and Pandey et al. [4]. Singh et al. [5] applied a perturbation method for the study of propagation of weak shock waves in radiative

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magnetogasdynamics. Anile [6] used a generalized wavefront expansion method for dealing with the propagation of weak shock waves. For moderately weak shock strengths, the complete history of the shock can be determined approximately by means of a method due to Courant and Friedrichs [7], Jeffrey and Taniuti [8] and Whitham [9]. Shankar [10], Keller [11] have used the well known ray theory to study the growth and propagation of shock and shock strength in radiative magnetogasdynamics flow under the assumption that the upstream flow is uniform but not at rest and without any restriction on the downstream flow. Ram [12] analysed the influence of radiative heat transfer on the process of steepening or flattening of acceleration waves and determined the shock formation time and distance. An interesting study of weak non-linear waves in real gas flows has been carried out by Chu [13], using a perturbation technique. Singh et al. [14] have used the method of Lie group analysis to determine an approximate analytical solution for the strong shock wave in a non-ideal gas. Vishwakarma and Nath [15] have determined similarity solutions for the problem of unsteady flow behind an exponential shock in a dusty gas.

In the present paper, we have considered the problem of propagation of a one dimensional unsteady planar and non-planar flow of an inviscid gas in a non-ideal magnetogasdynamics regime. To investigate the problem of propagation of acceleration waves along the characteristic path, the characteristics of the governing system are used as the reference coordinate system. An evolution equation governing the behaviour of acceleration waves has been obtained. It is noticed here that irrespective of their initial strength, all compressive waves terminate into a shock wave for planar and cylindrically symmetric case. Also, we compare the nature of solution in ideal and non-ideal magnetogasdynamic flow.

2. Governing equations and characteristics

The governing equations for one dimensional planar and cylindrically symmetric motion of a non-ideal gas in the presence of axial magnetic field with infinite electrical conductivity may be written in the form [16]

$$\rho_t + v\rho_x + \rho v_x + m\rho v/x = 0, \tag{1}$$

$$v_t + vv_x + \rho^{-1}(p_x + h_x) = 0, \tag{2}$$

$$p_t + vp_x - a^2(\rho_t + v\rho_x) = 0, \tag{3}$$

$$h_t + vh_x - e^2(\rho_t + v\rho_x) = 0. \tag{4}$$

where $a = (\gamma p/\rho(1 - b\rho))^{1/2}$ is the speed of sound, $e = (2h/\rho)^{1/2}$ is Alfvén speed, h the magnetic pressure defined as $h = \mu H^2/2$ with μ as the magnetic permeability and H the transverse magnetic field, p is the pressure, ρ is the gas density, v is the velocity along the x-axis, t the time, x is the single spatial co-ordinate being either axial in flows with planar ($m = 0$) geometry or radial in cylindrically symmetric ($m = 1$) flows and γ is the constant specific heat ratio. Here and throughout, non-numeric subscripts will denote partial differentiation with respect to the indicated variables unless stated otherwise.

The above equations are supplemented with the Van der Waals equation of state [17,1]

$$p(1 - b\rho) = \rho RT, \tag{5}$$

where R is the gas constant and b is the Van der Waals excluded volume which is known in terms of the molecular interaction potential in high temperature gases.

Eqs. (1)–(4) may be written in the matrix form as follows

$$f_t + Af_x + B = 0, \tag{6}$$

where f, B are column vectors and A is matrix of order 4×4, given as

$$f = \begin{bmatrix} \rho \\ v \\ p \\ h \end{bmatrix}, \quad A = \begin{bmatrix} v & \rho & 0 & 0 \\ 0 & v & 1/\rho & 1/\rho \\ 0 & \gamma p & v & 0 \\ 0 & 2h & 0 & v \end{bmatrix}, \quad B = \begin{bmatrix} m\rho v/x \\ 0 \\ m\gamma pv/x \\ 2mvh/x \end{bmatrix}.$$

Eq. (6) is a system of quasilinear hyperbolic partial differential equations having four real characteristics, along which acceleration waves are propagated. Thus the function $f(x, t)$ satisfies (6) everywhere except at a characteristic curve $\Omega(t)$, where $f(x, t)$ is continuous, but f_t and f_x undergo finite jump across $\Omega(t)$, such type of discontinuity is known as "acceleration wave". The jump of f across $\Omega(t)$ is denoted by $[f]$. Thus, we have

$$\frac{\partial}{\partial t}[f] = [f_t] + \frac{d\Omega(t)}{dt}[f_x], \tag{7}$$

where $\partial/\partial t$ represents time-derivative as observed from the wave front.

Taking jump in (6), using (7) and condition of continuity $[f] = 0$, yields

$$\left(A - \frac{d\Omega}{dt}I\right)[f_x] = 0, \tag{8}$$

where $I_{4\times4}$ is an identity matrix.

From Eq. (7), we observe that if there exists a finite discontinuity of acceleration along the characteristic curve, the characteristic speed of propagation $d\Omega/dt$ is the characteristic root of A. Hence the characteristic curves are given as

$$\frac{dx}{dt} = v \pm c, \quad \text{where } c = (a^2 + e^2)^{1/2}, \tag{9}$$

which represent the outgoing and incoming wavelets along the x-axis with a as the effective speed of sound in non-ideal medium and

$$\frac{dx}{dt} = v, \tag{10}$$

is the trajectory of the fluid particle.

3. Characteristic transformation

Now we introduce the characteristic variables ϕ and ξ as follows

I. ξ is a "particle tag" so that ξ is constant along the trajectory of the fluid particle $dx/dt = v$ in the (x, t) plane. The particle and its path will be labelled by $\xi = t'$, if the characteristic wave front traverses a particle at time t'.
II. ϕ is a wave tag so that ϕ is constant along an outgoing characteristics $dx/dt = v + c$ in (x, t) plane, which will be labelled by $\phi = t^*$, if an outgoing wave is generated at time t^*.

It is now clear that for each value of (ϕ, ξ) there is a corresponding pair (x, t) so that $x = x(\phi, \xi)$, $t = t(\phi, \xi)$. Thus we have

$$x_\phi = vt_\phi, \quad x_\xi = (v + c)t_\xi. \tag{11}$$

The above transformation yields

$$f_t = \frac{f_\xi x_\phi - f_\phi x_\xi}{J}, \tag{12}$$

$$f_x = \frac{f_\phi t_\xi - f_\xi t_\phi}{J}, \tag{13}$$

where J is the Jacobian of transformation given as

$$J = \frac{\partial(x,t)}{\partial(\phi,\xi)} = -ct_\phi t_\xi.$$

It is clear from the above relations that $J = 0$ if and only if $t_\phi = 0$, when two adjoining characteristics unify into a shock wave. Since doubling up or overlapping of fluid particles is prohibited from physical considerations, $t_\xi \neq 0$. Hence $J = 0$ gives us condition for the formation of shock wave. Using (12) and (13) in (1)–(4)we get the following relations

$$ct_\xi \rho_\phi - \rho t_\xi v_\phi + \rho v_\xi t_\phi + m\rho vct_\phi t_\xi / x = 0, \tag{14}$$

$$\rho ct_\xi v_\phi - t_\xi p_\phi + p_\xi t_\phi - h_\phi t_\xi + h_\xi t_\phi = 0, \tag{15}$$

$$ct_\xi p_\phi - \gamma pt_\xi v_\phi + \gamma pv_\xi t_\phi + \gamma pmvct_\phi t_\xi / x = 0, \tag{16}$$

$$ch_\phi t_\xi - 2h(v_\phi t_\xi - t_\phi v_\xi) + m2vcht_\phi t_\xi / x = 0. \tag{17}$$

Using (15)–(17)in (14) we have

$$p_\xi + \rho cv_\xi + h_\xi = -m\rho c^2 vt_\xi / x. \tag{18}$$

4. Boundary conditions

The boundary conditions at the shock front are

$$[p] = 0, \quad [\rho] = 0, \quad [v] = 0, \quad [T] = 0, \quad [h] = 0,$$
$$t = \xi \quad \text{at } \phi = 0. \tag{19}$$

Since the flow of the fluid ahead of the shock is homogeneous and at rest. Hence we have

$$p_\xi = 0, \quad \rho_\xi = 0, \quad v_\xi = 0, \quad T_\xi = 0, \quad h_\xi = 0,$$
$$t_\xi = 1, \quad \text{at } \phi = 0. \tag{20}$$

Now using (19) and (20) in Eqs. (15) and (11) we get

$$p_\phi = \rho_0 c_o v_\phi - (2h_0/c_0)v_\phi, \quad \text{at } \phi = 0, \tag{21}$$

$$x_\phi = 0, x_\xi = c_0, \quad \text{at } \phi = 0, \tag{22}$$

where the subscript "0" denotes flow variables associated with the undisturbed medium ahead of the wave. Using (20) in Eq. (13), we get

$$\left[\frac{\partial v}{\partial x}\right] = y = -\frac{v_\phi}{a_0 t_\phi}, \quad \text{at } \phi = 0, \tag{23}$$

where y is the amplitude of acceleration wave.

5. Solution of the problem

Now differentiating (11), (18), and (21) with respect to ϕ and ξ yields

$$\frac{v_{\phi\xi}}{t_\phi} = \frac{mc_0}{2\xi} y \quad \text{at } \phi = 0, \tag{24}$$

$$\frac{t_{\phi\xi}}{t_\phi} = \left[\frac{3}{2} - \frac{(2-\gamma) - 3b\rho_0}{2\varepsilon(1 - b\rho_0)}\right] y, \quad \text{at } \phi = 0. \tag{25}$$

where $\varepsilon = \left(1 + e_0^2/a_0^2\right)$.

On differentiating (23) with respect to ξ and using (24) and (25) we get

$$\frac{dy}{d\xi} + \frac{m}{2\xi} y + \frac{1}{2}\left(3 - \frac{(2-\gamma) - 3\bar{b}}{(1-\bar{b})\varepsilon}\right) y^2 = 0, \quad \text{at } \phi = 0. \tag{26}$$

where $\bar{b} = b\rho_0$.

We introduce the following dimensionless parameters [12]

$$\theta = \frac{y}{y^*}, \quad \eta = \frac{\xi - \xi^*}{2\xi^*}, \quad \omega = y^* \xi^*, \tag{27}$$

where θ, η and ω represent dimensionless parameter of the wave amplitude, dimensionless parameter of time and dimensionless parameter of initial acceleration respectively. The superscript "*" denotes initial wave level.

Now Eq. (26) may be written as

$$\frac{d\theta}{d\eta} + \frac{m}{2(2\eta + 1)}\theta + \frac{\omega}{2}\left(3 - \frac{(2-\gamma) - 3\bar{b}}{(1-\bar{b})\varepsilon}\right)\theta^2 = 0. \tag{28}$$

Eq. (28) may be reduced to a linear form so that its analytical solution is given as

$$\theta = \left\{(1 + 2\eta)^{m/2}\left(1 + \frac{\omega B(\eta)}{2}\left(3 - \frac{(2-\gamma) - 3\bar{b}}{(1-\bar{b})\varepsilon}\right)\right)\right\}^{-1}, \tag{29}$$

where $B(\eta) = \int_0^\eta \frac{1}{(1+2\eta)^{m/2}} d\eta$.

We conclude from Eqs. (23) and (29) that the shock wave will form when t_ϕ vanishes, i.e.

$$1 + \frac{\omega B(\eta)}{2}\left(3 - \frac{(2-\gamma) - 3\bar{b}}{(1-\bar{b})\varepsilon}\right) = 0. \tag{30}$$

Since $B(\eta) \geqslant 0$, the condition (30) shows that only compressive wave fronts ($\omega < 0$) may grow into shock waves.

6. Results and discussion

Eq. (30) shows that in case of ideal gas flows ($\bar{b} = 0$) at $\gamma = 2$, which corresponds to the plasma state, the magnetic field has no effect on the nature of solution and agrees with the results presented by Sharma et al.[18]. It may be noted here that the effect of magnetic field enters into the solution through the parameter ε. Also, $\varepsilon = 1.0$ corresponds to non-magnetic case and $\varepsilon > 1.0$ corresponds to magnetic case. The typical values of parameters are taken as

$\varepsilon = 1.0, 1.5, 1.6, 2.5.$
$\bar{b} = 0.0, 0.2, 0.4, 0.6.$
$\gamma = 1.67.$

For plane case ($m = 0$), the solution curves corresponding to Eq. (29) for compressive waves ($\omega < 0$) is presented in Fig. 1. Here, we observe that all compressive waves, irrespective of their initial strength, terminate into shock waves as in case of ideal non-magnetic case Schmitt [19]. This is in contrast with the corresponding case of ideal radiating gas where one always finds critical amplitude such that any compressive

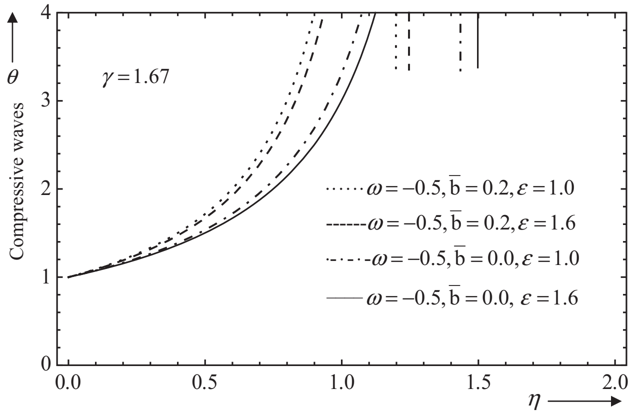

Figure 1 Growth of the compressive waves in ideal and non-ideal magnetogasdynamics with planar symmetry.

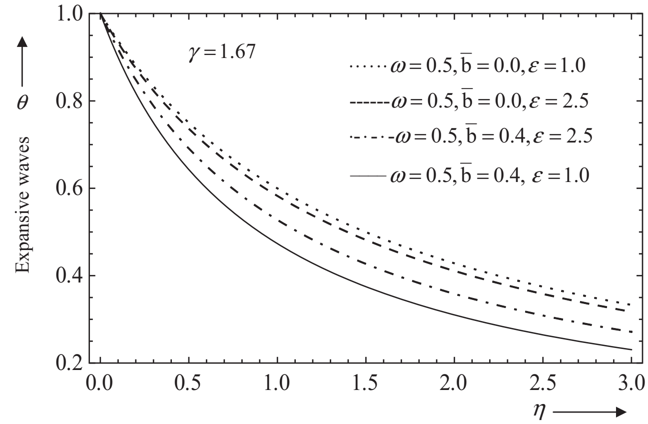

Figure 3 Decay of the expensive waves in ideal and non-ideal magnetogasdynamics with planar symmetry.

disturbance with initial amplitude greater than the critical one always terminates into a shock wave, while initial amplitude less than the critical one always results in a decay of the disturbance Ram [12]. In Fig. 1 the dotted, dashed, dot dashed and thin curves represent the solution in non-ideal, non-ideal magneto, ideal magneto and ideal gas flows respectively and corresponding vertical lines show the position of shock formation. We conclude that the presence of magnetic field in a non-ideal gas is to delay shock formation as compared to non-ideal non-magnetic case but in ideal magnetogasdynamic flow there is an early shock formation in comparison with the corresponding ideal gas flow. Further, the effect of non-idealness of the gas is to accelerate the process of shock formation which is illustrated by Fig. 2. In all cases the amplitude of expansive wave fronts ($\omega > 0$) decays and damp out ultimately. The rate of decay in non-ideal (ideal) gas is accelerated as compared to in a non-ideal magnetogasdynamics (ideal magnetogasdynamics) flow. This nature is exhibited by Fig. 3. It is also expected as the non-idealness of the medium has destabilizing effect on the process of steepening (flattening) of compressive (expansive) waves Wu and Roberts [17].

The solution curves corresponding to cylindrical symmetry ($m = 1$) are shown in Figs. 4–6. The behaviour of solution curves is similar to as in case of plane wave fronts however there is a slight variation in the sense that the process of steepening (flattening) of compressive (expansive) waves is accelerated as compared to plane case. These results are also in close agreement with the results presented by the various authors Ram [12] and Schmitt [19].

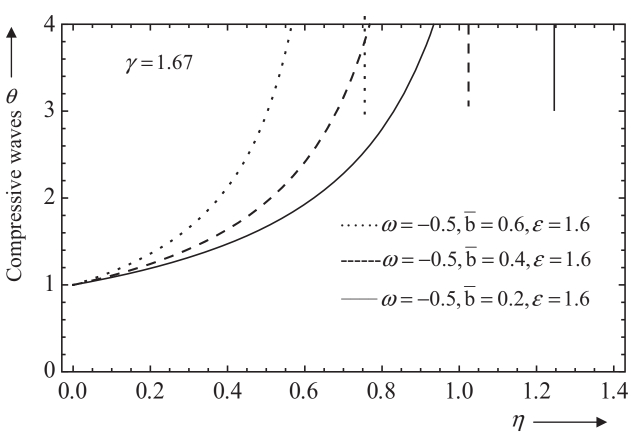

Figure 2 Growth of the compressive waves in non-ideal magnetogasdynamics with planar symmetry.

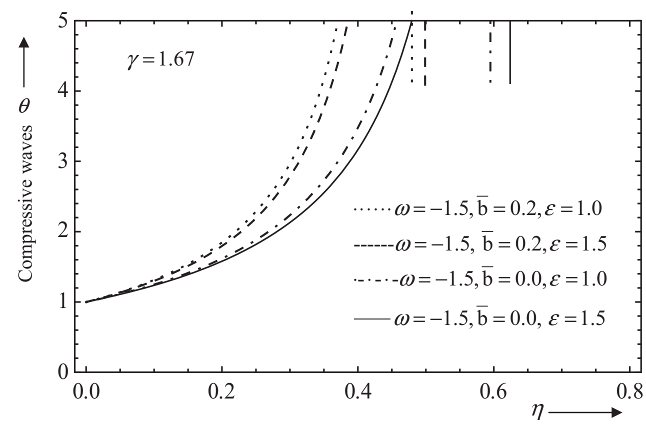

Figure 4 Growth of the compressive waves in ideal and non-ideal magnetogasdynamics with cylindrical symmetry.

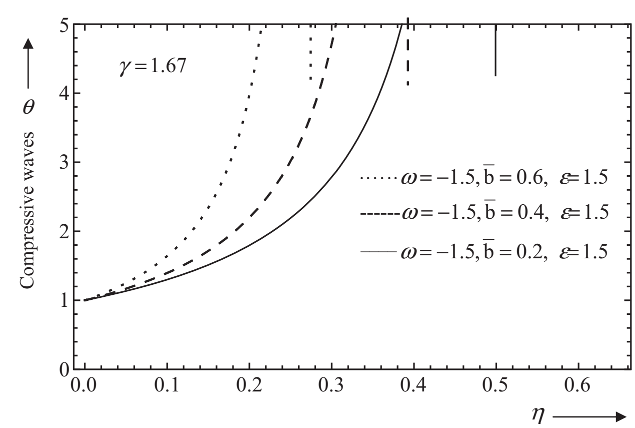

Figure 5 Growth of the compressive waves in non-ideal magnetogasdynamics with cylindrical symmetry.

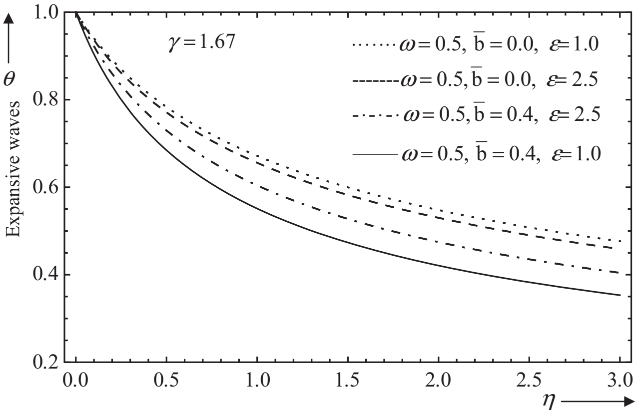

Figure 6 Decay of the expensive waves in ideal and non-ideal magnetogasdynamics with cylindrical symmetry.

7. Conclusion

The growth and decay behaviour of acceleration waves propagating in a plane and cylindrically symmetric motion of a non-ideal magnetogasdynamics regime is examined. It is shown that a linear solution in the characteristics plane may exhibit non-linear behaviour in the physical plane. Transport equations are derived which lead to determination of the first point of breaking into a shock. It is observed that all compressive waves, irrespective of their initial strength, terminate into shock waves. It is also noticed that the presence of magnetic field in a non-ideal gas is to delay shock formation as compared to non-ideal flow but in ideal magnetogasdynamics flow there is an early shock formation in comparison with ideal gas flow. In all cases the amplitude of expansive wave fronts ($\omega > 0$) decays and damp out ultimately. The rate of decay in non-ideal (ideal) gas is accelerated as compared to in a non-ideal magnetogasdynamics (ideal magnetogasdynamics) flow.

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