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Procedia Computer Science 79 (2016) 963 – 970

Procedia Computer Science

7th International Conference on Communication, Computing and Virtualization 2016

Synchronization between non-autonomous hyperchaotc systems with uncertainties using active control method

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Abstract

In this article authors have studied the synchronization between non-autonomous hyperchaotic systems viz., Liu and 4D hyperchaotic non-autonomous systems with parametric uncertainties using active control method. The numerical simulation and graphical results shows that the considered method is effective to synchronize non-autonomous hyperchaotic systems with parametric uncertain terms

Peer-review under responsibility of the Organizing Committee of ICCCV 2016

Keywords: Non-autonomous hyperchaotic systems; Synchronization; Uncertain parameters; Active control method.

1. Introduction

The applications of non linear dynamical systems have nowadays spread to a wide spectrum of disciplines including science, engineering, biology, sociology etc. Study and analysis of non-linear dynamics have gained immense popularity during the last few decades due to its important feature of any real-time dynamical system. Sometimes these may give rise to the complex behaviour called chaos. Chaos theory as a new branch of physics and mathematics has provided a new way of viewing the universe and is an important tool to understand the behaviour of the processes in the world. Chaotic behaviour have been observed in different areas of science and engineering such as mechanics, electronics, physics, medicine, ecology, biology, economy, and so on. Chaos is an interesting

1877-0509
Peer-review under responsibility of the Organizing Committee of ICCCV 2016
doi:10.1016/j.procs.2016.03.122

phenomenon of nonlinear systems. Thus a chaotic system is a non linear deterministic system with unpredictable complexity. In dynamical systems, the term chaos is applied to deterministic systems that are periodic and that exhibit sensitive dependence on initial conditions and parameter variations. The field of chaos has grabbed the attention of the researchers and this contributes to a significant amount of the ongoing research these days.The synchronization of two identical autonomous chaotic systems was first observed by Pecora and Carrol in 1990 [1]. Chaos synchronization is an interesting topic of research in the area of non linear sciences, which has been widely investigated in many different fields, such as chemical and ecological science [2-3], secure communication [4], etc. The concept of synchronization of chaotic systems is taking two or more chaotic systems. There are several types of synchronization schemes which can be used to synchronize of chaotic systems such as active control [5], adaptive control [6], nonlinear control [7], impulsive control [8], backstepping design [9] etc.The synchronization of non-autonomous chaotic circuits by using a feedback device to correct the phase of the periodic forcing in the response system was studied by Pecora and Carrol in 1993 [10]. Y. Lei et al. [11] studied robust synchronization of chaotic non-autonomous systems using adaptive-feedback control in 2007. In 2012 T. Botmart et al. [12] studied Synchronization of non-autonomous chaotic systems with time-varying delay via delayed feedback control, in the same year Z. Ye and C. Deng [13] studied Adaptive synchronization to a general non-autonomous chaotic system and its applications.The synchronization between chaotic systems with uncertainties is not easy jobs for researchers since there are always possibilities of destroying synchronization under the effects of those parameters. Jawaadaa et al. [14] studied robust active sliding mode anti-synchronization of hyperchaotic systems with uncertainties and external disturbances, in autonomous systems. In 2012, Chen et al. [15] have studied disturbance-observer-based robust synchronization control of uncertain autonomous chaotic systems. But the synchronization between non-autonomous Liu and 4D hyperchaotic systems with uncertainties using active control method is first of its kind.

In this article the authors have studied synchronization between non-autonomous Liu and 4D hyperchaotic systems in the presence of uncertain parameters through active control method. The numerical simulation and graphical results are carried out using Runge-kutta method with the help of Matlab.

2. Problem formulation and systems description

2.1 Problem formulation

Consider an uncertain non-autonomous chaotic system as a master system as

$$\frac{dx_i}{dt} = (A_1 + \Delta A_1)x_i + tf_1(x_i), \qquad i = 1, 2, ..., n. \tag{1}$$

and another uncertain non-autonomous chaotic system as the response system as

$$\frac{dy_i}{dt} = (A_2 + \Delta A_2)y_i + tf_2(y_i) + u_i(t), \qquad i = 1, 2, ..., n. \tag{2}$$

where $x_i = [x_1, x_2, ... x_n]^T \in R^n$ and $y_i = [y_1, y_2, ... y_n]^T \in R^n$ are the state vectors, $A_1, A_2 \in R^{n\times n}$ are constant matrices with proper dimensions, $f_1, f_2 : R^n \to R^n$ are the nonlinear functions of the systems, $\Delta A_1, \Delta A_2 \in R^{n\times n}$ are parametric uncertainties of chaotic systems with $|\Delta A_1| \le \delta_1$, $|\Delta A_2| \le \delta_2$, where δ_1 , δ_2 are positive constants and

$u_i(t) \in R^n$ is the control input vector of the uncertain chaotic system (2).

If we define the synchronization error as $e_i = y_i - x_i$, $i = 1, 2, ..., n.$ then the corresponding error dynamics can be obtained as $\frac{de_i}{dt} = (A_2 + \Delta A_2)y_i + tf_2(y_i) - (A_1 + \Delta A_1)x_i - tf_1(x_i) + u_i(t)$

$$= (A_2 + \Delta A_2 + \Delta A_1)e_i + F_1(x_i, y_i) + u_i(t), \qquad (3)$$

where $F_1(x_i, y_i) = tf_2(y_i) - tf_1(x_i) + ((A_2 + \Delta A_2) - A_1)x_i - \Delta A_1 y_i$

Now controller $u_i(t)$ is to be designed in such a way that the master and response systems are synchronized through the proper definitions of errors.

2.2 Systems description

2.2.1 Non-autonomous hyperchaotic Liu system

The non-autonomous hyperchaotic Liu system [16] is given by

$$\frac{dx_1}{dt} = a_1(x_2 - x_1),\ \frac{dx_2}{dt} = a_2x_1 - a_5x_1x_3 + x_4,\ \frac{dx_3}{dt} = a_3x_1^2 - a_4x_3,\ \frac{dx_4}{dt} = \sin(\omega t)x_2x_3 \qquad (4)$$

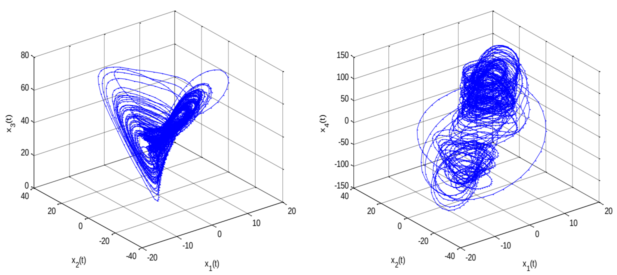
where ω is the angular frequency of the input signal. For the parametric values' $a_1 = 10, a_2 = 50, a_3 = a_4 = 4, a_5 = 2$ and $\omega \in [3.02, 3.13], [3.47, 3.915]$ or $[4.04, 5]$ the system (4) shows the hyperchaotic behaviour. Phase portraits of Liu hyperchaotic system is depicted through Fig. 1 for considered values of the parameters and initial condition $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5,\ 1,\ 2,\ 5)$.

Let us define the uncertain non-autonomous Liu hyperchaotic system as

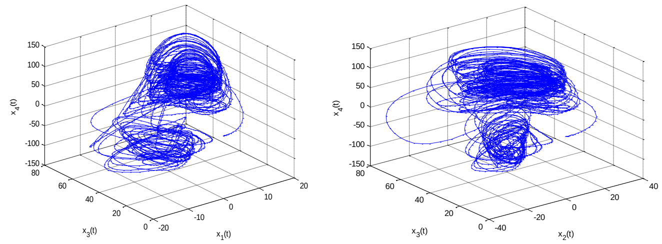
$$\frac{dx_1}{dt} = a_1(x_2 - x_1) + 0.1x_3 - 0.03x_4,\ \frac{dx_2}{dt} = a_2x_1 - a_5x_1x_3 + x_4 + 0.02x_1,\ \frac{dx_3}{dt} = a_3x_1^2 - a_4x_3 + 0.3x_4,$$

$$\frac{dx_4}{dt} = \sin(\omega t)x_2x_3 - 0.5x_2 \qquad (5)$$

where $\Delta A_1 = \begin{bmatrix} 0 & 0 & 0.1 & 0.03 \\ 0.02 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3 \\ 0 & 0.5 & 0 & 0 \end{bmatrix}$ is the uncertain parameter. Fig. 2 shows the phase portraits of non-autonomous Liu hyper chaotic system with uncertain parameter.

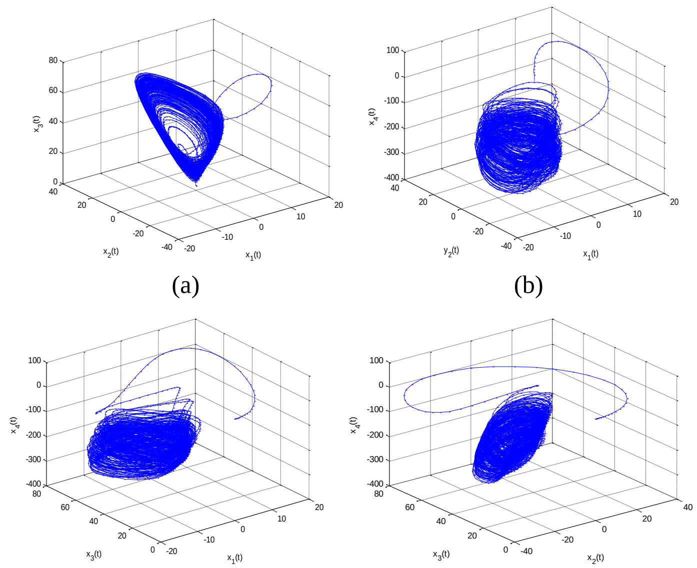


(a) (b)



(c) (d)

Fig.1. Phase portraits of non-autonomous Liu hyperchaotic system without uncertainties: (a) in $x_1 - x_2 - x_3$ space, (b) in $x_1 - x_2 - x_4$ space, (c) in $x_1 - x_3 - x_4$ space, (d) in $x_2 - x_3 - x_4$ space.



(a) (b)

(c) (d)

Fig.2. Phase portraits of non-autonomous Liu hyperchaotic system with uncertainties: (a) in $x_1 - x_2 - x_3$ space, (b) in $x_1 - x_2 - x_4$ space, (c) in $x_1 - x_3 - x_4$ space, (d) in $x_2 - x_3 - x_4$ space.

2.2.2 Non-autonomous 4D hyperchaotic system

The non-autonomous 4D hyperchaotic system [11] is given as

$$\frac{dy_1}{dt} = b_1(y_2 - y_1) + y_2 y_3 y_4, \quad \frac{dy_2}{dt} = b_2(y_1 + y_2) - y_1 y_3 y_4, \quad \frac{dy_3}{dt} = -[b_3 + b_4 \sin(t)]y_3 + y_1 y_2 y_4, \quad \frac{dy_4}{dt} = -b_5 y_4 + y_1 y_2 y_3$$

(6)

where b_1, b_2, b_3, b_4 and b_5 are the parameters. At the values of the parameters $b_1 = 30$, $b_2 = 10$, $b_3 = 95$, $b_4 = b_5 = 40$ and initial condition $(y_1(0),\ y_2(0),\ y_3(0),\ y_4(0)) = (3, -1, 8, 6)$ the 4D system shows hperchaotic behaviour. Phase portraits of equation (6) are depicted through Fig. 3.

The 4D hyerchaotic system with uncertain parameters is defined as

$$\frac{dy_1}{dt} = b_1(y_2 - y_1) + y_2 y_3 y_4 + 0.02 y_2 + 0.4 y_3, \quad \frac{dy_2}{dt} = b_2(y_1 + y_2) - y_1 y_3 y_4 - 0.1 y_4, \quad \frac{dy_3}{dt} = -[b_3 + b_4 \sin(t)]y_3 + y_1 y_2 y_4 + 0.05 y_1,$$

$$\frac{dy_4}{dt} = -b_5 y_4 + y_1 y_2 y_3 + 0.25 y_3 \tag{7}$$

where uncertain parameter $\Delta A_2 = \begin{bmatrix} 0 & 0.02 & 0.4 & 0 \\ 0 & 0 & 0 & 0.1 \\ 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0 \end{bmatrix}$ The phase portraits of 4D hyperchaotic system with uncertain parameter are displayed through Fig. 4.

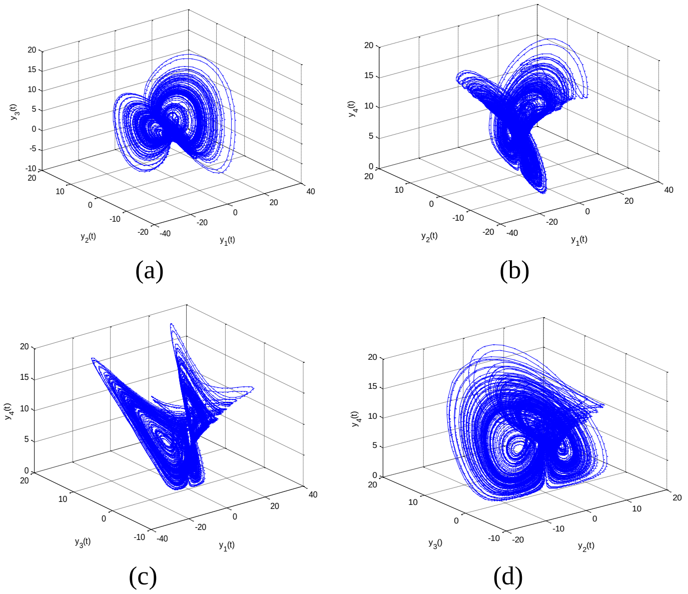


Fig.3. Phase portraits of non-autonomous 4D hyperchaotic system without uncertainties: (a) in $y_1 - y_2 - y_3$ space, (b) in $y_1 - y_2 - y_4$ space, (c) in $y_1 - y_3 - y_4$ space, (d) in $y_2 - y_3 - y_4$ space.

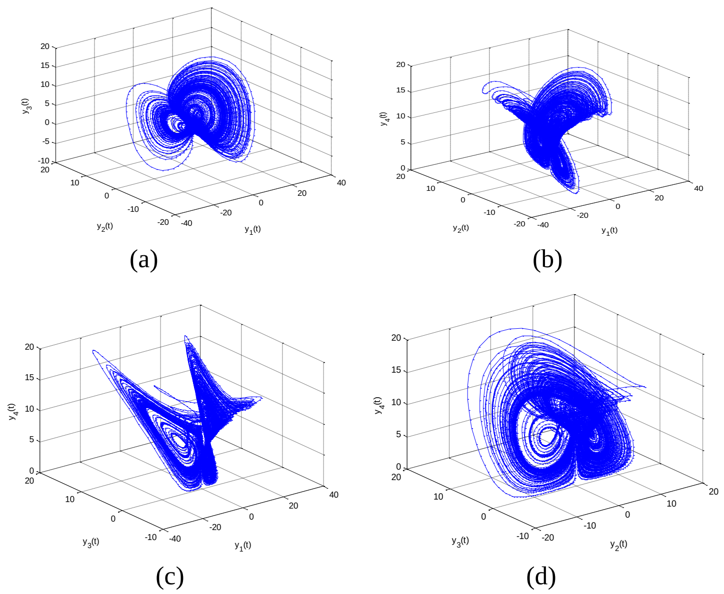


Fig.4. Phase portraits of non-autonomous 4D hyperchaotic system with uncertainties: (a) in $y_1 - y_2 - y_3$ space, (b) in $y_1 - y_2 - y_4$ space, (c) in $y_1 - y_3 - y_4$ space, (d) in $y_2 - y_3 - y_4$ space.

3. Synchronization between non-autonomous Liu and 4D hyperchaotic systems with uncertainties through active control method

The non-autonomous Liu hyperchaotic system with uncertainties (5) is considered as master system and 4D hyperchaotic system with uncertainties (7) is taken as response system as

$$\frac{dy_1}{dt} = b_1(y_2 - y_1) + y_2y_3y_4 + 0.02y_2 + 0.4y_3 + u_1(t),\ \frac{dy_2}{dt} = b_2(y_1 + y_2) - y_1y_3y_4 - 0.1y_4 + u_2(t),$$

$$\frac{dy_3}{dt} = -[b_3 + b_4\sin(t)]y_3 + y_1y_2y_4 + 0.05y_1 + u_3(t),\ \frac{dy_4}{dt} = -b_5y_4 + y_1y_2y_3 + 0.25y_3 + u_4(t) \tag{8}$$

where $u_1(t)$, $u_2(t)$, $u_3(t)$, $u_4(t)$ are the control functions to be designed. From equation (5) and equation (8) we get the following error systems as

$$\frac{de_1}{dt} = b_1(e_2 - e_1) + 0.02e_2 + 0.4e_3 + (b_1 - a_1)(x_2 - x_1) + 0.02x_2 + 0.3x_3 + 0.03x_4 + y_2y_3y_4 + u_1(t),$$

$$\frac{de_2}{dt} = b_2(e_1 + e_2) - 0.1e_4 - (a_2 + 0.02)x_1 - 1.1x_4 + a_5x_1x_3 + b_2(x_1 + x_2) - y_1y_3y_4 + u_2(t),$$

$$\frac{de_3}{dt} = 0.05e_1 - [b_3 + b_4\sin(t)]y_3 + y_1y_2y_4 + 0.05x_1 - a_3x_1^2 + a_4x_3 - 0.3x_4 + u_3(t),$$

$$\frac{de_4}{dt} = -b_5e_4 + 0.25e_3 + y_1y_2y_3 - b_5x_4 + 0.25x_3 + 0.5x_2 - \sin(\omega t)x_2x_3 + u_4(t) \tag{9}$$

where $e_i = y_i - x_i,\ i = 1,2,3,4.$ Here our goal is to design the control functions $u_1(t)$, $u_2(t)$, $u_3(t)$, $u_4(t)$ ([5]) as

$$u_1(t) = -(b_1 - a_1)(x_2 - x_1) - 0.02x_2 - 0.3x_3 - 0.03x_4 - y_2y_3y_4 + V_1(t),$$

$$u_2(t) = (a_2 + 0.02)x_1 + 1.1x_4 - a_5x_1x_3 - b_2(x_1 + x_2) + y_1y_3y_4 + V_2(t)$$

$$u_3(t) = [b_3 + b_4\sin(t)]y_3 - y_1y_2y_4 - 0.05x_1 + a_3x_1^2 - a_4x_3 + 0.3x_4 + V_3(t)$$

$$u_4(t) = -y_1y_2y_3 + b_5x_4 - 0.25x_3 - 0.5x_2 + \sin(\omega t)x_2x_3 + V_4(t)$$

which leads to error systems as

$$\frac{de_1}{dt} = b_1(e_2 - e_1) + 0.02e_2 + 0.4e_3 + V_1(t),\ \frac{de_2}{dt} = b_2(e_1 + e_2) - 0.1e_4 + V_2(t),\ \frac{de_3}{dt} = 0.05e_1 + V_3(t),$$

$$\frac{de_4}{dt} = -b_5e_4 + 0.25e_3 + V_4(t) \tag{10}$$

The error system (10) is a linear system with control inputs $V_1(t)$, $V_2(t)$, $V_3(t)$ and $V_4(t)$ as the functions of e_1, e_2, e_3 and e_4. Now design control inputs to stabilize the above error system.

Let us choose

$$\begin{bmatrix} V_1(t) \\ V_2(t) \\ V_3(t) \\ V_4(t) \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}$$

Where A is the 4×4 matrix. In order to make the closed loop system stable, matrix A should be selected in such a way that the feedback system will have the eigenvalues $\lambda_i, i = 1, 2, 3$ with negative real parts.

Choosing the matrix A as

$$A = \begin{bmatrix} b_1 - 1 & -b_1 - 0.02 & -0.4 & 0 \\ -b_2 & -b_2 - 1 & 0 & 0.1 \\ -0.05 & 0 & -1 & 0 \\ 0 & 0 & -0.25 & b_5 - 1 \end{bmatrix}$$

Thus the error system (10) reduced to

$$\frac{de_1}{dt} = -e_1, \ \frac{de_2}{dt} = -e_2, \ \frac{de_3}{dt} = -e_3, \ \frac{de_4}{dt} = -e_4 \tag{11}$$

All the eigenvalues of the error systems (11) are negative and hence the systems are stable and required synchronization between non-autonomous systems is obtained.

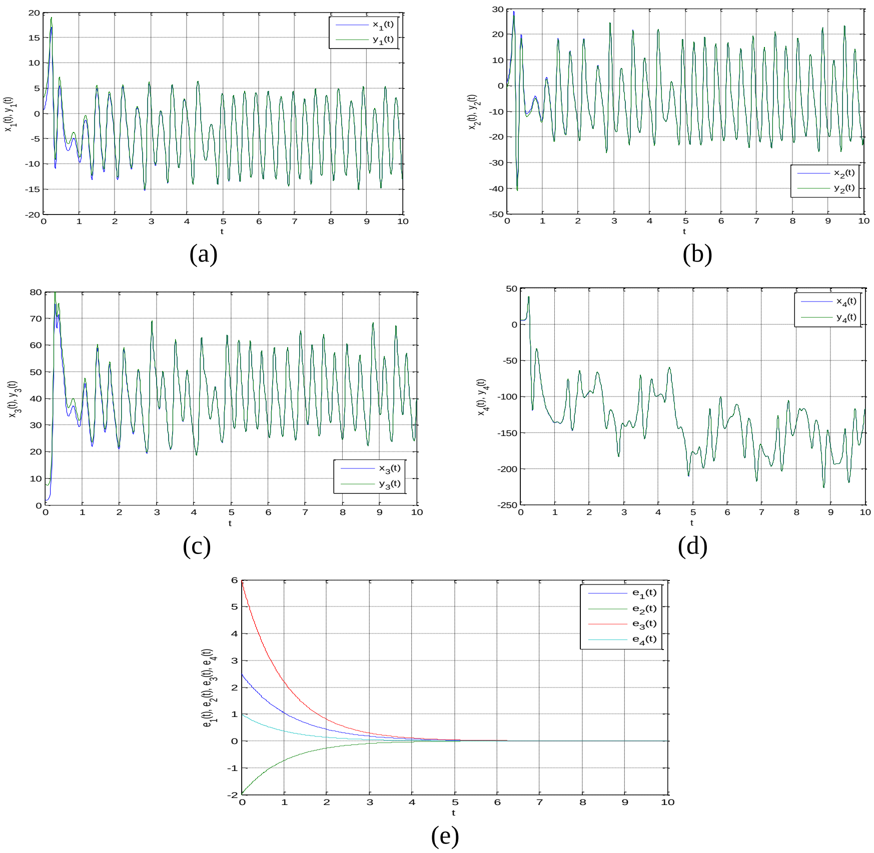


Fig.5. State trajectories of master system( ) and response system ( ): (a) between x_1 and y_1; (b) between x_2 and y_2; (c) between x_3 and y_3; (d) x_4 and y_4; (e) The evolution of the error functions $e_1(t), e_2(t)$, $e_3(t)$ and $e_4(t)$.

4. Numerical simulation and results

In this section, we take the earlier considered values of the parameters of both the systems. The initial conditions of drive and response systems are taken as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5, 1, 2, 5)$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (3, -1, 8, 6)$ respectively. Hence the initial conditions of error system are $(e_1(0), e_2(0), e_3(0), e_4(0)) = (2.5, -2, 6, 1)$. The synchronization between $x_1 - y_1$, $x_2 - y_2$, $x_3 - y_3$ and $x_4 - y_4$ are depicted through Fig.5(a)-5(d). The error functions are depicted through Fig. 5(e). We can see that from Fig. 5(a)-5(e) the considered non-autonomous hyperchaotic systems with uncertainties are synchronized after a small time of duration.

5. Conclusion

The authors have successfully used the active control method to achieve synchronization between the non-autonomous hyperchaotic systems with uncertainties along a desired trajectory, which clearly exhibits the reliability and potential of the method. The graphical representation of the numerical results is clearly shows that non-autonomous Liu and 4D hyperchaotic systems with uncertainties are synchronized when the error states tending to zero as time becomes large.

References

1. Pecora LM, Carroll TL. Synchronization in chaotic systems. Phys Rev Lett 1990; 64(8):821–4.
2. Han SK, Kerrer C, Kuramoto Y. Dephasing and bursting in coupled neural oscillators. Phys Rev Lett 1995; 75:3190–3.
3. Blasius B, Huppert A, Stone L. Complex dynamics and phase synchronization in spatially extended ecological system. Nature 1999; 399:354–9.
4. Feki M. An adaptive chaos synchronization scheme applied to secure communication. Chaos, Solitons & Fractals 2003; 18:141–8.
5. M.T. Yassen, Chaos synchronization between two different chaotic systems using active control, Chaos, Solitons and Fractals 23 (2005) 131–140.
6. Botmart T, Niamsup P. Adaptive control and synchronization perturbed Chuas system. Math Comput Simulat 2007; 75:37–55.
7. Park, J.H.: Chaos synchronization of a chaotic system via nonlinear control. Chaos, Solitons and Fractals 25, 579–584 (2005)
8. Cao J, Ho DWC, Yang Y. Prolective synchronization of a class of delayed chaotic systems via impulsive control. Phys Lett A 2009; 373:3128–33.
9. Lu JH, Zhang SC. Controlling Chen's chaotic attractor using backstepping design based on parameters identification. PhysLett A 2001; 286:148–52.
10. Carroll TL, Pecora LM. Synchronizing non-autonomous chaotic circuits. IEEE Trans Circuits Syst II 1993; 40:646–50.
11. Youming Lei, Wei Xu, Jianwei Shen, Robust synchronization of chaotic non-autonomous systems using adaptive-feedback control, Chaos, Solitons and Fractals 31 (2007) 371–379.
12. T. Botmart, P. Niamsup, X. Liu, Synchronization of non-autonomous chaotic systems with time-varying delay via delayed feedback control. Commun Nonlinear Sci Numer Simulat 17 (2012) 1894–1907.
13. Zhiyong Ye, Cunbing Deng, Adaptive synchronization to a general non-autonomous chaotic system and its applications. Nonlinear Analysis: Real World Applications 13 (2012) 840–849.
14. Jawaadaa W, Noorani MSM, Al-sawalha MM (2012) Robust active sliding mode anti-synchronization of hyperchaotic systems with uncertainties and external disturbances. Nonlinear Analysis: Real World Applications 13: 2403–2413
15. Chen M, Wu Q, Jiang C (2012) Disturbance-observer-based robust synchronization control of uncertain chaotic systems. Nonlinear Dyn. 70: 2421–2432. doi: 10.1007/s11071-012-0630-9.
16. Luo Xiao-Hua, Circuitry implementation of a novel non-autonomous hyperchaotic Liu system based on sine input. Chinese Physics B, Vol 18 No 8, 3304-08, August 2009.