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Study of Residual Stresses in I Sectioned bars of Non-Linear Work-Hardening Materials under Torsion

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Abstract

This paper deals with the springback problems of I section bars of non-linear work-hardening materials under the torsional loading. Using the deformation theory of plasticity, a numerical scheme based on the finite difference approximation has been proposed. The growth of the elastic-plastic boundary and the resulting stresses while loading, and the springback and the residual stresses after unloading are calculated.

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1. Introduction

In forming operation, springback is an important consideration in designing the punch and die set. During the forming process, when the load is applied, the sheet is deformed plastically and the contour of the sheet section matches that of the die. On removal of the applied load, the sheet section takes up a different shape due to elastic recovery on removal of the applied load is commonly known as springback. Torsional springback is the measure of

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elastic recovery of the angle of twist on removal of the applied torque after twisting the section beyond the elastic limit. So, in designing the die-set, these factors should be taken into consideration to avoid mismatching while assembling different formed sections.

Initially springback studies were limited to sheet bending operations only. Sachs[1], Schroeder[2], Gardiner[3], Singh and Johnson[4] and others studied the springback considering bending of sheets of different shapes, and depicted springback as a function of material thickness, length and width of the sheets taken. Their studies were limited to V and U-shaped dies for applying bending loads and they predicted the springback as a measure of change in the curvature distribution. Huch[5], Nadai[6] and Upadhyay[7], have all done a number of excellent works on the elasto plastic torsion of bars with rectangular sections, but their work has been limited to monotonically increasing loads only. Dwivedi *et al* [8,9] analytically predicted the residual angle of twist and torque relation etc. for bars of elastic strain-hardening materials with narrow rectangular sections. This works, however, has the limitation that it is valid for thin rectangular strips only. Dwivedi *et al* [10,11] dealt with the torsional springback of square-section bars of linear and non linear work-hardening materials. Dwivedi *et al* [12] also dealt with the torsional springback of L-shaped section bars of non linear work-hardening materials.

An accurate analysis of springback has been made in the past on sheet bending and tube bending operations through experiment [13, 14]. Torsional springback in thin tubes with non-linear work hardening analysis by Choubey *et al*[15-17]. Springback analysis of rectangular sectioned bar of non linear work- hardening materials under torsional loading analysis by Lal *el al.* [18].Sharad *et. al.* [19] predicted the springback by using finite element analysis and Srinivas*et. al* [20]review the severe plastic deformation.

In the following, a numerical scheme has been prepared for analysing the problem of springback and elastic-plastic boundary in I-sectioned bars of non linear work-hardening materials under torsional loading. Work-hardening behaviour has been considered by assuming a Ramberg-Osgood type of stress-strain relation.

2. Basic Theory

2.1 Elastic Torsion

Consider a prismatic bar under elastic torsion [21]. Let u, v and w be the small displacements of a point (x,y,z) relative to its initial position, in the X-,Y-, Z- directions respectively. At a section z = constant, the cross-section rotates about the Z axis, and so

 $u = -yz\theta$, $v = xz\theta$ and $w = \theta f(x, y)$ (1)

where θ is the angle of twist per unit length. For elastic deformation, θ is small and is constant along the length of bar; $\theta f(x, y)$ is called the warping function and is assumed to be independent of z. It then follows that the only non-zero components of strain are the shear strains.

$$\gamma_{xz} = -\Theta y + \frac{\partial w}{\partial x} , \quad \gamma_{yz} = \Theta x + \frac{\partial w}{\partial y} ,$$
 (2)

The equilibrium equations reduce to single equations in τ_{xz} and τ_{yz} which are satisfied identically when these stresses are given in terms of the stress function (ψ) such that

$$\tau_{xz} = \frac{\partial \psi}{\partial y}, \quad \tau_{yz} = -\frac{\partial \psi}{\partial x},$$
(3)

After elimination of the warping function from eqn. (2), we get

$$\frac{\partial \tau_{xz}}{\partial y} - \frac{\partial \tau_{yz}}{\partial x} = -2G\theta \tag{4}$$

Once the appropriate constitutive equation, relating the stresses and strains is chosen, then eqn. (4) in conjunction with eqn. (3) will give the equation for the stress function ψ . For elastic torsion $\tau_{xz} = G\gamma_{xz}$ and $\tau_{yz} = G\gamma_{yz}$, so that eqn. (4) reduces to

$$\begin{split} &\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = -2G\theta \\ &\nabla^2\psi = -2G\theta \end{split} \tag{5}$$

with $\psi = a$ constant (taken to be zero) along the boundary of the cross-section.

The torque T is given by

$$T = 2 \iint_{A} \psi \, dx \, dy,$$
 (6)

Where A is the cross-section of the bar.

2.2 Plastic Torsion

It has been assumed that the basic assumptions in the theory of elastic torsion are still valid for the elastioplastic case except for the constitutive equations which are different. To derive the equations for plastic torsion, a generalized form of the Ramberg-Osgood stress-strain law has been used for the deformation theory of Nadai.

If the stresses are non-dimensionalized by the yield stress, σ_y and the strain by the corresponding yield strain $\varepsilon_y = \sigma_y/E$, then a generalized form of the constitutive equation is given by

$$\overline{\varepsilon}_{ij} = (1+\nu)\overline{S}_{ij} + \frac{3}{2}\alpha(\overline{\sigma}_e)^{n-1}\overline{S}_{ij} + \left(\frac{1-2\nu}{3}\right)\overline{\sigma}_{kk}\delta_{ij}$$
⁽⁷⁾

where $\overline{\mathbf{E}}_{ij}$ and $\overline{\mathbf{S}}_{ij}$ are respectively the normalized strain and non-dimensional deviatoric stress components, $\overline{\sigma}_{ij} = \sigma_{ij} / \sigma_y$ and $\alpha = 0.02$ corresponding to the usual engineering definition of yield. Using the von Mises criterion, the yielding of the material is characterized by $\overline{\sigma}_e = 1$

For uniaxial tension, equation (7) is reduces to

$$\mathcal{E} = \frac{\sigma}{E} \left[1 + \left(\frac{\sigma}{\sigma_v}\right)^{n-1} \right] \tag{8}$$

Whose variation is shown in Fig. 1 for different values of the strain-hardening index (n).

In this work, only the simplest deformation theory has been employed. The plastic material has been considered to be incompressible and viscosity effects have been neglected. For moderate strain rates both assumptions are well justified for structural metals.

Since only two components of the stress, namely τ_{xz} , τ_{yz} have been assumed to non-zero, the first invariant of the stress.

 $I=\sigma_{kk}=0$

Substituting zero for σ_{kk} in equation (7) gives



Fig. 1 Ramberg-Osgood Stress Strain curve.

Plastic deformation is assumed to be independent of the hydrostatic component of the stress $1/3\sigma_{kk}$ and is also assumed to be completely determined by the second invariant of the stress deviator,

$$S_{ij} = \sigma_{ij} - \frac{1}{3} (\sigma_{kk} \, \delta_{ij}) \tag{10}$$

The invariant has been introduced in the form of the effective stress defined by

$$\bar{\sigma}_e^2 = \frac{3}{2} \bar{S}_{ij} \cdot \bar{S}_{ij} \tag{11}$$

Replacing Sij by σ ij from (9) and expanding equation (11), the value of the effective stress comes out to be

$$\bar{\sigma}_{e} = \sqrt{3} [\tau_{xz}^{2} + \tau_{yz}^{2}]^{1/2} \tag{6}$$

Assuming a stress function ψ exists such that

$$\overline{\tau}_{xz} = \frac{\psi_y}{\sigma_y}; \overline{\tau}_{yz} = -\frac{\psi_x}{\sigma_y}$$
(13)

the expression for $\bar{\sigma}_e$ reduced to

$$\bar{\sigma}_{e} = \frac{\sqrt{3}}{\sigma_{y}} [\psi_{x}^{2} + \psi_{y}^{2}]^{1/2}$$
(14)

The generalized stress-strain relation given by equation (9) gives

$$\overline{\varepsilon}_{xz} = \left\{ (1+v) + \frac{3}{2} \alpha (\overline{\sigma}_e)^{n-1} \right\} \overline{\tau}_{xz}
\overline{\varepsilon}_{yz} = \left\{ (1+v) + \frac{3}{2} \alpha (\overline{\sigma}_e)^{n-1} \right\} \overline{\tau}_{yz}$$
(15)

which on substituting the values of the stress components $\overline{\tau}_{xz}$ and $\overline{\tau}_{yz}$ from equation (13), reduces to

$$\overline{\varepsilon}_{xz} = \left\{ (1+\nu) + \frac{3}{2} \alpha (\overline{\sigma}_e)^{n-1} \right\} \psi_y / \sigma_y$$

$$\overline{\varepsilon}_{yz} = -\left\{ (1+\nu) + \frac{3}{2} \alpha (\overline{\sigma}_e)^{n-1} \right\} \psi_x / \sigma_y$$
(16)

Next, by expressing the strain components in terms of the displacement and its derivatives, eqn. (2) and eqn. (16) gives

$$\frac{1}{2} \left(\frac{\partial w}{\partial x} - \theta y \right) E = \left\{ (1+v) + \frac{3}{2} \alpha (\bar{\sigma}_e)^{n-1} \right\} \psi_y$$
(17)

and

$$\frac{1}{2}\left(\frac{\partial w}{\partial y} + \theta x\right) E = -\left\{ (1+v) + \frac{3}{2}\alpha (\bar{\sigma}_e)^{n-1} \right\} \psi_x$$
(18)

Differentiating equation (17) with respect to y and equation (18) with respect to x and subtracting, we get

$$-\theta E = \left\{ (1+\nu) + \frac{3}{2} \alpha (2-n) (\bar{\sigma}_e)^{n-1} \right\} \nabla^2 \psi$$

$$+ \frac{3}{2} \alpha (n-1) (\bar{\sigma}_e)^{n-2} \left\{ (\bar{\sigma}_e \psi_x)_x + (\bar{\sigma}_e \psi_y)_y \right\}$$
(19)

This is the final form of the differential equation which is supposed to govern torsion in the plastic region, and is analogous to equation (5) for the elastic torsion.

Unloading of the bar from a plastic stage corresponding to torque T_p is equivalent to applying the torque T_p in the opposite direction to that of twisting. The untwisting or unloading being elastic, the stress-strain curve during unloading is a straight line parallel to the elastic loading line.

From Fig. 2, the amount of spring back in twist is given by

$$\theta_S = \theta_P - \theta_R$$

Since the slopes of the elastic loading line (AB) and that of the unloading line (XY) are the same, hence

12)

(20)

$$\theta_S = \frac{T_P}{T_O} \theta_0 \tag{21}$$

Corresponding to this recovered angle of twist, shear stresses $(\tau_{xz})_s$ and $(\tau_{yz})_s$ are calculated. From eq. (5)



Fig. 2 Loading-Unloading curve

$$\frac{\partial^2(\psi_S)}{\partial x^2} + \frac{\partial^2(\psi_S)}{\partial y^2} = -2G\theta_S$$
(22)

If the stress function ψ_P corresponding to the plastic twisting θ_p at point X (Figure 2) be known by solving equation (19), then after unloading, the resulting ψ surface on the cross-section is given by

$$\psi_{R} = \psi_{P} - \psi_{S}$$
 (23)
and residual shear stress, $(\tau_{ii})_{R}$, are given by

$$\partial \psi_R$$
 $\partial \psi_R$

$$(\tau_{xz})_R = \frac{\partial \varphi_R}{\partial y}, (\tau_{yz})_R = -\frac{\partial \varphi_R}{\partial x}$$
(24)

In a non-dimensionalized form, relations (24) become

$$(\bar{\tau}_{xz})_R = \frac{(\tau_{xz})_R}{\sigma_y}, (\bar{\tau}_{yz})_R = \frac{(\tau_{yz})_R}{\sigma_y}$$
(25)

The residual torque T_R is determined with the help of

$$T_R = 2 \iint_A (\psi_R) dx \, dy \tag{26}$$

This should be zero in the case of loading within elastic limit.

3. Scheme of numerical solution

Before coming to a numerical solution, coordinates are non-dimensionalized as

$$\xi = \frac{x}{L}, \eta = \frac{y}{L} \tag{27}$$

where L is a characteristic length of the prismatic bar. Further, a new stress function ϕ defined by

$$\phi(\xi,\eta) = 1/L^2(\psi(x,y)) \tag{28}$$

On replacing the coordinates and stress function by non-dimensionalized coordinate and ϕ , equation (5) and equation (19) reduces to

$$\nabla^2 \phi = -2G\theta \tag{29}$$

(elastic deformation)

and

$$-\theta E = \{(1+\nu) + \frac{3}{2}\alpha(2-n)(\bar{\sigma}_e)^{n-1}\}\nabla^2\phi + \frac{3}{2}\alpha(n-1)(\bar{\sigma}_e)^{n-2}\{(\bar{\sigma}_e\phi_\xi)_\xi + (\bar{\sigma}_e\phi_\eta)_\eta\}$$
(30)

(Plastic deformation)

Stress components are given by $\tau_{xz} = L\phi_{\eta}, \tau_{yz} = L\phi_{\xi}$



Fig.3 Definition of grid system

and therefore

$$\overline{\sigma}_{e} = \frac{\sqrt{3}}{\sigma_{y}} L[(\phi_{\xi})^{2} + (\phi_{\eta})^{2}]^{1/2}$$
(32)

Equation (6), which gives the value of the torque becomes

5

$$T = 2L^4 \iint_A(\phi) d\xi \, d\eta \tag{33}$$

For the solution of the elasto-plastic problem, the finite difference method has been used. This technique leads to the replacement of differential equations by a system of difference equations which are a set of simultaneous algebraic equations involving the unknown variables at nodes only. The solution of these simultaneous equations gives the numerical values of the unknowns at these nodal points. The central difference operator is used here for writing the difference equations.

Let O be one of the typical nodes (Fig. 3) at which the difference equation has to be satisfied and 1, 2, 3 and 4 be the neighboring nodes around O. Then by the finite difference approximation, the differential equation (29), for the elastic torsion, can be written at point O as algebraic equations

$$\nabla^2 \phi_0 = -2\left[\frac{1}{h_1 h_3} + \frac{1}{h_2 h_{41}}\right] \phi + \frac{2}{h_1 + h_3} \left[\frac{\phi_1}{h_1} + \frac{\phi_3}{h_3}\right] + \frac{2}{h_2 + h_4} \left[\frac{\phi_2}{h_2} + \frac{\phi_4}{h_4}\right] + \text{error term O(h)}^2$$

= - 2G0 (34)

Where h_1 , h_2 , h_3 and h_4 are the distance of the neighboring nodes 1, 2, 3 and 4 from O respectively. If the neighboring points are equal distance from the point O, say h, equation (34) can further be reduced after neglecting second order term O (h^2) to

$$4\phi_0 - \phi_1 - \phi_2 - \phi_3 - \phi_4 = 2G\theta h^2$$
(35)

The differential equation for plastic torsion equation (30) is written as

$$\begin{cases} (1+\nu) + \frac{3}{2}\alpha(2-n)(\chi_{0})^{n-1} \\ \left[-2\{\frac{1}{h_{1}h_{3}} + \frac{1}{h_{2}h_{4}}\}\phi_{0} + \frac{2}{h_{1}+h_{3}}\{\frac{\phi_{1}}{h_{1}} + \frac{\phi_{3}}{h_{3}}\} + \frac{2}{h_{2}+h_{4}}\{\frac{\phi_{2}}{h_{2}} + \frac{\phi_{4}}{h_{4}}\} \\ + \frac{3}{2}\alpha(n-1)(\chi_{0})^{n-2}[\frac{\chi_{0}+\chi_{1}}{h_{1}(h_{1}+h_{3})}\phi_{1} + \frac{\chi_{0}+\chi_{2}}{h_{2}(h_{2}+h_{4})}\phi_{2} + \frac{\chi_{0}+\chi_{3}}{h_{3}(h_{3}+h_{1})}\phi_{3} + \frac{\chi_{0}+\chi_{4}}{h_{4}(h_{4}+h_{2})}\phi_{4}] \\ - \{\chi_{0}(\frac{1}{h_{1}h_{3}} + \frac{1}{h_{2}h_{4}}) + \frac{\chi_{1}}{h_{1}(h_{1}+h_{3})} + \frac{\chi_{2}}{h_{2}(h_{2}+h_{3})} + \frac{\chi_{3}}{h_{3}(h_{3}+h_{1})} + \frac{\chi_{4}}{h_{4}(h_{4}+h_{2})}\}\phi_{0}] = -\theta E \end{cases}$$

$$(36)$$

4. Results and Discussion

(31)

Numerical scheme is based on finite difference approximation, it is necessary to decide upon an approximate mesh size which will give a solution converging to the actual one. For the I- section bar, the solution was obtained using different mesh size and mesh size h=1/16 was an optimum choice from point of view of accuracy and computational time. This case will be analyzed in detail for different values of angle of twist and work hardening index and result will be presented in non-dimensional form in Figs 3-11.

In figure-4, Elasto-plastic boundaries of an I-section is shown for fixed value of nwith $\hat{\theta}_p$ as parameter, it is clear from figure-4 that elasto-plastic boundaries increases as the angle of twist increases. As it can be shown from the figure that at the points where the web is connected to the flanges, the stress concentration takes place so the elasto-plastic boundaries starts earlier.

Figure-5 is drawn mainly to show the variation of equivalent stress ($\overline{\sigma}_e$) along a line LP of an I-section for

fixed values of n ,taking $\bar{\theta}_p$ as parameter .In loading ,the equivalent stress ($\bar{\sigma}_e$) increases becomes maximum and again becomes equivalent to zero then again increases rapidly due to stress concentration near the corners . Similar behaviour is obtained for unloading case.

Figure- 6 shows the variation of equivalent stress ($\bar{\sigma}_e$) along the line LP of an I- cross- section for fixed

value of $(\bar{\theta}_p) = 2.5$ and taking n (n=7,9,12,15) as the parameter.

Figure-7 shows the variations of equivalent stress $\bar{\sigma}_e$ for a fixed value of n =9 and taking $\bar{\theta}_p$ ($\bar{\theta}_p$ =1.5, 2.0,

2.5, 3.0) as the parameter along the edge KN. It can be seen from fig 7 equivalent stress σ_e becomes maximum almost at the middle of the edge KN.

Figure-8 shows the pattern of variation of equivalent stress $\bar{\sigma}_e$ for a fixed value of $\bar{\theta}_p$ =2.5 and taking n (n=7,9,12,15) as the parameter along the edge KN.

Figure-9 shows the variation of stress function ϕ along the line LP of I -cross section for a fixed value of n=9 and taking $\bar{\theta}_p$ ($\bar{\theta}_p$ =1.5, 2.0, 2.5, 3.0) as the parameter . It is noted that the value of stress function becomes zero at boundary.

Figure-10 shows the variation of stress function ϕ along the line LP of I -cross section for a fixed value of $\bar{\theta}_p$ =2.5 and taking n (n=7,9,12,15) as the parameter. In figure- 11, Spring back/ Residual Vs angle of twist with n as the parameter is shown.



Fig 4 -Elastic-Plastic boundaries of a I-section for n=9 with θ_p as the parameter



Fig 5-Equivalent stress $\bar{\sigma_e}$ along the LP of a I-section for n=9



Fig 6- Equivalent stress $\bar{\sigma}_e$ along LP of a I-cross section $\bar{\theta}_p$ =2.5



Fig 7- equivalent stress of $\overline{\sigma}_e$ along edge KN for n=9



Fig 8- Equivalent stress $\bar{\sigma}_e$ along the edge KN for $\bar{\theta}_p$ =2.5



Fig 9- stress function ϕ along the LP for n=9



Fig 10- stress function along the LP for $\bar{\theta}_p = 2.5$



Fig 11- Spring back/ Residual Vs angle of twist with n as the parameter

5. Conclusion

The proposed numerical scheme is found to predict the torsional springback quite successfully .This is supported by the excellent agreement with the theoretical results. The accuracy of the theoretical results, of course, depends on the mesh size. The non linear strain hardening index (n) has little effect on the elasto-plastic boundary. It means that even If one commits some error in determining the value of n the elasto-plastic boundaries are hardly affected. The method can be used for any other cross-section such as elliptical, triangular, etc.

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