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Revisiting Approximation Techniques to Reduce Order of Interval System

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Abstract: Large interconnected modules result in complex system of higher order and often of interval structure, making the overall study and analysis, time consuming and complicated. Accepting the challenge to state an approximate model of such system, both system analysts and control engineers, headed towards the model order reduction. Continuing the same, this paper revisits few noteworthy estimation techniques for simplification of discrete-time interval system. In particular, denominator is derived through reciprocal algorithm and numerator by two varied algorithms. The proposed algorithms are validated with examples from literatures and real-time test systems via assessment of error computation. Limitation encountered during the course is also taken into count.

Keywords: Discrete-time Interval System, Bilinear Transformation, Routh Approximation, Direct Truncation, Pade Approximation, Mixed Approach, Order Reduction.

1. INTRODUCTION

Emergence of order reduction methodologies is an attractive field of research till date as seen in the survey papers [Bultheel and Barel, 1986; Genesio and Milanese, 1976; Gugercin and Antoulas, 2004; Hwang and Lee, 1997]. With the time span, system's complexity increased for accounting unmodelled dynamics, parameter variation, disturbances, actuators, etc. leading to ambiguity. Systems as flight vehicles, electric motors, and robots are formulated under continuous-time domain of vague structure. These fears are handled by considering interval system, instead of fixed coefficient mathematical representation. Algorithms notably Routh-Pade [Bandyopadhyay, Ismail and Gorez, 1994], Pade approximation [Bandyopadhyay and Ismail, 1995], Routh approximants [Sastry, Raja Rao and Rao, 2000], $\gamma-\delta$ Routh approximation [Bandyopadhyay, Upadhye and Ismail, 1997], are available for the reduction of continuous-time interval systems.

The outburst of discrete-time signals and systems, grabbed the interest of order reduction of discrete-time interval system for being considerably simple and computationally easy. Techniques for such systems are few but are proficient, specifically Pade approximation allowing dominant poles retention [Ismail, Bandyopadhyay and Gorez, 1997], accessing higher-order integrators [Hsu and Wang, 2000], μ-dependent approach [Zhang, Boukas and Shi, 2009], and pole retention with direct series expansion [Singh and Chandra, 2011]. In [Dolgin and Zeheb, 2004], finite impulse response is used for order reduction of discrete-time interval system. In recent past, algorithms by [Choudhary and Nagar, 2013 (a, b)] are applied to discrete-time interval system, showing their acceptable extension to interval structure from fixed coefficient system. Freshly, in [Choudhary and Nagar, 2015] a glimpse of algorithm similar to the methodology briefed in this paper is discovered with a significant difference between them.

Add on to the existing algorithms of discrete-time interval system is proposed in this paper. The techniques discussed here exist for fixed coefficient and continuous-time interval system, yet, state to be novel for order reduction of discrete-time interval system. The attempt in the paper is to propose new mixed methods. Precisely, denominator is computed by a new algorithm of reciprocity and numerator by two different prevailing techniques namely direct truncation and Pade approximation as explained in the next section. The algorithms are illustrated through examples from the literatures and are compared for their validation based on the error sum between the original and reduced representations in section 3. Two real-time test systems are also considered to strengthen the algorithm. Section 4 discusses the competence of the proposed algorithms taking the affordable limitation into count. Finally, paper concludes with an emergence of two varied simple and efficient mixed algorithms for obtaining reduced model of interval form.

2. METHODOLOGY

Conceive the higher order interval transfer function and its approximate lower order transfer function with $k<n$ be

$$G_n(z)=\frac{\left[M_{n-1}^-,M_{n-1}^+\right]z^{n-1}+\left[M_{n-2}^-,M_{n-2}^+\right]z^{n-2}+....+\left[M_0^-,M_0^+\right]}{\left[N_n^-,N_n^+\right]z^n+\left[N_{n-1}^-,N_{n-1}^+\right]z^{n-1}+.....+\left[N_0^-,N_0^+\right]}=\frac{M_n(z)}{N_n(z)} \tag{1}$$

$$R_k(z)=\frac{\left[m_{k-1}^-,m_{k-1}^+\right]z^{k-1}+\left[m_{k-2}^-,m_{k-2}^+\right]z^{k-2}+.....+\left[m_0^-,m_0^+\right]}{\left[n_k^-,n_k^+\right]z^k+\left[n_{k-1}^-,n_{k-1}^+\right]z^{k-1}+.....+\left[n_0^-,n_0^+\right]}=\frac{M_k(z)}{N_k(z)} \tag{2}$$

Proceeding towards the derivation of a simple representation of higher order system, Routh Approximation is considered for obtaining denominator polynomial. Prime concern for applying this approximation is its computational simplicity and possibility to attain stable reduced model.

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Conduct of continuous-time domain algorithm over $G_n(z)$, insist its transformation to an appropriate domain and is ended by substituting $z = \frac{1+w}{1-w}$, known as bilinear or Tustin transformation. This transforms the z-domain system to its w-domain equivalent; much closer to continuous-time domain system. The transformation result in

$$G_n(w) = \frac{\left[A_n^-, A_n^+\right]w^n + \left[A_{n-1}^-, A_{n-1}^+\right]w^{n-1} + +\ldots + \left[A_0^-, A_0^+\right]}{\left[B_n^-, B_n^+\right]w^n + \left[B_{n-1}^-, B_{n-1}^+\right]w^{n-1} + \ldots + \left[B_0^-, B_0^+\right]} = \frac{M_n(w)}{N_n(w)} \tag{3}$$

Consider the reciprocal form of the above denominator polynomial $N_n(w)$ to obtain the reduced denominator polynomial represented as $\hat{N}_n(w)$;

$$\hat{N}_n(w) = \frac{1}{w} N_n\left(\frac{1}{w}\right) = \left[B_0^-, B_0^+\right]w^n + \left[B_1^-, B_1^+\right]w^{n-1} + \ldots + \left[B_n^-, B_n^+\right] \tag{4}$$

Use $\hat{N}_n(w)$ to draft the first two rows of the Routh array as shown in Table I.

Table I: Routh Approximation for Denominator

$\left[B_0^-, B_0^+\right] = \left[B_{1,1}^-, B_{1,1}^+\right]$	$\left[B_2^-, B_2^+\right] = \left[B_{1,2}^-, B_{1,2}^+\right]$	$\left[B_4^-, B_4^+\right] = \left[B_{1,3}^-, B_{1,3}^+\right]$	..
$\left[B_1^-, B_1^+\right] = \left[B_{2,1}^-, B_{2,1}^+\right]$	$\left[B_3^-, B_3^+\right] = \left[B_{2,2}^-, B_{2,2}^+\right]$	$\left[B_5^-, B_5^+\right] = \left[B_{2,3}^-, B_{2,3}^+\right]$	..
$\left[B_{3,1}^-, B_{3,1}^+\right]$	$\left[B_{3,2}^-, B_{3,2}^+\right]$		
….			
$\left[B_{n,1}^-, B_{n,1}^+\right]$			

Entries down the third row in the table is computed by

$$\left[B_{i,j}^-, B_{i,j}^+\right] = \left[B_{i-2,j+1}^-, B_{i-2,j+1}^+\right] - \left[\alpha_{i-2}^-, \alpha_{i-2}^+\right]\left[B_{i-1,j+1}^-, B_{i-1,j+1}^+\right] \tag{5}$$

where $i=3,4,\ldots\ldots,n$ and $j=1,2,\ldots\ldots$

with $\left[\alpha_i^-, \alpha_i^+\right] = \dfrac{\left[B_{i,1}^-, B_{i,1}^+\right]}{\left[B_{i+1,1}^-, B_{i+1,1}^+\right]}$ $\quad i=1,2,\ldots k,\ldots..n$ (6)

provided $\left[B_{i+1,1}^-, B_{i+1,1}^+\right] \notin [0]$

The reduced denominator, $\hat{N}_k(w)$ is obtained according to equation (7) as stated for definite system [Hutton and Friedland, 1975]

$$\hat{N}_k(w) = \left[\alpha_k^-, \alpha_k^+\right] w \hat{N}_{k-1}(w) + \hat{N}_{k-2}(w) \tag{7}$$

with $\hat{N}_{-1}(w) = 1$, $\quad \hat{N}_0(w) = 1$

For instance, if $k=1,\ 2$ then denominator polynomial is

$$\hat{N}_1(w) = \left[\alpha_1^-, \alpha_1^+\right]w + [1,1] \tag{8a}$$

and $\hat{N}_2(w) = \left[\alpha_1^-, \alpha_1^+\right]\left[\alpha_2^-, \alpha_2^+\right]w^2 + \left[\alpha_2^-, \alpha_2^+\right]w + [1,1]$ (8b)

The resulting $\hat{N}_k(w)$ is reciprocated back to $N_k(w)$ which on inverse bilinear transformation give the required $N_k(z)$.

The numerator $M_k(z)$ is computed by implicating two algorithms discussed below;

2.1 Algorithm 1

Direct Truncation [Choudhary and Nagar, 2013 b] is hired for obtaining the reduced numerator polynomial declared as

$$M_k(z) = \left[m_{k-1}^-, m_{k-1}^+\right]z^{k-1} + \left[m_{k-2}^-, m_{k-2}^+\right]z^{k-2} + \ldots + \left[m_0^-, m_0^+\right] \tag{9}$$

2.2 Algorithm 2

Another prevailing technique; Pade approximation [Bultheel and Barel, 1986] used for obtaining the numerator polynomial is illustrated here. Once the denominator $N_k(w)$ exist, numerator $M_k(w)$ is obtained by matching first t time moments and l Markov parameters, such that $t+l=k$.

Assume the reduced model of order k be

$$\frac{\left[C_0^-, C_0^+\right] + \left[C_1^-, C_1^+\right]w + \ldots + \left[C_{k-1}^-, C_{k-1}^+\right]w^{k-1}}{\left[D_0^-, D_0^+\right] + \left[D_1^-, D_1^+\right]w + \ldots + \left[D_k^-, D_k^+\right]w^k} = \frac{M_k(w)}{N_k(w)} \tag{10}$$

Equate (10) and (3), cross multiply and compare left & right hand side for similar coefficients.

$$\frac{M_k(w)}{N_k(w)} = \frac{M_n(w)}{N_n(w)} \tag{11a}$$

$$\left[\left[C_0^-, C_0^+\right] + \ldots + \left[C_{k-1}^-, C_{k-1}^+\right]w^{k-1}\right]N_n(w) = M_n(w)\left[\left[D_0^-, D_0^+\right] + \ldots + \left[D_k^-, D_k^+\right]w^k\right] \tag{11b}$$

Place the obtained coefficient in equation (10) and apply inverse bilinear transformation, $w = \frac{z-1}{z+1}$ to obtain $R_k(z)$.

Reduced models are validated for their acceptable existence through the error sum computation known as performance index and expressed as

$$J = \sum_{k=0}^{\infty}\left[y(k) - y_k(k)\right]^2 \tag{12}$$

where, $y(k)$ and $y_k(k)$ are the unit step responses of original system $G_n(z)$ and its reduced model $R_k(z)$.

3. ILLUSTRATIVE EXAMPLES

The algorithms proposed are examined on the examples from literatures and two real-time test systems. The obtained results are then compared with the existing techniques to validate their potential.

Example 1: Consider the higher order system available from literatures [Ismail, Bandyopadhyay and Gorez, 1997; Singh and Chandra, 2011; Choudhary and Nagar, 2013 (a, b), 2015] be

$$G_3(z)=\frac{[1,2]z^2+[3,4]z+[8,10]}{[6,6]z^3+[9,9.5]z^2+[4.9,5]z+[0.8,0.85]} \quad (13)$$

By the proposed algorithm, its *w*-domain representation is

$$G_3(w)=\frac{[-9,-5]w^3+[17,27]w^2+[-34,-24]w+[12,16]}{[0.55,1.2]w^3+[5.9,6.65]w^2+[19.45,20.2]w+[20.7,21.35]} \quad (14)$$

$$\hat{N}_3(w)=\frac{1}{w}N_3\left(\frac{1}{w}\right)=[20.7,21.35]w^3+[19.45,20.2]w^2+[5.9,6.65]w+[0.55,1.2] \quad (15)$$

From $\hat{N}_3(w)$, required Routh array is outlined in Table II;

Table II: Denominator Table

w^3	$[20.70,21.35]$	$[5.90,6.65]$
w^2	$[19.45,20.20]$	$[0.55,1.20]$
w^1	$[4.58,6.08]$	
w^0	$[0.55,1.20]$	

Required parameters procured from the above table are

$[\alpha_1^-,\alpha_1^+]=[1.02,1.09]$, $[\alpha_2^-,\alpha_2^+]=[3.19,4.40]$

The second order reduced denominator polynomial by equation (8) result in

$$\hat{N}_2(w)=[3.27,4.83]w^2+[3.19,4.40]w+[1,1] \quad (16)$$

On appropriate reciprocal transformation and inverse bilinear transformation gives the reduced denominator as

$$N_2(z)=[7.46,10.24]z^2+[4.54,7.66]z+[-0.13,2.63] \quad (17)$$

Numerators by the two varied algorithms resulting to the overall reduced model are as;

Algorithm 1:

Direct truncation, result the reduced model as

$$R_2(z)=\frac{[3,4]z+[8,10]}{[7.46,10.23]z^2+[4.54,7.66]z+[-0.12,2.63]} \quad (18)$$

Algorithm 2:

Pade approximation through equation (11) provide $[C_0^-,C_0^+]=[1.83,3.73]$ and $[C_1^-,C_1^+]=[-9.73,-2.05]$ which result, $M_2(w)$ as

$$M_2(w)=[-9.7351,-2.0504]w+[1.8389,3.7368] \quad (19)$$

And the overall reduced model after inverse bilinear transformation as

$$R_2(z)=\frac{[-7.89,1.68]z^2+[3.67,7.47]z+[3.88,13.47]}{[7.46,10.24]z^2+[4.54,7.66]z+[-0.13,2.63]} \quad (20)$$

Table III, is used to display the results obtained by the proposed algorithms and the existing ones.

Table III. Error for 1st and 2nd Order Reduced Models of the Best Case with the Prevailing Techniques

Methods	Error	
	Lower Limit	Upper Limit
Proposed Algorithm 1	*0.0553*	*0.0033*
Proposed Algorithm 2	*1.1265*	*0.2183*
Choudhary and Nagar 2015	0.1079	0.0342
Ismail, Bandyopadhyay and Gorez, 1997	0.1810	0.0741
Singh and Chandra, 2011	0.3237	0.3229
Choudhary and Nagar, 2013 a	0.1292	0.0443
Choudhary and Nagar, 2013 b	0.0278	0.0077

Example 2: Consider another example from [Ismail, Bandyopadhyay and Gorez, 1997] be

$$G_2(z)=\frac{[0.3428,0.4117]z+[0.2301,0.3372]}{z^2-[1.1866,1.5114]z+[0.3012,0.5488]} \quad (21)$$

with its *w*-domain equivalent as

$$G_2(w)=\frac{[-0.18,-0.01]w^2+[-0.67,-0.46]w+[0.57,0.74]}{[2.48,3.06]w^2+[-0.21,0.36]w+[1.32,1.39]} \quad (22)$$

Table IV: Denominator Table

w^2	$[1.32,1.39]$	$[2.48,3.06]$
w^1	$[-0.21,0.36]$	
w^0	$[2.48,3.06]$	

Above table give $[\alpha_1^-,\alpha_1^+]=[-6.64,3.85]$ to obtain

$$N_1(z)=[-5.64,4.85]z+[-7.64,2.85] \quad (23)$$

Algorithm 1:

Reduced model derived is

$$R_1(z)=\frac{[0.23,0.33]}{[-5.64,4.85]z+[-7.64,2.85]} \tag{24}$$

Algorithm 2:

The reduced model of system described in (21) is

$$R_1(z)=\frac{[-3.75,2.17]}{[-5.64,4.85]z+[-7.64,2.85]} \tag{25}$$

Cumulative error for comparing the algorithms with the prevailing one is made known in Table V.

Table V: Cumulative Errors of Example 2

Methods	Error Summation	
	Lower Limit	Upper Limit
Proposed Algorithm 1	*0.1471*	*0.1172*
Proposed Algorithm 2	*0.1038*	*0.0014*
Ismail, Bandyopadhyay and Gorez, 1997	2.888	0.1411

Example 3: Consider a real-time digital control system shown in figure 1, where

$$D(z)=\frac{1.68z^6-0.566z^5+0.356z^4-0.204z^3-0.312z^2+0.05z-0.006}{z^6+1.159z^5+0.76z^4+0.466z^3+0.096z^2-0.016z+0.003} \tag{26}$$

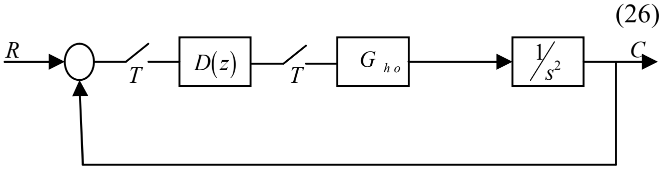

Figure 1: Digital Control system

With $T=(0.5)^{1/2}sec$, and robustness of the system into count, the overall transfer function alters to

$$G_8(z)=\frac{\begin{array}{l}[1.6484,1.7156]z^7+[1.0937,1.1383]z^6+[-0.2142,-0.2058]z^5\\+[0.1490,0.1550]z^4+[-0.5263,-0.5057]z^3+[-0.2672,-0.2568]z^2\\+[0.0431,0.0449]z+[-0.0061,-0.0059]\end{array}}{\begin{array}{l}[23.52,24.48]z^8+[-1.7156,-1.6484]z^7+[-1.1383,-1.0937]z^6\\+[0.2058,0.2142]z^5+[-0.1550,-0.1490]z^4+[0.5057,0.5263]z^3\\+[0.2568,0.2672]z^2+[-0.0449,-0.0431]z+[0.0059,0.0061]\end{array}} \tag{27}$$

By the algorithms, the reduced models are obtained as

Algorithm 1:

$$R_1(z)=\frac{[-0.006,-0.005]}{[1.11,1.13]z+[-0.88,-0.86]} \tag{28}$$

$$R_2(z)=\frac{[0.04,0.04]z+[-0.006,-0.005]}{[1.38,1.45]z^2+[-1.91,-1.89]z+[0.64,0.71]} \tag{29}$$

Algorithm 2:

$$R_1(z)=\frac{[0.01,0.01]z+[0.01,0.01]}{[1.11,1.13]z+[-0.88,-0.86]} \tag{30}$$

$$R_2(z)=\frac{[0.02,0.05]z^2+[0.007,0.01]z+[-0.04,-0.01]}{[1.38,1.45]z^2+[-1.91,-1.89]z+[0.64,0.71]} \tag{31}$$

Table VI, present the cumulative error computed for the obtained reduced models.

Example 4: Let automatic voltage regulator be defined by the block diagram in Fig. 2, with $T_1=5$, $T_2=2$, $T_3=0.07$, $T_4=0.04$, $T_5=0.1$, $a_1=2.5$, $a_2=3.2$, $a_3=6$, $a_4=3$, $a_5=3$, through zero order hold equivalent with sampling time $t=0.1sec$. Allowing the perturbation in the system result the overall transfer function as

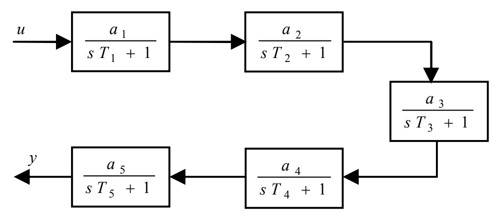

Figure 2: Higher Order Model of AVR

$$G_{AVR}(z)=\frac{\begin{array}{l}[0.98,1.02]z^4+[11.4562,11.9238]z^3\\+[11.9266,12.4134]z^2+[1.8228,1.8972]z\\+[0.0323,0.0337]\end{array}}{\begin{array}{l}[173.5384,180.6216]z^5+[-459.6936,-441.6664]z^4\\+[360.9634,375.6966]z^3+[-91.4328,-87.8472]z^2\\+[-3.9576,-3.8024]z+[-1.2138,-1.1662]\end{array}} \tag{31}$$

The reduced order models obtained are

Algorithm 1

$$R_1(z)=\frac{[0.03,0.03]}{[0.51,1.48]z+[-1.48,-0.51]} \tag{32}$$

$$R_2(z)=\frac{[1.82,1.89]z+[0.03,0.03]}{[0.85,1.11]z^2+[-2.09,-1.90]z+[0.88,1.14]} \tag{33}$$

Algorithm 2

$$R_1(z)=\frac{[-0.60,0.60]z+[-0.60,0.60]}{[0.51,1.48]z+[-1.48,-0.51]} \tag{34}$$

$$R_2(z)=\frac{[-0.35,0.35]z^2+[-0.11,0.11]z+[-0.35,0.35]}{[0.85,1.11]z^2+[-2.09,-1.90]z+[0.88,1.14]} \tag{35}$$

Cumulative error is made known in Table VII.

Table VI: Cumulative Error for 1^{st} and 2^{nd} Order Reduced Models Example 3

Methods	Error Summation			
	1st Order		2nd Order	
	Lower Limit	Upper Limit	Lower Limit	Upper Limit
Proposed Algorithm 1	*0.0057*	*0.0057*	*0.0015*	*0.0015*
Proposed Algorithm 2	*0.0021*	*0.0016*	*0.0011*	*0.0021*

Table VII: Cumulative Error for 1^{st} and 2^{nd} Order Reduced Models Example 4

Methods	Error Summation			
	1^{st} Order		2^{nd} Order	
	Lower Limit	Upper Limit	Lower Limit	Upper Limit
Proposed Algorithm 1	*0.0032*	$2.9294*10^{-4}$	*4.4863*	*2.8770*
Proposed Algorithm 2	*3.1492*	*1.0709*	*2.5712*	*1.0126*

4. DISCUSSION

From the above tables of cumulative error, limitation of getting a relatively higher error sum is clearly observed (*For example: In Table 1; lower limit of Proposed Algorithm 1 is higher than Choudhary and Nagar 2013b; Upper limit of Proposed Algorithm 2 is higher than Ismail, Bandyopadhyay and Gorez, 1997; Choudhary and Nagar, 2013 a, b*). Similar limitation is observed for example 2. This limitation is considered with a confrontation that these error differences are very minute and the algorithms proposed are computationally simple and easy relative to the prevailing ones. Negligence of this limitation is also strengthened, when these proposed algorithms are applied to the real-time systems and error sum obtained is minimal as desired.

Moreover, an attempt to check the stability of this uncertain system is also performed. The Khariton software package is used to check the stability. When checked, it is found that example 2, 3 and 4 results in stable reduced model but example 1 do not. This leaves a problem for future research as to find a suitable reduction methodology that result in a stable reduced model.

5. CONCLUSION

An attempt to provide new techniques for order reduction of discrete-time interval system is achieved successfully. Though the considered methodologies are in existence for their sole importance, yet, proofs themselves to be new as per the elaboration in this paper. The method to find the reduced denominator polynomial is fresh. From the two algorithms for finding the numerator polynomial, *Algorithm 1* uses Direct Truncation which earlier exists for discrete-time interval system but in this paper; it's used in mixed form. For *Algorithm 2* again a mixed nature is used for obtaining the reduced model. During the course of computing the reduced model a limitation derived is also discussed. The paper leaves a possible future work for researchers and control engineers to develop an algorithm based on this simple methodology to generate stable reduced models. Finally, the paper ends with a confrontation of two mixed approaches to derive reduced models.

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