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Application of Various Order Reduction Methodologies Over Power System Components

Amit Kumar Choudhary* and Shyam Krishna Nagar

Department of Electrical Engineering,
Indian Institute of Technology (Banaras Hindu University) Varanasi
*Varanasi, U.P., India (e-mail: *amit.rs.eee@iitbhu.ac.in)*

Abstract: Study and analysis of highly interconnected electrical system is time consuming and difficult; and appearance of inner uncertainty, result in system complexity with higher order, posing a great challenge to both system analysts and control engineers. Simplification of such design to their lower order equivalent via order reduction accomplishes a good approximation to the system for analysis. This paper attempts to discuss few noteworthy approximation techniques relevant to power system components. The test result based on the error computation between the original and reduced systems through the varied algorithms validate the models obtained to be a good approximant.

Keywords: Power system design, uncertain system, order reduction, direct truncation, gamma-delta approximation, differentiation.

1. INTRODUCTION

Electrical power system is efficiently modelled and analyzed by classification of key elements and eradication of spurious one. Some of the basic power system modules are supplies, loads, conductors, capacitors, reactors, protective devices and SCADA systems. All together, they result in a very high order of state variables for simulation, trajectory sensitivity analysis, control and others making their overall investigation cumbersome. Here, develop a loophole for control engineers to implement an optimal control strategy for such problem to forbid the computational burden, maintaining the accuracy and dynamic behaviour of the original system. At this juncture, application of model order reduction is inevitable to reduce computational effort and process time with an aim of deriving an approximate reduced model. Order reduction is an important tool in power system to deal with size and complexity, since it provides a simplified representation of the system, while preserving the dynamic characteristics of interest. A bulk of model reduction techniques in power system are tailored for controller design and transient/small signal stability analysis, both in time domain as well as in frequency domain [Troullinos, Dorsey, Wong, and Myers, (1988); Martins, Lima, and Pinto, (1996); Gustavsen, and Semlyen, (1999); Noda, Semlyen, and Iravani, (2003); Chaniotis, and Pai, (2005), Sambariya, and Prasad, (2012)].

Parametric uncertainty in real system is an unavoidable case and must be considered because of nonlinear effects, environmental conditions, tolerance of the equipments, measurement faults and many others. Uncertain or interval system is an emerging field of research from the day of its discovery. They are defined as the system varying within a finite range instead of being deterministic. This set the motive of the paper, to deal with higher order system of uncertain nature. The uncertainties in the plant representation are demonstrated via an interval bound for each numerator and denominator polynomial coefficients. Order reduction of such systems also grabbed the interest for investigation; justified by the methodologies available notably Routh Approximation Method, Pade Approximation methods and others [Ismail, Bandyopadhyay, and Gorez, (1997); Bandyopadhyay, Upadhye, and Ismail, (1997); Choudhary, and Nagar, (2013 a, b)].

With an aim to propose relevant methodologies for power system components of uncertain form, the paper is stretched over seven sections; introduction to power system design with a review of reduction techniques applied above them in section 1 followed by the description of a block diagram of physically available power system design in section 2. This section states the components of interest to apply order reduction algorithm. Section 3 accommodates the representation of higher and lower order system with an illustration to the reduction methodologies of interest namely direct truncation (*DT*); gamma-delta approximation (*GD*) and differentiation method (*DM*). This section also state the validation tool used for assessment of the algorithms. Section 4 present an example taken from the available literatures to show the algorithms supremacy. In section 5, these algorithms are applied on the components of interest considered from power system design. The finding from the illustration is complied through a brief discussion in section 6. Finally, section 7; conclude with an emergence of an acceptable approximation techniques for power system design/component of uncertain structure.

2. BLOCK DIAGRAM OF POWER SYSTEM DESIGN

An excitation control of synchronous generator is considered from [Anderson, and Fouad, (1977)] as shown in the Figure 1. It consists of Automatic Voltage Regulator (AVR), Power System Stabilizer (PSS), Exciter, Governor integrated to Turbine and Generator. Briefly this system is explained as; input to the turbine is governed by the governor and output of

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the turbine is fed to the generator which then is passed on to the transmission lines. In this combination, the generator is excited by the exciter that takes input from the generator itself. There are many set connections for exciter to the generator but are beyond our interest. At the governor point also, its governance is performed by the output of the turbine through a proper check between the reference torque ω_r and the incoming input from the turbine ω. Both of this governor and the exciter are externally taken care of by the AVR and the PSS for constant input without any break. A PSS installed in the AVR of the Generator improves the power system stability. It has an excellent cost performance compared to other power system modifications or additions. All of these individual components when taken together result in a very high order system which is not analysable at user end. Considering each of the components will be lengthy and tedious. Thus, only AVR and PSS are taken into consideration for evaluating the techniques of order reduction. Taking uncertainties into account of the parameters, their transfer function provide more realistic design, that result in rewriting the transfer function in uncertain form as stated in section 5.

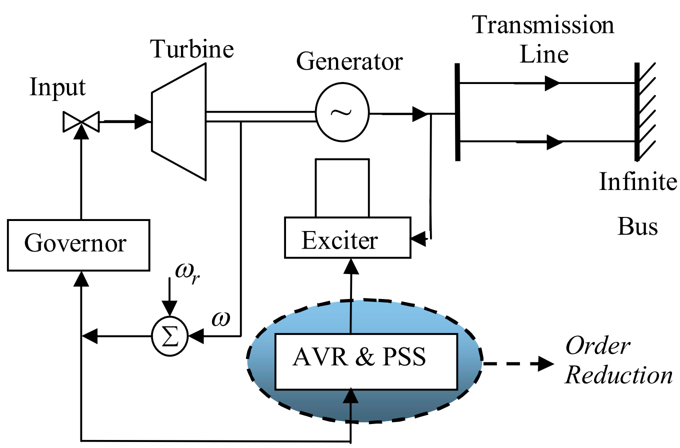

Fig. 1. Block Diagram of Excitation Control of Synchronous Generator

3. BRIEF DISCUSSION ON ALGORITHMS

This section is a discourse of the representation of higher and lower order system followed by the three varied reduction methodologies. Two of the illustrated algorithms are existing namely *DT* and *GD* and one among them designated as *DM* is novel for its outcome. Among the three algorithms; *GD* and *DM* call for an appropriate transformation to their continuous-time equivalent representation. This is mandatory in order to apply continuous-time algorithm on discrete-time domain. To meet this requirement *p*-domain transformation, where $z=1+p$ is performed for being simple and computationally easy. Inverse transformation applicable for these two algorithms to obtain the desired reduced model in *z*-domain is also executed. Tools to validate these algorithms are also discussed in this section.

3.1. Representation

Consider the higher order discrete-time uncertain system transfer function as;

$$G_n(z)=\frac{\left[N_1^-,N_1^+\right]z^{n-1}+\left[N_2^-,N_2^+\right]z^{n-2}+\ldots+\left[N_n^-,N_n^+\right]}{\left[D_0^-,D_0^+\right]z^{n}+\left[D_1^-,D_1^+\right]z^{n-1}+\ldots+\left[D_n^-,D_n^+\right]}=\frac{N_n(z)}{D_n(z)} \tag{1}$$

And its reduced order transfer function with $r<n$ be expressed as

$$H_r(z)=\frac{\left[n_1^-,n_1^+\right]z^{r-1}+\left[n_2^-,n_2^+\right]z^{r-2}+\ldots+\left[n_r^-,n_r^+\right]}{\left[d_0^-,d_0^+\right]z^{r}+\left[d_1^-,d_1^+\right]z^{r-1}+\ldots+\left[d_r^-,d_r^+\right]}=\frac{N_r(z)}{D_r(z)} \tag{2}$$

3.2. Reduction methodologies

3.2.1. Direct Truncation Method [Choudhary and Nagar (2013 a)]

The denominator and numerator polynomial of r^{th} order as stated in equation (2) is

$$D_r(z)=\left[D_r^-,D_r^+\right]z^{r}+\left[D_{r-1}^-,D_{r-1}^+\right]z^{r-1}+\ldots..+\left[D_0^-,D_0^+\right] \tag{3a}$$

$$N_r(z)=\left[N_{r-1}^-,N_{r-1}^+\right]z^{r-1}+\left[N_{r-2}^-,N_{r-2}^+\right]z^{r-2}+\ldots.+\left[N_0^-,N_0^+\right] \tag{3b}$$

3.2.2. Gamma-Delta Approximation [Choudhary and Nagar (2013 b)]

Required *p*-domain transformation result in

$$G_n(p)=\frac{\left[b_1^-,b_1^+\right]p^{n-1}+\left[b_2^-,b_2^+\right]p^{n-2}+\ldots+\left[b_n^-,b_n^+\right]}{\left[a_0^-,a_0^+\right]p^{n}+\left[a_1^-,a_1^+\right]p^{n-1}+\ldots+\left[a_n^-,a_n^+\right]}=\frac{B_n(p)}{A_n(p)} \tag{4}$$

Using the numerator and denominator polynomials from the above transfer function (4), the first two rows of the Routh tables are drafted as shown in Table 1 and 2.

Table 1: Denominator Array for γ Parameters

$\left[a_n^-,a_n^+\right]=\left[a_{0,0}^-,a_{0,0}^+\right]$	$\left[a_{n-2}^-,a_{n-2}^+\right]=\left[a_{0,1}^-,a_{0,1}^+\right]$	$\left[a_{n-4}^-,a_{n-4}^+\right]=\left[a_{0,2}^-,a_{0,2}^+\right]$	…..
$\left[a_{n-1}^-,a_{n-1}^+\right]=\left[a_{1,0}^-,a_{1,0}^+\right]$	$\left[a_{n-3}^-,a_{n-3}^+\right]=\left[a_{1,1}^-,a_{1,1}^+\right]$	$\left[a_{n-5}^-,a_{n-5}^+\right]=\left[a_{1,2}^-,a_{1,2}^+\right]$	…..
….			
$\left[a_{n-1,0}^-,a_{n-1,0}^+\right]=\left[a_{n,0}^-,a_{n,0}^+\right]$			

Entries down the third row of the tables and the $\gamma-\delta$ parameters (both of uncertain structure) are calculated as

$$\gamma_r = \frac{\left[a_{r-1,0}^-, a_{r-1,0}^+\right]}{\left[a_{r,0}^-, a_{r,0}^+\right]} \text{ where } r=1,2,3.... \tag{5a}$$

and

$$\left[a_{i,j}^-, a_{i,j}^+\right] = \frac{\left[a_{i-2,j+1}^-, a_{i-2,j+1}^+\right]\left[a_{i-1,0}^-, a_{i-1,0}^+\right] - \left[a_{i-2,0}^-, a_{i-2,0}^+\right]\left[a_{i-1,j+1}^-, a_{i-1,j+1}^+\right]}{\left[a_{i-1,0}^-, a_{i-1,0}^+\right]} \tag{5b}$$

with $i=2,3,4.....$ and $j=0,1,2......$

Table 2: Numerator Array for δ Parameters

$\left[b_n^-, b_n^+\right] = \left[b_{1,0}^-, b_{1,0}^+\right]$	$\left[b_{n-2}^-, b_{n-2}^+\right] = \left[b_{1,1}^-, b_{1,1}^+\right]$	$\left[b_{n-4}^-, b_{n-4}^+\right] = \left[b_{1,2}^-, b_{1,2}^+\right]$	..
$\left[b_{n-1}^-, b_{n-1}^+\right] = \left[b_{2,0}^-, b_{2,0}^+\right]$	$\left[b_{n-3}^-, b_{n-3}^+\right] = \left[b_{2,1}^-, b_{2,1}^+\right]$	$\left[b_{n-5}^-, b_{n-5}^+\right] = \left[b_{2,2}^-, b_{2,2}^+\right]$	..
…..			
$\left[b_{n-1,0}^-, b_{n-1,0}^+\right] = \left[b_{n,0}^-, b_{n,0}^+\right]$			

$$\delta_r = \frac{\left[b_{r,0}^-, b_{r,0}^+\right]}{\left[a_{r,0}^-, a_{r,0}^+\right]} \text{ where } r=1,2,3.... \tag{6a}$$

and

$$\left[b_{i,j}^-, b_{i,j}^+\right] = \frac{\left[b_{i-2,j+1}^-, b_{i-2,j+1}^+\right]\left[a_{i-2,0}^-, a_{i-2,0}^+\right] - \left[b_{i-2,0}^-, b_{i-2,0}^+\right]\left[a_{i-2,j+1}^-, a_{i-2,j+1}^+\right]}{\left[a_{i-2,0}^-, a_{i-2,0}^+\right]} \tag{6b}$$

with $i=2,3,4.....$ and $j=0,1,2......$

The obtained $\gamma-\delta$ parameters are used to derive the r^{th} order model which in its general form is

$$H_r(p) = \frac{B_r(p)}{A_r(p)} \tag{7}$$

where

$$A_r(p) = p^2 A_{r-2}(p) + \left[\gamma_r^-, \gamma_r^+\right] A_{r-1}(p) \tag{8a}$$

$$B_r(p) = \left[\delta_r^-, \delta_r^+\right] p^{r-1} + p^2 B_{r-2}(p) + \left[\gamma_r^-, \gamma_r^+\right] B_{r-1}(p) \tag{8b}$$

with $A_{-1}(p) = \frac{1}{p}$, $A_0(p) = 1$, $B_{-1}(p) = 0$, $B_0(p) = 0$

3.2.3. Differentiation Method

According to this algorithm, consider equation (4) and differentiate it to the desired reduced order in *p*-domain.

3.2.4 Inverse Transformation

The reduced *p*-domain models obtained in sections (3.2.2) and (3.2.3) is transformed back to the desired *z*-domain by $p = z-1$ transformation.

3.3 Validation Tools

Performance of the obtained reduced model is validated by computation of error *J*, defined as the sum of squared error over a fixed interval of time and is determined by the error between the transient responses of the higher order system, and the lower order system, expressed as;

$$J = \sum_{k=0}^{\infty} \left[y(k) - y_r(k)\right]^2 \tag{9}$$

where, $y(k)$ and $y_r(k)$ are the unit step responses of original system $G_n(z)$ and reduced order system $H_r(z)$.

Computation of minimum *J* confers the obtained model to be an approximate one.

Tracking of step response of the higher order system and the lower order model is also considered to support the dominance of the projected algorithms.

4. EXCELLENCE OF THE DISCUSSED ALGORITHMS

Confrontation of the algorithms superiority over the existing techniques allow their application to the power system component and is attained by an example discussed;

Consider the transfer function from literature [Choudhary et. al. (2013 a, b), Ismail et. al. (1997)] as

$$G_3(z) = \frac{[1,2]z^2 + [3,4]z + [8,10]}{[6,6]z^3 + [9,9.5]z^2 + [4.9,5]z + [0.8,0.85]} \tag{10}$$

A. Direct Truncation

Reduced model by this algorithm is

$$H_2(z) = \frac{[3,4]z + [8,10]}{[9,9.5]z^2 + [4.9,5]z + [0.8,0.85]} \tag{11}$$

B. Gamma-Delta Approximation

By this algorithm, the reduced model obtained is

$$H_2(z) = \frac{[0.181,0.296]z + [-0.577,-0.392]}{z^2 + [-0.513,-0.445]z + [0.731,0.811]} \tag{12}$$

C. Differentiation Method

This result the lower order equivalent as

$$H_2(z) = \frac{[2,4]z + [1,6]}{[18,18]z^2 + [18,19]z + [3.9,6]} \tag{13}$$

The comparison between the squared error sums of the above models to display their merits is shown in Table 3. The error computed by the illustrated algorithms is acceptable to the prevailing one.

Figure 2; demonstrate the tracking of the step response of the obtained reduced models as a support to the adoption of the algorithms. The figure in its normalized form (*removes the y-axis values*) is to show the responses in a clearer manner because it is observed that the responses are very near to the unity line (*horizontal dotted line*) which in its original figure would not be clear. Though the tracking is not exact but is considered for giving the minimum required error, leaving an ambiguity to control engineers to design an appropriate controller for approximate tracking of the original response. The response in the figure state that the obtained reduced model is of non-minimum phase although the original response is of minimum phase. This again leaves a loophole for researchers to get an appropriate reduction methodology.

Table 3: Comparison between the Methods Example 1

Methods	Error	
	Lower Limit	Upper Limit
Direct Truncation [*Choudhary and Nagar (2013 a)*]	0.0278	0.0077
Gamma-Delta [*Choudhary and Nagar (2013 b)*]	0.0370	0.0014
Differentiation	0.0031	0.0123
Routh Pade Method [*Ismail, Bandyopadhyay, and Gorez, (1997)*]	0.1810	0.0741

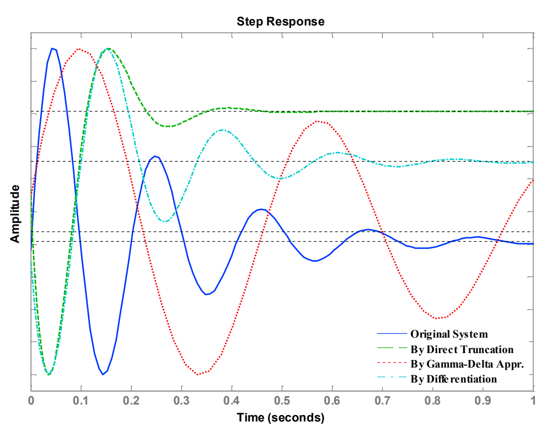

Fig. 2. Step Response of the Reduced Model for the example used for validation

Both the validation tools present a satisfactory result for the acceptance of the algorithms for their implementation on the physical available power system components chosen from section 2.

5. ORDER REDUCTION ON POWER SYSTEM COMPONENTS

Component 1: Consider Automatic Voltage Regulator [Sannuti, and Kokotovic, (1969)] defined by the block diagram in Figure 3, with $T_1=5$, $T_2=2$, $T_3=0.07$, $T_4=0.04$, $T_5=0.1$, $a_1=2.5$, $a_2=3.2$, $a_3=6$, $a_4=3$, $a_5=3$; (T_1, T_2, T_3, T_4 and T_5 in *sec*) in combination. The regulator through zero order hold circuit having sampling time *t=0.1sec* (selection of sampling time is discussed in Appendix) result in

$$G_{AVR}(z)=\frac{z^4+11.69z^3+12.17z^2+1.86z+0.033}{177.08z^5-450.68z^4+368.33z^3-89.64z^2-3.88z-1.19} \tag{14}$$

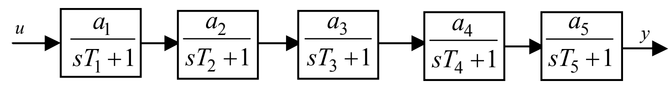

Fig. 3. Higher Order Model of AVR

Considering perturbation in the system, modifies the above transfer function to

$$G_{AVR}(z)=\frac{[0.95,1.05]z^4+[11.10,12.27]z^3+[11.56,112.77]z^2+[1.76,1.95]z+[0.03,0.03]}{[168.22,185.93]z^5+[-473.21,-428.14]z^4+[349.91,386.74]z^3+[-94.12,85.15]z^2+[-4.07,-3.68]z+[-1.24,-1.13]} \tag{15}$$

Reduced model of the above equation by the three varied algorithms is stated in Table 4, which prove to be a good approximate by the computation of the performance measures.

Component 2: Consider Power System Stabilizer [Sambariya, and Prasad, (2012)], in Figure 4, where K_{PSS} is PSS gain, T_ω is the time constant of washout stage, which is to prevent a steady state voltage shift. T_1, T_2, T_3, T_4 are the time constants of the two phase lead stages. In some cases, more than two phase lead stages are required and a filter and an output limiter is added to PSS in application. Double stage lead lag PSS data's are $T_\omega=2.0sec$, $T_1=0.5sec$, $T_2=0.1sec$, $T_3=0.2sec$, $T_4=0.02sec$ and $K_{PSS}=0.5$. Finally, the transfer function with ZOH equivalent at *t=0.1sec* result to

$$G_{PSS}(z)=\frac{25z^3-58.6z^2+43.1z-9.5}{z^3-1.3z^2+0.36z-0.002} \tag{16}$$

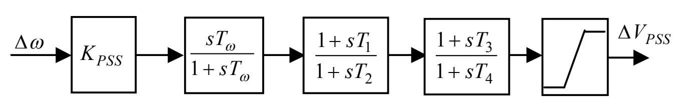

Fig. 4. Higher Order Model of PSS

Uncertainty in the system alters the transfer function as (17) and its reduced models by various algorithms as shown in Table 5.

$$G_{PSS}(\Delta z)=\frac{[22.5,27.5]z^3+[-64.46,-52.74]z^2+[38.79,47.41]z+[-10.45,-8.55]}{[0.9,1.1]z^3+[-1.43,-1.17]z^2+[0.32,0.39]z+[-0.002,-0.001]} \tag{17}$$

The above examples present a clear understanding of order reduction techniques to approximate the higher order uncertain power system components for their application at user end.

6. BRIEF DISCUSSION

Order reduction of higher order power system components of uncertain nature is cut down to user convenient order for their analysis. This is performed by two existing and one novel methodology. These algorithms are first stated to show their supremacy over the other prevailing technique and thereafter they are applied to the components under consideration. Validation of the algorithms is made through the computation of squared error sum and tracking of the step responses. The minimum errors obtained depict their supremacy. Algorithms are also strengthened through the tracking of the step response that portrayed a limitation for emergence of a non-minimum phase model. Reason for this outcome is the shifting of the zeros to the right hand side of the continuous-time domain plane. Though, the limitation is present but is of negligible nature to accomplish the goal. This does not hamper the preservation of dynamic characteristic of the higher order systems. Since, the motive is to obtain an approximate model; this is attained by reaching the minimum error of the reduced models.

7. CONCLUSION

The motive to propose relevant methodologies for power system components of uncertain form is achieved satisfactorily in the paper. Existing techniques Direct Truncation and Gamma-Delta Approximation and one fresh Differentiation Method is implemented for the order reduction. These algorithms are successfully applied to the components of interest Automatic Voltage Regulator and Power System Stabilizer, which are validated through error computation. Further work for control engineers to design a controller that track the original system step response approximately is asserted in the paper. Also stated is to derive reduction methodologies that would not result for non-minimum phase models.

Table 4: Reduced Order Models of AVR and their performance measures

		Error	
Method	Reduced Transfer Function	Lower Bound	Upper Bound
Direct Truncation	$H_2(z)=\dfrac{[1.767,1.953]z+[0.031,0.034]}{[-94.122,-85.158]z^2+[-4.070,-3.686]z+[-1.249,-1.130]}$	$5.9637*10^{-4}$	$8.1687*10^{-4}$
Gamma Delta Approximation	$H_2(z)=\dfrac{[-0.340,0.187]z+[-1.157,-0.836]}{z^2+[-3.143,-1.036]z+[-0.284,2.463]}$	0.1195	0.0329
Differentiation	$H_2(z)=\dfrac{[23.4,25.2]z+[64.233,75.447]}{[10093.56,11156.04]z^2+[-13482.09,-8150.54]z+[-1107.11,5527.07]}$	$1.1081*10^{-5}$	$1.1481*10^{-5}$

Table 5: Reduced Order Models of PSS

Method	Reduced Transfer Function
Direct Truncation	$H_2(z)=\dfrac{[38.79,47.41]z+[-10.45,-8.55]}{[-1.43,-1.17]z^2+[0.324,0.396]z+[-0.0022,-0.0018]}$
Gamma Delta	$H_2(z)=\dfrac{[-17.81,19.23]z+[-89.60,87.60]}{z^2+[-1.924,-0.933]z+[-1.421,3.032]}$
Differentiation	$H_2(z)=\dfrac{[135,165]z+[-158.92,-75.48]}{[2.7,3.3]z^2+[-4.06,-1.14]z+[-1.396,2.116]}$

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APPENDIX: SELECTION OF SAMPLING TIME

Any sampler has a definite set of possible sampling periods, depending on its clock. Too high, sampling rate lead to unnecessary computational difficulty and too low, lead to loss data fidelity. It is to the designer to choose an appropriate microcontroller to cater a desirable sampling time. Precisely, the sampling frequency for digital control systems is chosen according to the desired bandwidth of the closed loop system. Note that, no matter how the desired performances are specified, these can always be related to the closed loop system bandwidth. [Landau and Zito, (2006)]

In the paper, sampling time $t= 0.1\ sec$ is considered. The reason for this selection is detailed below;

The various representations obtained for varied t are

For $t=0.1$

$$G_{AVR}(z)=\frac{z^4+11.69z^3+12.17z^2+1.86z+0.033}{177.08z^5-450.68z^4+368.33z^3-89.64z^2-3.88z-1.19}$$

For $t=0.01$

$$G_{AVR}(z)=\frac{1.18e^{-7}z^4+2.82e^{-6}z^3+6.60e^{-6}z^2+2.39e^{-6}z+8.47e^{-8}}{z^5-4.51z^4+8.16z^3-7.38z^2+3.34z-0.61}$$

For $t=0.05$

$$G_{AVR}(z)=\frac{z^4+17.15z^3+28.39z^2+7.29z+0.18}{3775z^5-11927z^4+14037z^3-7686z^2+2111z-310}$$

For $t=0.001$

$$G_{AVR}(z)=\frac{1.27e^{-12}z^4+3.28e^{-11}z^3+8.27e^{-11}z^2+3.23e^{-11}z+1.23e^{-12}}{z^5-4.95z^4+9.80z^3-9.70z^2+4.80z-0.95}$$

As seen from the above representations for various sampling time, it is clear less t will give accurate representation for better analysis of system as compared to the higher t. But, considering the same less t, for deriving the uncertain structure would be very tedious resulting in an abrupt solution. Thus, for convince of reduced order computation, $t=0.1\ sec$ is considered.