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Numerical Study on non-Fourier Bio heat Transfer during Thermal Ablation

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Abstract

In this paper, a mathematical model describing the process of non-Fourier heat transfer in biological tissues for generalized coordinate system during thermal ablation by electromagnetic radiation with Gaussian external heat source is studied. The boundary value problem governing this process has been solved using hybrid numerical method taking Legendre wavelet basis function. The result obtained from finite element Legendre wavelet Galerkin method (FELWGM) is compared with exact analytical solution and shows a good agreement. The effect of variability of time, generalized coordinate system, location of tumor or cancer, relaxation time and external heat source coefficient on temperature distribution is discussed in detail. It has been observed that treatment of tumor or cancerous cell is independent of the generalized coordinate system at the thermal ablation position.

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Keywords: Non-Fourier; Thermal ablation; Bioheat transfer; Finite element Legendre wavelet Galerkin method.

1. Introduction

The term tumor ablation is defined as the direct application of thermal therapies to a specific tumor in an attempt to achieve abolition of substantial tumor destruction [1]. The techniques of tumor ablation most commonly used in current practice are divided into two categories, namely, hyperthermic ablation, in which heat may be generated by ultrasound or electromagnetic energy (i.e. radiofrequency, microwave, laser), and cryoablation to destroy tumor by freezing it.

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Thermal ablation has many roles in the palliative treatment of benign and malignant tumors. Cell death is a result of irreversible coagulation of proteins, including enzymes, intranuclear proteins, DNA proteins, and DNA [2]. Radiofrequency (RF), microwave (MW), laser, and ultrasound (US) ablate tissue by heating tissue to cytotoxic temperatures. The last few years have seen a rapid expansion in the use and availability of thermal ablation techniques with a lot of papers published. They studied the different type thermal ablation technique such as radiofrequency, microwave ablation, ultrasound ablation. Temperatures in excess of 60°C are known to cause relatively instantaneous cell death, while temperature from 50 to 60°C will induce coagulation and cell death in a matter of minutes [3]. Successful thermal ablation treatment of tumors requires understanding the attendant thermal process in both diseased and healthy tissues.

In 1948, Pennes [4] studied the temperature distribution in the forearm skin temperature, which means that the equation is amenable to analysis by various methods commonly used to solve the heat transfer model for infinite heat propagation, which is based on heat conduction of classical Fourier i.e.

$$q(r,t) = -k\nabla T(r,t),$$

where q and ∇T are heat flux and temperature gradient in same instant time and space.

In fact, heat is always found to propagate with a finite speed within living biological tissues as they have highly non-homogeneous inner structure. To solve the paradox occurred in the Pennes bio-heat equation, thermal wave model of bio-heat transfer is introduced which is based on thermal wave constitutive relation given as follows: [5, 6]

$$q(r,t+\tau_q) = -k\nabla T(r,t),$$

where τ_q is thermal lagging time. q and ∇T at the same point (r) are in different instant of time. As the value of τ_q tends to zero, thermal wave model reduces to Pennes model. Thermal wave model gives a suitable required thermal data for describing temperature distribution in living biological tissue.

Modified Pennes bio-heat equation is solved using different type of numerical methods which are available in literature. The finite-decomposition method [7], homotopy perturbation method [8], Galerkin approach with variation iteration method [7, 8] and finite element Legendre wavelet Galerkin method [9,10]. Kumar et al. [11] solved analytic solution of both parabolic and hyperbolic bio-heat transfer equation for finite domain. Steady state bioheat equation is solved by Legendre wavelet collocation method [12].

In the present study, we have obtained the solution of the hyperbolic (thermal wave) bio-heat transfer equation under metabolic and modified Gaussian external heat source. The hybrid numerical method (FELWGM) is used to solve the thermal wave model. The approximate solution is verified with exact analytical solution. Semi discrtizing in space co-ordinate, the problem is converted into a system of second order differential equation (S.O.D.E's) with initial conditions. This system of S.O.D.E's in unknown variables has been also solved by wavelet Galerkin approach with Legendre wavelet as basis function. Thus, our problem reduces into Sylvester matrix equation. Solution of this Sylvester matrix equation gives temperature distribution inside living biological tissue during thermal ablation.

2. Mathematical formulation of the problem

In thermal ablation, the body tissue, which is initially at a constant temperature T_0 (= 37°C) is heated by electromagnetic radiation with modified Gaussian external heat source using high power antenna. During heating process the surface of the tissue is always maintained at normal wall temperature. The generalized coordinate system is considered to study the temperature distribution in the tissue during thermal ablation. In this study, we used hyperbolic bioheat transfer equation under the above assumptions; the differential equation governing the process of heat transfer in the tissue can be expressed as

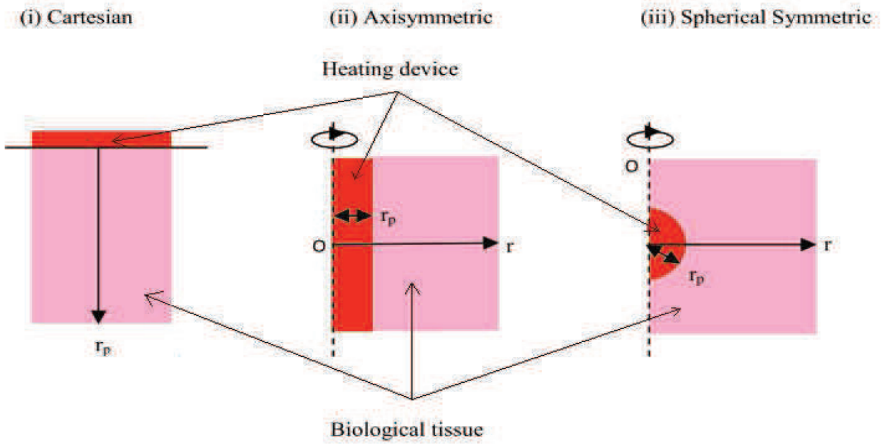

Fig.1 Schematic geometry of one dimensional generalised coordinate system [7, 12].

$$\left(1+\tau_q\frac{\partial}{\partial t}\right)\left(\rho c\frac{\partial T(r,t)}{\partial t}-Q_b-Q_m-Q_s\right)=\frac{k}{r^{\Gamma}}\frac{\partial}{\partial r}\left(r^{\Gamma}\frac{\partial T(r,t)}{\partial t}\right),0<r<L,t>0 \tag{1}$$

with initial conditions

$$T(r,0)=T_0 \quad \text{and} \quad \frac{\partial T(r,0)}{\partial t}=0\,, \tag{2}$$

boundary conditions

$$T(0,t)=T_w \tag{3}$$

and symmetric condition

$$\frac{\partial T(L,t)}{\partial r}=0\,, \tag{4}$$

where T is the local tissue temperature; r, the space coordinate; t, the time; ρ, the density; c, the specific heat; Γ is the number to classify coordinates i.e., $\Gamma = 0, 1, 2$ to indicate cartesian, axisymmetric and spherical symmetric coordinates, respectively [7, 12].

The term Q_b *r*epresents heat source due to blood perfusion, it may be expressed as [7, 8]

$$Q_b=\omega_b\rho_b c_b\left(T_a-T\right),$$

where T_a is a constant arterial blood temperature.

For more practical purposes, metabolic heat generation can be approximated as a linear function of local tissue temperature as observe by Mitchel et al., which is given as follows [8]

$$Q_m=Q_{m0}\left(1+\frac{T-T_0}{10}\right),$$

where Q_{m0} is constant metabolic heat source.

Heat transfer in tissues when external heat source is taken as a modified Gaussian heat source [9] i.e.

$$\mathrm{Q_s}\left(\mathrm{r,t}\right)=Q_{r0}\,\mathrm{e}^{-a_0^2(r-r_0)^2},$$

where $Q_{r0}=\rho SP$, S is antenna power; P is transmitted power, a_0 is the scattering parameter and r_0 is the location of tumor, where the heating power is applied.

3. Solution of the problem

Introducing the non-dimensional variable and similarity criteria

$x = \frac{r}{L}, x_0 = \frac{r_0}{L}, \theta = \frac{T-T_0}{T_0}, \theta_b = \frac{T_b-T_0}{T_0}, P_f = \sqrt{\frac{W_b c_b}{k} L^2}, P_{m0} = \frac{Q_{m0}}{T_0 k} L^2, P_{r0} = \frac{Q_{r0} L^2}{T_0 k}, \theta_w = \frac{T_w-T_0}{T_0},$

$\theta_a = \frac{T_a-T_0}{T_0}, F_o = \frac{k\,t}{\rho c L^2}, F_{oq} = \frac{k\,\tau_q}{\rho c L^2}, a = a_0 \times L,\ d = 0.1 \times T_0$. (5)

The Eqs.(1) - (4) reduce to the following form

$$F_{oq}\frac{\partial^2\theta}{\partial F_o^2} + \left(1 + F_{oq}P_f^2 - F_{oq}P_{m0}d\right)\frac{\partial\theta}{\partial F_o} - \left(P_{m0}d - P_f^2\right)\theta = \frac{\partial^2\theta}{\partial x^2} + \frac{\Gamma}{x}\frac{\partial\theta}{\partial x} + P_f^2\theta_a + P_m \quad (6)$$

$+P_{r0}e^{-a^2(x-x_0)^2}, 0 < x < 1, F_o > 0,$

$\theta(x,0) = 0$, and $\frac{\partial\theta(x,0)}{\partial F_o} = 0$, (7)

$\theta(0, F_o) = \theta_w$, (8)

$\frac{\partial\theta(1, F_o)}{\partial x} = 0$. (9)

Firstly, we descrtized Eqs. (6) - (9) in space coordinate; the equations are converted into system of second order ordinary differential equation with initial conditions i.e.

$$F_{oq}\frac{d^2\theta(F_o)}{dF_o^2} + \left(1 + F_{oq}P_f^2 - F_{oq}P_{m0}d\right)\frac{d\theta(F_o)}{dF_o} - A\theta(F_o) = B, \quad (10)$$

$\theta(0) = 0$ and $\frac{d\theta(0)}{dF_o} = 0$, (11)

where $\theta(F_o)$ and B are $n\times 1$ matrix; A be the tridigonal matrix of order n.

3.1 Legendre Wavelet Galerkin Method:

Let us assume that

$$\frac{d^2\theta(F_o)}{dF_o^2} = C^T\psi(F_o)\,, \quad (12)$$

where $C^T\psi(\mathrm{F_o}) = \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\,\psi_{\mathrm{n,m}}(\mathrm{F_o})$ and (13)

$c_{n,m} = \int_0^1 f(\mathrm{F_o})\psi_{\mathrm{n,m}}(\mathrm{F_o})\mathrm{dF_o}$. (14)

C and $\psi(\mathrm{F_o})$ are $2^{k-1}M \times 1$ matrices given by

$C = [c_{1,0}, c_{1,1}, \ldots, c_{1,M-1}, c_{2,0}, c_{2,1}\ldots, c_{2,M-1}, c_{2^{k-1},0}c_{2^{k-1},1}, \ldots, c_{2^{k-1},M-1}]^T$ and (15)

$\psi(\mathrm{F_o}) = [\psi_{1,0}(\mathrm{F_o}), \psi_{1,1}(\mathrm{F_o}), \ldots, \psi_{1,M-1}(\mathrm{F_o}), \psi_{2,0}(\mathrm{F_o}), \ldots, \psi_{2,M-1}(\mathrm{F_o}), \ldots, \psi_{2^{k-1},0}(\mathrm{F_o})$

$\psi_{2^{k-1},1}(\mathrm{F_o}), \ldots, \psi_{2^{k-1},M-1}(\mathrm{F_o})]^T$. (16)

The Legendre wavelets $\psi_{\mathrm{n,m}}(\mathrm{F_o}) = \psi(k, \hat{n}, m, \mathrm{F_o}), \hat{n} = 2n-1,$ n = 1, 2,…, 2^{k-1} , k is any positive integer, m is the order of Legendre polynomials and '$\mathrm{F_o}$' is defined on the interval [0, 1) by

$$\psi_{\mathrm{n,m}}(\mathrm{F_o}) = \begin{cases} \sqrt{(m+1/2)}\,2^{\frac{k}{2}}P_m(2^k\mathrm{F_o} - \hat{n}), & \frac{\hat{n}-1}{2^k} \le \mathrm{F_o} < \frac{\hat{n}+1}{2^k} \\ 0\,, & otherwise \end{cases}, \quad (17)$$

where m = 0, 1, …, M -1 and n = 1, 2,…, 2^{k-1}. Here $P_m(F_o)$ is the well known Legendre polynomials i.e. $P_0(F_o) = 1,\ P_1(F_o) = F_o,\ P_{m+1}(F_o) = \frac{2m+1}{m+1}F_o\,P_m(F_o) - \frac{m}{m+1}\,P_{m-1}(F_o)$ *m =1, 2, 3,..., M-1.* (18)

Integrating Eq. (12) with respect to F_o from 0 to F_o, we have

$$\frac{d\theta(F_o)}{dF_o} = C^T P\psi(F_o)\,, \quad (19)$$

where P is $2^{k-1}M \times 2^{k-1}M$, operational matrix of integration given in ([13])

$$P=\frac{1}{2^k}\begin{bmatrix} 1 & \frac{1}{\sqrt{3}} & 0 & 0 & & 0 & 0 \\ \frac{-1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{15}} & 0 & \cdots & 0 & 0 \\ 0 & \frac{-1}{\sqrt{15}} & 0 & \frac{1}{\sqrt{35}} & & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{35}} & 0 & & & \\ & & \vdots & & \ddots & \vdots & \\ 0 & 0 & 0 & 0 & & 0 & \frac{\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \\ 0 & 0 & 0 & 0 & \cdots & & \\ & & & & & \frac{-\sqrt{2M-1}}{(2M-1)\sqrt{2M-3}} & 0 \end{bmatrix}. \quad (20)$$

Again integrating Eq. (19) with respect to F_o from 0 to F_o, we have

$$\theta(F_o) = C^T P^2 \psi(F_o). \quad (21)$$

Substituting $\frac{d^2\theta(F_o)}{dF_o^2}$, $\frac{d\theta(F_o)}{dF_o}$ and $\theta(F_o)$ in Eq. (10), we get

$$A\,C^T B^T - C^T + D = 0, \quad (22)$$

where $B = \left(P^2 \left(F_{oq} I_n + \left(1 + F_{oq} P_f^2 - F_{oq} P_{m0} d\right) P - (P_{m0} d - P_f^2) P^2 \right)^{-1} \right)^T$,

$\mathrm{D}=\mathrm{M}_1^{-1}\mathrm{B}\mathrm{d}_1^T \left(F_{oq} I_n + \left(1 + F_{oq} P_f^2 - F_{oq} P_{m0} d\right) P - (P_f^2 + P_{m0} d) P^2 \right)^{-1}$,

where I_n is identity matrix of order n and d_1 =*[1;0;0;0;0;0;...n-times]*. Solving Sylvester matrix Eq. (22), we obtain value of *C*. Substituting the values of *C* in equation (21), then it gives required tissue temperature.

4. Numerical computation and analysis

In this work, the temperature distribution in biological tissues obtained from thermal wave model is studied under modified external heat source as a Gaussian heat source and the most generalized coordinate system. We are also taking temperature dependent metabolic heat source and blood perfusion in tissue. The short term heating (4-6 min.) process is known as thermal ablation. During this therapy, the tumors are killed immediately. The computation has been made using MATLAB (R2012a) software and results are presented graphically. Only the parameters whose values different from reference value are indicated. The selected reference values for temperature distribution in living biological tissue in finite domain are taken as follows:

$\rho = 1000\ kg\ m^{-3}, C = 4.18 \times 10^3 J\ kg^{-1}\ K^{-1}, T_0 = 37\ ℃$, $T_w = 37\ ℃$, $L = 0.05\ m$, $T_a = 37\ ℃$, $L = 0.05\ m$, $Q_{r0} = 7.85 \times 10^5\ \mathrm{W\ m^{-3}}$, $Q_{m0} = 1.091 \times 10^3 \mathrm{W\ m^{-3}}$, $k = 0.5\ W\ m^{-1}$, $W_b = 0.5\ kg\ m^{-3}$, $a_0 = 1 \times 10^{-5}\ m^{-1}$, $r_0 = 0.025\ m$, $\tau_q = 20\ s$, $t = 6\ min.$, $\Gamma = 0,1,2$.

For approximate numerical solution of thermal wave model, we have used the finite element Legendre wavelet Galerkin method (FELWGM). The numerical scheme is based on Legendre wavelet that combines the multi-resolution and multi-scale computational property of Legendre wavelet with element wise analysis. Multi-resolution analysis of Legendre wavelet localizes small scale variation of solution and fast switching of functional bases in the case of thermal wave model. FELWGM minimizes error and produces higher degree of accuracy [9, 10]. When the lagging time $\tau_q = 0$ and absence of modified Gaussian external heat source then the bioheat transfer equation is solved by analytically (Laplace transfer technique), which is given as follows:

$$\theta(x, F_o) = \left(P_f^2 \theta_b + P_m\right) \left\{ P_f^{-2} - \frac{\cosh\left(P_f (x-1)\right)}{\cos\left(P_f\right)} \right\} + \sum_{n=0}^{\infty} \frac{2 \exp(S_n^2 F) \cosh\left(\sqrt{S_n^2 - P_f^2}\,(x-1)\right)}{S_n^2 \sqrt{S_n^2 - P_f^2}\, \sinh\left(\sqrt{S_n^2 - P_f^2}\right)},$$

where $S_n = \sqrt{-P_f^2 - (4n+1)\frac{\pi}{2}} F_0, n = 0, 1, \ldots, \infty$. The approximate solution (FELWGM) has compared with analytical solution (Laplace transform technique) and shows good agreement, which is shown in Fig.2.

Thermal ablation (high temperature hyperthermia) is a technique to treat cancerous cells in the short time at high temperature. The high temperature occurs at the target region for 4 to 6 min during the thermal ablation process [14]. In this process the mechanism of killing cells are protein denaturation / coagulation and ablation / vaporization. Fig.

3 shows the effect of time on temperature with respect to space coordinate. We observed that temperature increases from 54 to 70 °C as the time increases to 4-6 min for the value of $Q_{r0} = 7.85\times10^{5}$ W m^{-3}, $Q_{m0} = 1.091\times 10^{3}$W m^{-3} during thermal ablation. So, the time duration affects the treatment process.

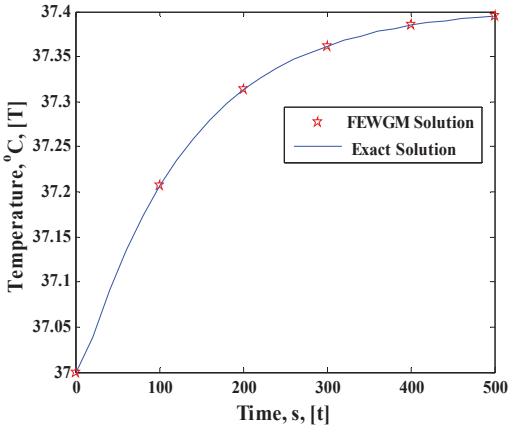

Fig.2 plot of temperature vs. time of FELWGM solution and Exact solution.

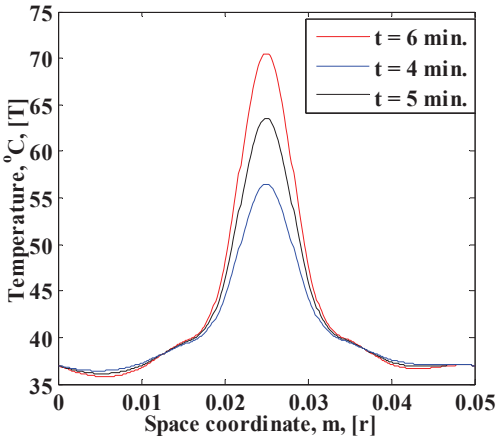

Fig.3 plot of temperature vs. space coordinate for different times.

The effect of generalised coordinate system ($\Gamma = 0,1,2$) on the treatment of tumor during thermal ablation has shown in Fig. 4. We observed from this figure, that the temperature distribution is independent of coordinate systems at the thermal ablation position, while it plays an important role in heating region.

Fig. 5 shows the heating position *(r=0.025 m)* where the tumor exists. In this figure, heating position is changed according to the existence of tumor. If we identified the tumor in living biological tissue and organ then the value of parameter is fixed accordingly. Thus, the parameter (r_0) is important in treatment of tumor.

The comparison of temperature distribution in Pennes and thermal wave models of bioheat transfer are shown in Fig. 6. We observed that temperature profile in thermal wave model is less than in comparison to Pennes model.

As Q_{r0} increases, temperature increases which is shown in Fig. 7. According to the estimation of Q_{r0}, temperature is taken in the range of thermal ablation. Thus, we can say that the external heat source plays an important role in treatment.

As $\tau_q \rightarrow 0$, temperature distribution obtained for Pennes model ($\tau_q = 0$ s) and thermal bioheat transfer model ($\tau_q = 0.01$ s) coincides mutually. So, in limiting case (i.e. $\tau_q \rightarrow 0$), thermal wave model reduces to Pennes model which is shown in Fig.8. I hope, this study will prove very beneficial for the clinical therapeutic application in treatment of cancerous cells.

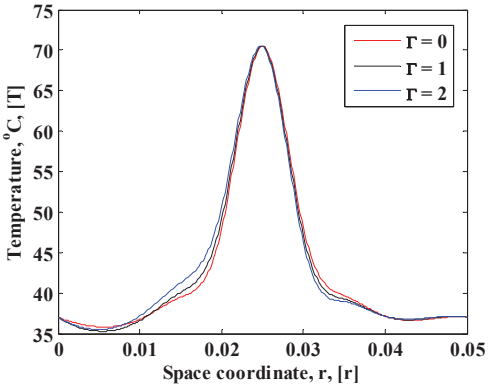

Fig.4 plot of temperature vs. space coordinate for $\Gamma = 0,1,2$.

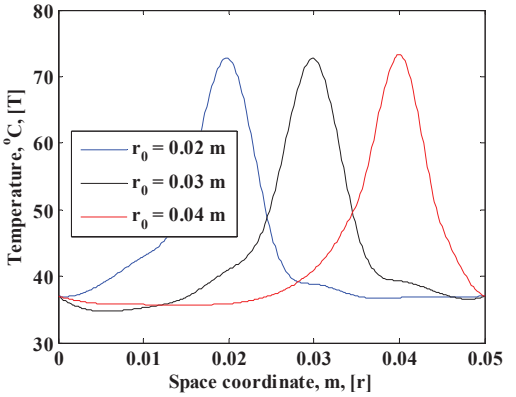

Fig.5 plot of temperature vs. space coordinatefor different location of tumor.

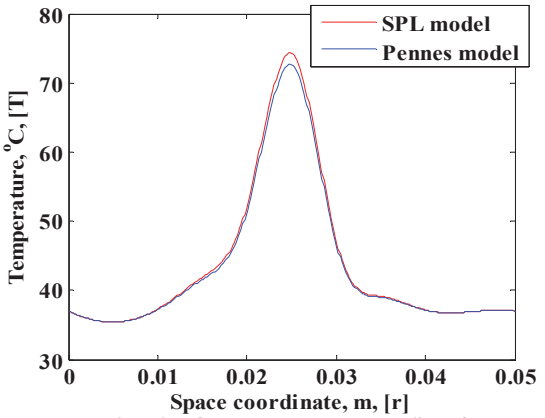

Fig.6 plot of temperature vs. space coordinate for different value of relaxation times.

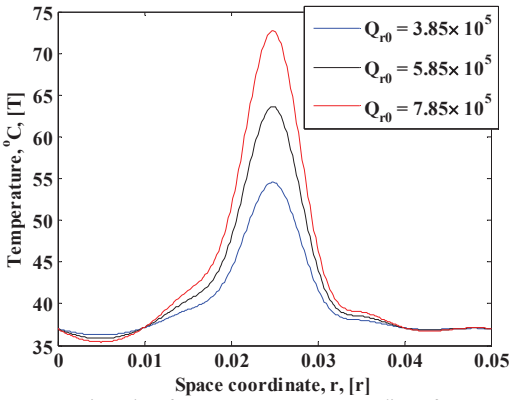

Fig.7 plot of temperature vs. space coordinate for different value of external heat sources.

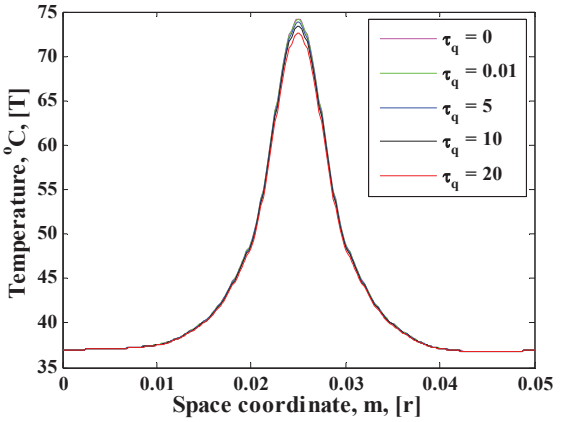

Fig.8 plot of temperature vs. space coordinate for different values of relaxation time (limiting ca τ_q).

5. Concusion

We have presented FELWGM for the solution of thermal wave bioheat transfer model in presence of temperature dependent metabolic heat source and modified Gaussian external heat source. The multi-resolution analysis of Legendre wavelet in case of present problem localizes small scale variations of solution and fast switching of functional bases. FELWGM has the capability to minimize error and produce higher degree of accuracy. FELWGM results are compared with analytical results in a particular case and gives good accuracy (Fig. 2). Temperature at the site of tumor increases as time increases (suggested from the Fig. 3). Treatment process is independent of coordinate system ($\Gamma = 0,1,2$) at thermal ablation position (Fig. 4). Treatment at specific thermal ablation position can be done fixing the source parameter r_0 accordingly, which is shown in Fig. 5. The temperature in thermal wave model is less in comparison to Pennes model at thermal ablation position (Fig. 6). The value of external heat source parameter $\left(Q_{r0}\right)$ increases as temperature increases also (Fig. 7). So, we produce the require temperature at the thermal ablation position. The source parameters allow us to specific heating (r_0) and control of temperature $\left(Q_{r0}\right)$ at thermal ablation position. In limiting case (i.e. $\tau_q \rightarrow 0$), thermal wave model reduces to Pennes model which is shown in Fig. 8. The relaxation time affects significantly in treatment during thermal ablation. So, our study will give thermal treatment a new future direction in clinical therapeutic applications.

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