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Exponentiated Weibull distribution approach based inflection S-shaped software reliability growth model

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KEYWORDS

Weibull distribution;
Software reliability growth model;
Reliability approximation;
Coefficient of multiple determinations;
Relative prediction error;
Mean square fitting error

Abstract The aim of this paper was to estimate the number of defects in software and remove them successfully. This paper incorporates Weibull distribution approach along with inflection S-shaped Software Reliability Growth Models (SRGM). In this combination two parameter Weibull distribution methodology is used. Relative Prediction Error (RPE) is calculated to predict the validity criterion of the developed model. Experimental results on actual data from five data sets are compared with two other existing models, which expose that the proposed software reliability growth model predicts better estimation to remove the defects. This paper presents best software reliability growth model with including feature of both Weibull distribution and inflection S-shaped SRGM to estimate the defects of software system, and provide help to researchers and software industries to develop highly reliable software products.

1. Introduction

The Software reliability model specifies the general form of the dependence of the failure process on the factors mentioned. Most software reliability models (SRM) are based on using a stable programme in a stable way. This means that neither the code nor the operational profile is changing. If the programme and environment do change, they often do so and are usually handled in a piecewise fashion. Thus the models focus mainly on fault removal. If either fault introduction, fault removal or operational profile changes are occurring,

Abbreviations: DS, data set; SPSS, Statistical Package for Social Sciences; MSE, Mean Square Fitting Error; R^2, Coefficient of Multiple Determination; RPE, Relative Prediction Error; SS, sum of squares; GFC, Goodness of Fit Criterion; PVC, Predictive Validity Criterion

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Table 1 Parameter estimation for 5 data sets according to distinct SRGMs.

Data set	Model discretion	Parameter estimation		
		a	*b*	*r*
DS-I	Yamada delay S-shaped SRGM	103.984	0.265	–
	Ohba inflection S-shaped SRGM	110.829	0.172	0.837
	Developed SRGM	116.733	0.273	0.169
DS-II	Yamada delay S-shaped SRGM	127.399	0.242	–
	Ohba inflection S-shaped SRGM	124.445	0.254	0.209
	Developed SRGM	129.708	0.332	0.110
DS-III	Yamada delay S-shaped SRGM	76.695	0.288	–
	Ohba inflection S-shaped SRGM	62.630	0.568	0.058
	Developed SRGM	63.240	0.595	0.051
DS-IV	Yamada delay S-shaped SRGM	47.229	0.207	–
	Ohba inflection S-shaped SRGM	43.363	0.279	0.134
	Developed SRGM	44.575	0.354	0.076
DS-V	Yamada delay S-shaped SRGM	1689.370	0.090	–
	Ohba inflection S-shaped SRGM	1331.053	0.201	0.047
	Developed SRGM	1485.927	0.222	0.036

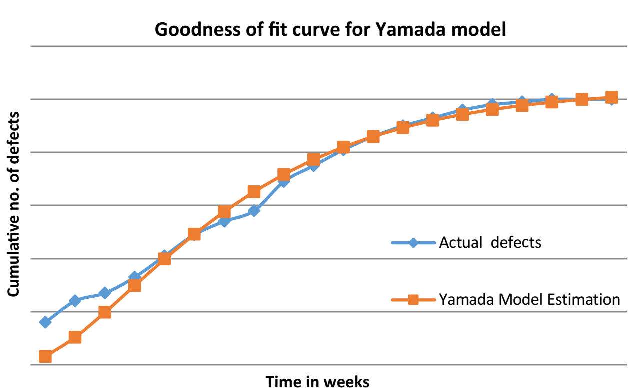

Figure 1a Goodness of fit of Yamada model on DS-I.

the failure intensity will be constant, and the model should simplify to accommodate this fact. In general terms, a good model enhances communication on a project and provides a common framework of understanding for the software development process developing a software reliability model that is useful in practice involves substantial theoretical work, tool building and the accumulation of a body of loss from practical experience. Research on software reliability engineering has been conducted during the past three decades and numerous statistical models have been proposed for estimating software reliability [1]. Most existing models for predicting software

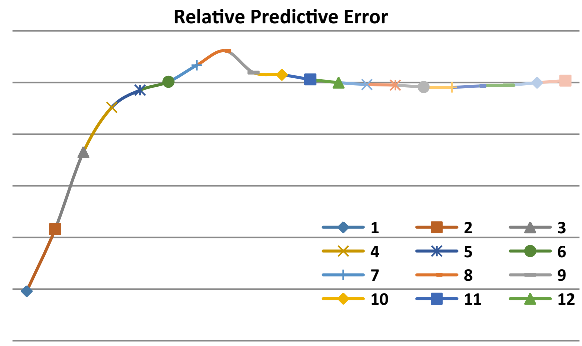

Figure 1b RPE of Yamada model on DS-I.

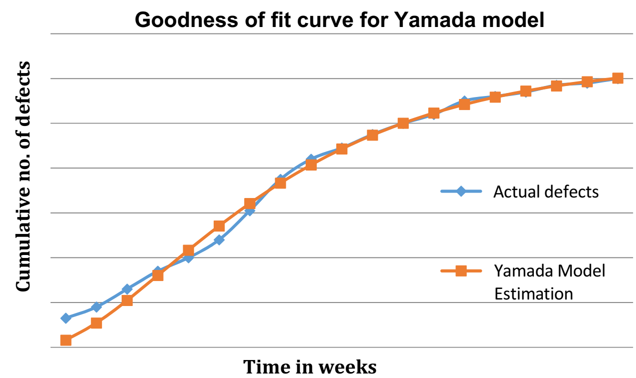

Figure 2a Goodness of fit of Yamada model on DS-II.

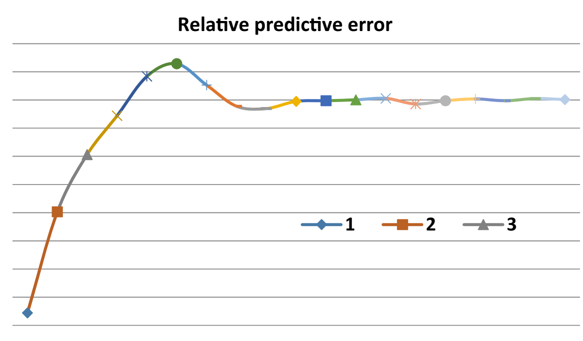

Figure 2b RPE of Yamada model on DS-II.

reliability are based purely on the observation of software product failures where they require a considerable amount of failure data to obtain an accurate reliability prediction. To estimate the failure and faults in software products Software Reliability Growth Models (SRGM) have been developed to measuring the growth of reliability of software which is being improved. The component based software system reliability increases as the component reliability increases [2]. Software reliability modelling and estimation are a measure concern in the software development process particularly during the software testing phase as unreliable software can cause a failure in the computer system that can be hazardous [3]. The software error detection phenomenon in software testing in model by a Nonhomogeneous Poisson Process (NHPP) is presented.

Table 2 Yamada model estimation on DS-I.

Test weeks	Execution hrs/efforts	Actual defects	Yamada model estimation	Estimated defects–actual defects	(Estimated defects–actual defects)^2	MSE	R^2	RPE	RPE for curve
1	519	16	3.073942722	−12.92605728	167.0829567	5.051276	0.969	−0.02043	−0.80787858
2	968	24	10.36422371	−13.63577629	185.934395				−0.568157345
3	1430	27	19.73830936	−7.261690641	52.73215097				−0.268951505
4	1893	33	29.82872145	−3.17127855	10.05700764				−0.09609935
5	2490	41	39.79224889	−1.207751107	1.458662737				−0.029457344
6	3058	49	49.1398433	0.139843302	0.019556149				0.002853945
7	3625	54	57.61732946	3.617329462	13.08507244				0.066987583
8	4422	58	65.12248382	7.122483819	50.72977575				0.122801445
9	5218	69	71.64800816	2.648008159	7.011947212				0.03837693
10	5823	75	77.24283193	2.242831928	5.030295056				0.029904426
11	6539	81	81.98629708	0.986297078	0.972781927				0.012176507
12	7083	86	85.97132241	−0.028677594	0.000822404				−0.00033346
13	7487	90	89.29376389	−0.706236109	0.498769442				−0.007847068
14	7846	93	92.04599732	−0.954002681	0.910121116				−0.010258093
15	8205	96	94.31333292	−1.686667078	2.844845831				−0.017569449
16	8564	98	96.17229048	−1.827709521	3.340522092				−0.018650097
17	8923	99	97.69006237	−1.309937628	1.715936589				−0.013231693
18	9282	100	98.92470453	−1.075295474	1.156260357				−0.010752955
19	9641	100	99.92574508	−0.07425492	0.005513793				−0.000742549
20	10,000	100	100.7350057	0.735005691	0.540233365				0.007350057
		1389	1360.626465		505.1276266				

Table 3 Yamada model estimation on DS-II.

Test weeks	Execution hrs/efforts	Actual defects	Yamada model estimation	Estimated defects–actual defects	(Estimated defects–actual defects)^2	MSE	R^2	RPE	RPE for curve
1	384	13	3.172376338	−9.827623662	96.58218685	2.080806	0.990	−0.01099	−0.755971051
2	1186	18	10.85509886	−7.144901136	51.04961225				−0.396938952
3	1471	26	20.9651963	−5.034803703	25.34924833				−0.193646296
4	2236	34	32.10621472	−1.893785285	3.586422706				−0.055699567
5	2772	40	43.36944409	3.369444089	11.35315347				0.084236102
6	2967	48	54.18885232	6.188852318	38.30189302				0.128934423
7	3812	61	64.23582747	3.235827466	10.47057939				0.053046352
8	4880	75	73.34332153	−1.656678471	2.744583557				−0.022089046
9	6104	84	81.45162271	−2.548377291	6.49422682				−0.030337825
10	6634	89	88.56996558	−0.430034421	0.184929603				−0.004831847
11	7229	95	94.74967806	−0.250321944	0.062661076				−0.002634968
12	8072	100	100.0656809	0.065680883	0.004313978				0.000656809
13	8484	104	104.6039909	0.603990893	0.364804999				0.005807605
14	8847	110	108.4535028	−1.546497176	2.391653516				−0.014059065
15	9253	112	111.7007885	−0.299211473	0.089527505				−0.002671531
16	9712	114	114.4269967	0.426996735	0.182326212				0.003745585
17	10,083	117	116.7061911	−0.293808862	0.086323648				−0.002511187
18	10,174	118	118.6046521	0.60465207	0.365604126				0.00512417
19	10,272	120	120.1808047	0.180804651	0.032690322				0.001506705
		1478	1461.750206		249.6967414				

Least Square estimation (LSE) and Maximum likelihood estimation (MLE) are used for the reliability parameters. The software reliability data analysis used actual data sets. Several SRGM have been developed by a literature to monitor the relationship between expected faults removal and execution calendar time [4]. A fault identified from one release on a failure reported by the user is also expected to occur in the other existing software release and can be simultaneously removed if present. Software reliability growth models are the tool used to evaluate software quantitatively and estimate and predict the reliability of the software during testing operational environment. SRGM described the failure occurrence and\or failure removal phenomenon of the testing process and consequently enhancement in the reliability with respect to time (CPU time, calendar time or execution time or test cases) [5].

Most of the existing research in this area considers that similar testing efforts and strategy are required on debugging efforts. However this may not be true in practice. Different

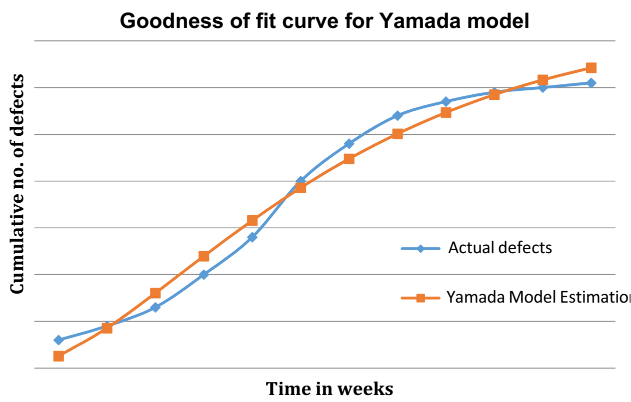

Figure 3a Goodness of fit of Yamada model on DS-III.

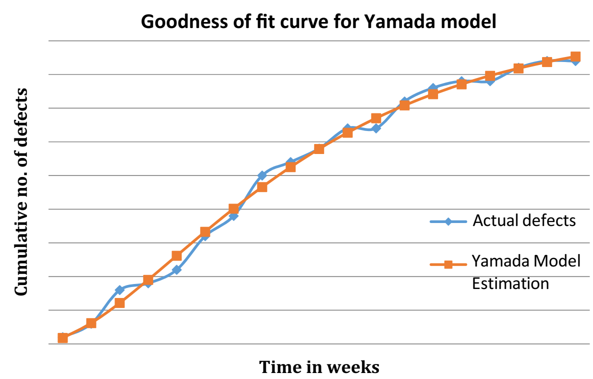

Figure 4a Goodness of fit of Yamada model on DS-IV.

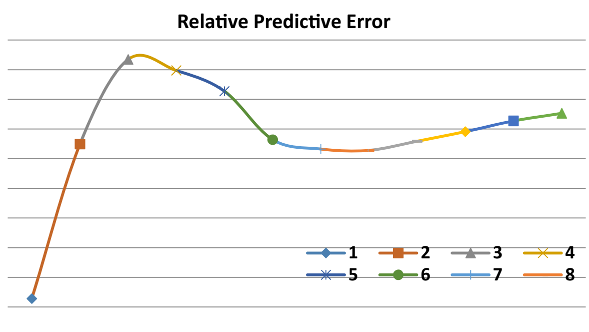

Figure 3b RPE of Yamada model on DS-III.

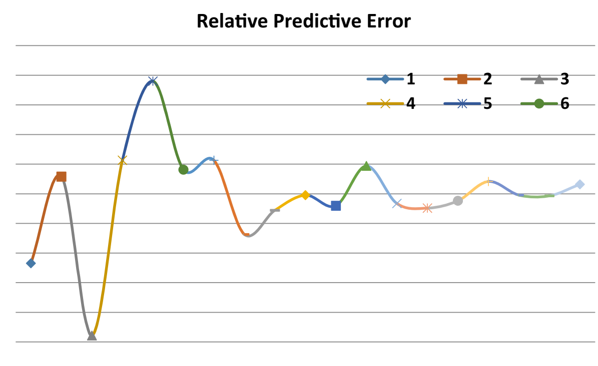

Figure 4b RPE of Yamada model on DS-IV.

faults may require different amount testing efforts and testing strategy for their removal [6]. The logistic testing efforts' functions into both exponential type and S-shaped software reliability model have been incorporated [7]. Most of the SRGM belongs to one of the two categories: exponential and S-shaped. In exponential SRGM software reliability growth model is defined by the mathematical relationship that exists between the time span of using a programme and the cumulative number of error discovered while S-shaped reliability growth is more often observed in real projects. There are many reasons why observed software reliability growth curves often become S-shaped. S-shape Software reliability growth curve is typically caused by the definition of failure [8]. The Weibull distribution is one of the most widely used lifetime distributions in reliability engineering (Hribar Lovre). Two parameter Weibull is the most popular distribution for analysing any lifetime data [9,10]. Weibull distribution has been applied in the area of reliability quality control duration, and failure time modelling. This distribution can be widely and effectively used

Table 4 Yamada model estimation on DS-III.

Test weeks	Execution hrs/efforts	Actual defects	Yamada model estimation	Estimated defects–actual defects	(Estimated defects–actual defects)^2	MSE	R^2	RPE	RPE for curve
1	162	6	2.570151037	−3.429848963	11.76386391	1.577445	0.981	0.00034	−0.571641494
2	499	9	8.542716316	−0.457283684	0.209108368				−0.050809298
3	715	13	16.05107681	3.051076812	9.309069712				0.234698216
4	1137	20	23.95047072	3.950470719	15.6062189				0.197523536
5	1799	28	31.57342782	3.573427822	12.7693864				0.127622422
6	2438	40	38.56244604	−1.437553958	2.066561382				−0.035938849
7	2818	48	44.75655551	−3.243444488	10.51993215				−0.06757176
8	3574	54	50.11526993	−3.884730066	15.09112768				−0.071939446
9	4234	57	54.66831071	−2.331689287	5.436774931				−0.04090683
10	4680	59	58.48296256	−0.51703744	0.267327715				−0.008763346
11	4955	60	61.64338111	1.643381107	2.700701464				0.027389685
12	5053	61	64.23791235	3.237912348	10.48407637				0.05308053
		455	455.1546809		96.22414899				

Table 5 Yamada model estimation on DS-IV.

Test weeks	Execution hrs/efforts	Actual defects	Yamada model estimation	Estimated defects–actual defects	(Estimated defects–actual defects)^2	MSE	R^2	RPE	RPE for curve
1	254	1	0.88268485	−0.11731515	0.013762844	0.443288	0.995	−3.95954E−05	−0.11731515
2	788	3	3.086805743	0.086805743	0.007535237				0.028935248
3	1054	8	6.08727767	−1.91272233	3.658506712				−0.239090291
4	1393	9	9.509188365	0.509188365	0.259272791				0.056576485
5	2216	11	13.08999535	2.089995352	4.36808057				0.189999577
6	2880	16	16.65060991	0.650609911	0.423293257				0.040663119
7	3593	19	20.07336619	1.073366194	1.152114987				0.056492958
8	4281	25	23.2853116	−1.714688404	2.940156323				−0.068587536
9	5180	27	26.24559864	−0.754401361	0.569121414				−0.027940791
10	6003	29	28.93602872	−0.063971279	0.004092325				−0.002205906
11	7621	32	31.35400981	−0.645990186	0.41730332				−0.020187193
12	8783	32	33.50735599	1.507355987	2.272122071				0.047104875
13	9604	36	35.41048615	−0.589513849	0.347526578				−0.016375385
14	10,064	38	37.0816806	−0.918319404	0.843310528				−0.0241663
15	10,560	39	38.54113261	−0.458867389	0.21055928				−0.01176583
16	11,008	39	39.80959378	0.809593778	0.655442085				0.020758815
17	11,237	41	40.90745902	−0.092540977	0.008563832				−0.002257097
18	11,243	42	41.85417434	−0.145825657	0.021265122				−0.003472039
19	11,305	42	42.66787851	0.667878512	0.446061706				0.015901869
		489	488.9806379		18.61809098				

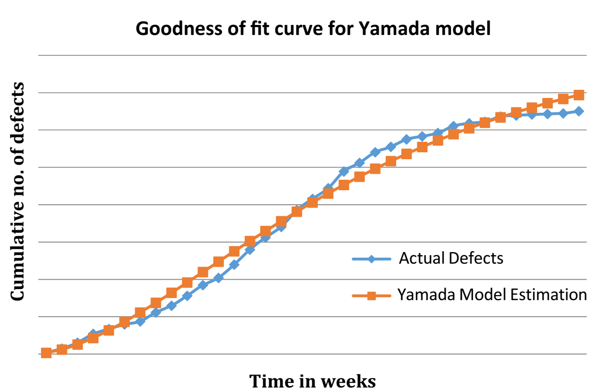

Figure 5a Goodness of fit of Yamada model on DS-V.

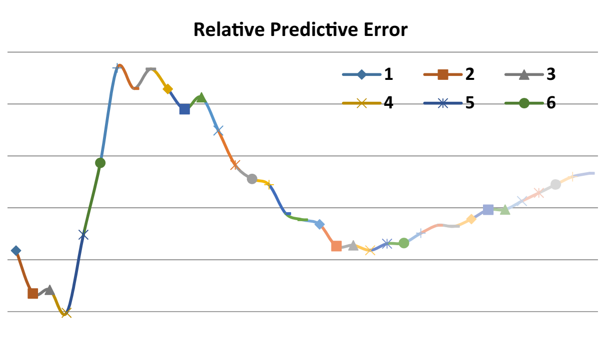

Figure 5b RPE of Yamada model on DS-V.

in reliability applications because it has wide variety of shapes in its density and failure rate functions making it useful for fitting much type of data. The amount of testing effort spent on software testing can be depicted as a Weibull type curve. In the modelling software development of effort was often described by the Weibull type curves. The discrete Weibull distribution can describe flexibility stochastic behaviour of the failure occurrence times. The Weibull distribution has the following properties: decrease software failure rate for $0 < \beta < 1$, constant failure rate $\beta = 1$, and increasing software failure rate $\beta > 1$ [11]. Maximum availability is illustrated in the case where the system failure obeys the Weibull distribution [12]. The Weibull based method is significantly better than the Laplacian based rate prediction [13]. Both logistic and Weibull distributions will result in a cumulative distribution function with an S-shaped for the lifetime software product [14]. Weibull distribution becomes special cases of the EW distribution. Thus the EW distribution is the useful and widely applicable reliability model for optimal accelerated life test [15]. The Weibull distribution fits the actual data, and it more appropriately describes the distribution of the software faults' full distribution [16].

This paper describes a new approach for estimation of the software reliability using Weibull S-shaped software reliability growth model. For this model based on new approach prediction established using curve fitting method and regression analysis. The MLE is used for fitting the Weibull probability function in actual defect data and regression analysis has been used to estimate the fault defects. This paper has been organized as follows: Section 2 presents Basic assumptions and Model descriptions by Software reliability growth model. Section 3 describes parameter estimation and various data sets used to check the validation of the model. Various Data analysis and compression between developed model and two other reliability growth models: Delay S-shaped software reliability growth model (Yamada model) and inflection S-shaped software reliability growth model (Ohba model) have been presented in Section 4. Discussion on various results and conclusion is incorporated in Section 5.

Table 6 Yamada model estimation on DS-V.

Test weeks	Execution hrs/efforts	Actual defects	Yamada model estimation	Estimated defects–actual defects	(Estimated defects–actual defects)^2	MSE	R^2	RPE	RPE for curve
1	7.25	7	6.42387355	−0.57612645	0.331921687	72.9920357	0.987	0.007861599	−0.082303779
2	10.42	29	24.21799158	−4.78200842	22.86760453				−0.164896842
3	17.5	61	51.38082997	−9.619170028	92.52843204				−0.157691312
4	24.83	108	86.17088907	−21.82911093	476.5100838				−0.202121397
5	32.08	134	127.0767851	−6.923214916	47.93090478				−0.051665783
6	44.66	159	172.7905615	13.79056151	190.1795867				0.086733091
7	64.58	175	222.1838881	47.18388814	2226.3193				0.269622218
8	117.08	223	274.2868484	51.28684842	2630.34082				0.229985867
9	164.26	259	328.269046	69.26904595	4798.200727				0.267448054
10	259.36	312	383.4227885	71.42278847	5101.214714				0.228919194
11	315.11	369	439.1481315	70.14813149	4920.760352				0.190103337
12	374.36	408	494.9395863	86.93958633	7558.491672				0.213087221
13	417.94	479	550.3743169	71.37431694	5094.293119				0.149006925
14	462.69	559	605.1016677	46.10166771	2125.363765				0.082471677
15	505.02	624	658.8338807	34.83388074	1213.399248				0.055823527
16	580.02	681	711.3378755	30.33787551	920.3866903				0.04454901
17	642.85	771	762.4279767	−8.572023257	73.47958272				−0.011118059
18	716.43	831	811.9594885	−19.04051153	362.5410792				−0.02291277
19	759.18	888	859.8230223	−28.17697768	793.9420711				−0.031730831
20	799.85	978	905.939498	−72.06050203	5192.715953				−0.073681495
21	896.6	1024	950.2557421	−73.74425787	5438.215569				−0.072015877
22	985.18	1081	992.7406201	−88.25937989	7789.718139				−0.08164605
23	1041.93	1110	1033.381641	−76.61835914	5870.372957				−0.069025549
24	1121.18	1150	1072.181983	−77.81801719	6055.643799				−0.067667841
25	1194.68	1166	1109.157893	−56.84210689	3231.025116				−0.048749663
26	1260.01	1184	1144.336418	−39.66358181	1573.199722				−0.033499647
27	1327.84	1221	1177.753428	−43.24657219	1870.266006				−0.035418978
28	1444.76	1236	1209.451899	−26.54810098	704.8016658				−0.021479046
29	1532.84	1244	1239.48043	−4.519570243	20.42651519				−0.003633095
30	1610.92	1272	1267.891956	−4.108044417	16.87602894				−0.003229595
31	1648.84	1278	1294.742645	16.74264532	280.3161724				0.013100661
32	1689.92	1283	1320.090955	37.09095454	1375.738908				0.028909551
33	1744.42	1286	1343.996818	57.99681778	3363.630872				0.045098614
34	1807.42	1289	1366.520963	77.52096277	6009.499668				0.06014039
35	1846.92	1301	1387.724332	86.72433154	7521.10968				0.066659748
		26,180	26385.81667		94962.63845				

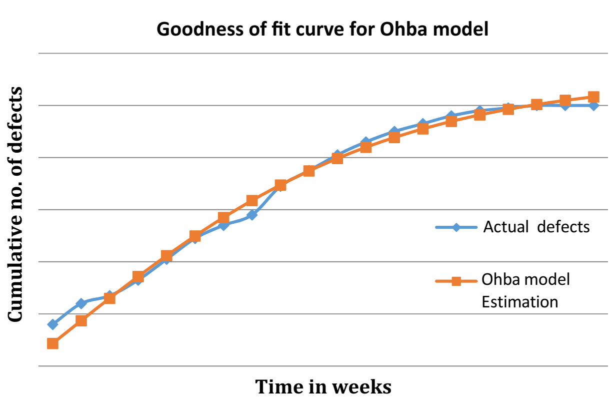

Figure 6a Goodness of fit of Ohba model on DS-I.

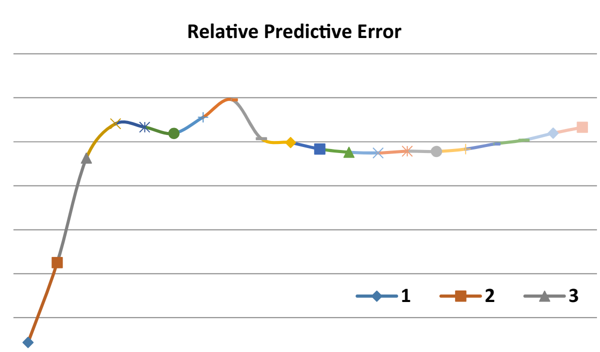

Figure 6b RPE of Ohba model on DS-I.

Table 7 Ohba model estimation on DS-I.

Test weeks	Execution hrs/efforts	Actual defects	Ohba model estimation	Estimated defects–actual defects	(Estimated defects–actual defects)2	MSE	R^2	RPE	RPE for curve
1	519	16	8.697695695	−7.302304305	53.32364816	1.795843	0.989	−0.006267571	−0.456394019
2	968	24	17.40593238	−6.594067624	43.48172784				−0.274752818
3	1430	27	25.99716534	−1.002834658	1.00567735				−0.037142024
4	1893	33	34.3506062	1.350606199	1.824137105				0.040927461
5	2490	41	42.35878041	1.358780411	1.846284206				0.033140986
6	3058	49	49.93254789	0.932547886	0.86964556				0.01903159
7	3625	54	57.00415646	3.004156464	9.02495606				0.055632527
8	4422	58	63.52823976	5.528239763	30.56143487				0.095314479
9	5218	69	69.48097541	0.480975406	0.231337341				0.006970658
10	5823	75	74.85782788	−0.142172118	0.020212911				−0.001895628
11	6539	81	79.67039072	−1.329609276	1.767860828				−0.016414929
12	7083	86	83.94282817	−2.057171833	4.231955949				−0.023920603
13	7487	90	87.70833022	−2.291669783	5.251750395				−0.025462998
14	7846	93	91.00587539	−1.994124607	3.976532949				−0.0214422
15	8205	96	93.87747405	−2.12252595	4.505116409				−0.022109645
16	8564	98	96.36596233	−1.634037673	2.670079116				−0.016673854
17	8923	99	98.5133414	−0.486658599	0.236836592				−0.004915743
18	9282	100	100.3596091	0.35960913	0.129318726				0.003596091
19	9641	100	101.942007	1.942007006	3.771391213				0.01942007
20	10,000	100	103.2945983	3.294598251	10.85437764				0.032945983
		1389	1380.294344		179.5842812				

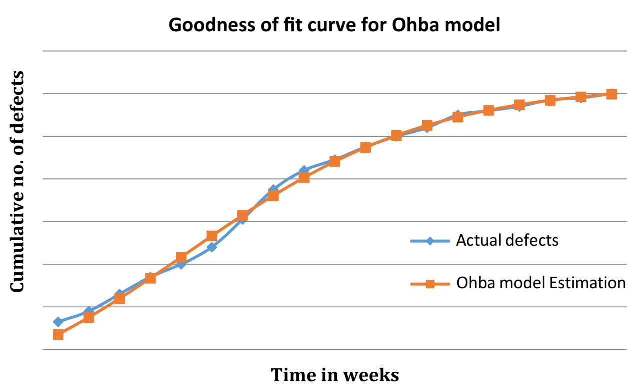

Figure 7a Goodness of fit of Ohba model on DS-II.

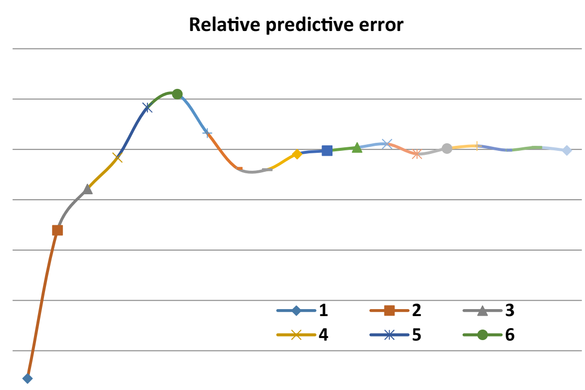

Figure 7b RPE of Ohba model on DS-II.

2. Software reliability growth model

Basic assumptions of the Software Reliability Growth Model (SRGM) are as follows:

- All faults in a programme are mutually independent from the failure detection point of view.
- The probability of failure detection at any time is proportional to the current number of faults in a programme.
- The proportionality of failure detections and fault isolations is constant.
- The probability of fault isolation at any time is proportional to the current number of faults not isolated.
- The detected faults can be entirely removed.
- The software system is subject to failures at random times caused by error remaining in the system.
- Error removal phenomenon in software testing is modelled by NHPP.

Basic descriptions on SRGMs are given next.

2.1. Exponential software reliability growth model

The most widely used model developed by Goel and Okumoto to analyse software failure data in a NHPP is exponential SRGM with mean value function [17].

$$m(t) = a(1 - e^{-\varphi t}) \quad (1)$$

2.2. Delay S-shaped software reliability growth model

The delay S-shaped SRGM defining the failure observation and fault removal as a two phase process consists of failure detection and its removal on isolation. It takes into account the time taken to isolate and remove a fault. It is further

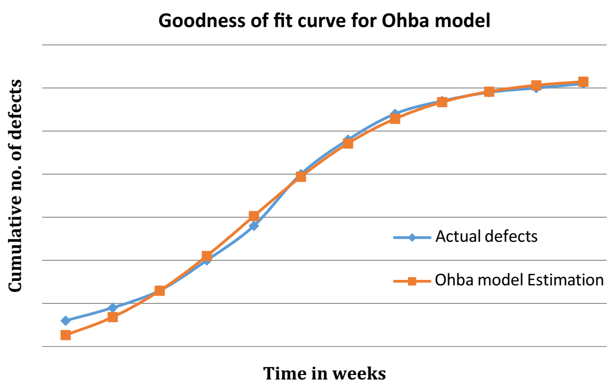

Figure 8a Goodness of fit of Ohba model on DS-III.

assumed that the number of faults isolated and removed at any time instant is proportional to the remaining number of detected faults to be removed from the software. Failure detection, fault isolation and removal rate per fault are assumed to be same and equal to b.

$$m_f'(t) = b(a - m_f(t)) \tag{2}$$

$$m_r'(t) = b(m_f(t) - m_r(t)) \tag{3}$$

where $m_f(t)$ is the expected number of failures in $(0, t]$. Solving (2) and (3) we get the mean value function as [17]

$$m_r(t) = a(1 - (1 + bt)e^{-bt}) \tag{4}$$

2.3. Inflection S-shaped software reliability growth model

The inflection S-shaped software reliability growth model has been developed to analyse the software failure detection process and its underlying reasons by modifying the logistic curve model which is widely used by Japanese computer makers for assessing the reliability growth of their software products. The underlying concept is that the observed software reliability growth becomes S-shaped if faults in a programme are mutually dependent [18].

$$m(t) = a\frac{1 - e^{-\varphi t}}{1 + \psi e^{-\varphi t}} \tag{5}$$

where φ is the failure detection rate in the sense of the Jelinski–Moranda model, ψ is the inflection parameter and $m(t)$ is the number of failures detected up to time t. The inflection parameter is defined for given r by the following equation:

$$\psi(r) = \frac{1 - r}{r}, \quad r > 0$$

where r is the inflection rate which indicates the ratio of the number of detectable faults to total number of faults in the programme.

$$m(t) = a\frac{(1 - e^{-bt})}{1 + \psi e^{-bt}}$$

$$m(t) = a\frac{(1 - e^{-bt})}{1 + \left(\frac{1-r}{r}\right)e^{-bt}} \tag{6}$$

An alternate life distribution model that is also widely used is the Weibull distribution. The Weibull distribution function may be stated in several ways, but the most general way is

$$F_T(t) = 1 - e^{-\left(\frac{t-\delta}{\theta-\delta}\right)^{\beta}} \tag{7}$$

This is called three parameter distribution functions. The parameter δ is a minimum life parameter which is after assumption to have value zero (0) and β is a shape parameter. The interpretation of the parameter δ is that it is the time before which no failures occur. When expressed in this manner, it seems reasonable to set $\delta = 0$. Then the form of the two parameter Weibull distribution function is

Table 8 Ohba model estimation on DS-II.

Test weeks	Execution hrs/efforts	Actual defects	Ohba model estimation	Estimated defects–actual defects	(Estimated defects–actual defects)^2	MSE	R^2	RPE	RPE for curve
1	384	13	7.086181539	−5.913818461	34.97324879	0.950217	0.995	−0.00437	−0.454909112
2	1186	18	15.10839391	−2.891606089	8.361385772				−0.160644783
3	1471	26	23.95908822	−2.040911784	4.16532091				−0.078496607
4	2236	34	33.45029828	−0.549701718	0.302171979				−0.016167698
5	2772	40	43.32325855	3.323258553	11.04404741				0.083081464
6	2967	48	53.27344944	5.273449443	27.80926903				0.10986353
7	3812	61	62.98677302	1.98677302	3.947267031				0.03257005
8	4880	75	72.17816501	−2.821834986	7.962752688				−0.037624466
9	6104	84	80.62293337	−3.37706663	11.40457902				−0.040203174
10	6634	89	88.17409138	−0.825908621	0.682125051				−0.009279872
11	7229	95	94.76416602	−0.235833981	0.055617667				−0.002482463
12	8072	100	100.3946609	0.394660864	0.155757197				0.003946609
13	8484	104	105.1186919	1.118691864	1.251471486				0.010756653
14	8847	110	109.0221493	−0.977850697	0.956191985				−0.008889552
15	9253	112	112.2070875	0.207087481	0.042885225				0.001848995
16	9712	114	114.779093	0.779092967	0.606985851				0.006834149
17	10,083	117	116.8388728	−0.161127164	0.025961963				−0.001377155
18	10,174	118	118.4774492	0.477449205	0.227957744				0.00404618
19	10,272	120	119.774036	−0.225964012	0.051059735				−0.001883033
		1478	1471.538839		114.0260565				

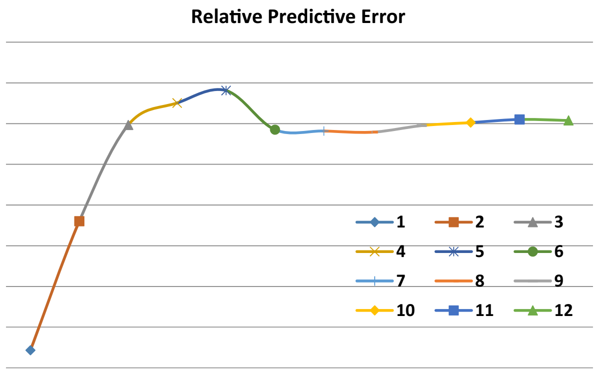

Figure 8b RPE of Ohba model on DS-III.

$$F_T(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^{\beta}} \quad (8)$$

This (8) is called Weibull distribution with two scale parameter. β is a shape parameter and θ is a scale parameter and assumed the value of $\theta = 1$, the form Weibull distribution becomes

$$F_T(t) = 1 - e^{(-t)^{\beta}}$$

Or

$$m(t) = 1 - e^{(-t)^{\beta}} \quad (9)$$

Compare Eq. (9) with Eq. (1).

$$m(t) = a(1 - e^{(-bt)}) = 1 - e^{(-t)^{\beta}}$$

Replace the value of nominator $(1 - e^{(-bt)})$ in Eq. (6) by (9).

$$m(t) = a\frac{(1 - e^{(-t)^{b}})}{1 + \left(\frac{1-r}{r}\right)e^{-bt}} \quad (10)$$

Eq. (10) is a developed model of this paper. Reliability indices comparisons among various SRGMs have been illustrated in the graphical presentation in the next sections.

Model notations

a: Total number of defects observed.
b: Error detection rate.
r: Inflection parameter.
t: Test time in weeks.
δ and θ: Scale parameter.
β: Shape parameter.

3. Parameter estimation

Parameter estimation and model validation are an important aspect of modelling. The mathematical equations of the proposed SRGMs are Nonlinear. Technically, it is more difficult to find the solution for nonlinear models using Least Square Method and requires numerical algorithms to solve it. Statistical software packages such as SPSS help to overcome this problem. SPSS is a *Statistical Package for Social Sciences*. For the estimation of the parameters of the proposed model, Method of Least Square (nonlinear regression method) has been used. Non-Linear Regression is a method of finding a nonlinear model of the relationship between the dependent variable and a set of independent variables. Variables of the developed software reliability growth model are as follows: 1. Total number of defects observed (a), 2. Error detection rate (b) and Inflection parameter (r), and these variables have been estimated for better goodness of fit and relative prediction error. Unlike traditional linear regression, which is restricted to estimating linear models, nonlinear regression can estimate model with arbitrary relationships between independent and dependent variables.

3.1. Model validation

To check the validity of the proposed model to describe the software reliability growth, it has been tested on different data sets, 4 (four) data sets cited from [19], which have 4 (four) releases and another from [20]. These data sets represent significant changes in fault detection as testing progress and hence suit better for analysis purpose.

3.2. DS-I, DS-II, DS-III, and DS-IV

These data are cited from [19] from the Release-1 (The software was tested for 20 weeks in which 100 faults were discovered.), Release-2 (The software was tested for 19 weeks in

Table 9 Ohba model estimation on DS-III.

Test weeks	Execution hrs/efforts	Actual defects	Ohba model estimation	Estimated defects–actual defects	(Estimated defects–actual defects)^2	MSE	R^2	RPE	RPE for curve
1	162	6	2.659044572	−3.340955428	11.16198317	0.412171	0.995	−0.0086	−0.556825905
2	499	9	6.840980399	−2.159019601	4.661365637				−0.239891067
3	715	13	12.95666659	−0.043333406	0.001877784				−0.003333339
4	1137	20	21.01168533	1.011685332	1.023507211				0.050584267
5	1799	28	30.27506689	2.275066893	5.175929368				0.081252389
6	2438	40	39.40287742	−0.597122584	0.356555381				−0.014928065
7	2818	48	47.12556899	−0.874431006	0.764629585				−0.018217313
8	3574	54	52.86054359	−1.139456414	1.298360919				−0.021101045
9	4234	57	56.72014364	−0.279856362	0.078319583				−0.004909761
10	4680	59	59.14856238	0.148562381	0.022070781				0.002518006
11	4955	60	60.61237611	0.612376111	0.375004502				0.010206269
12	5053	61	61.47204164	0.472041641	0.222823311				0.007738388
		455	451.0855576		25.14242723				

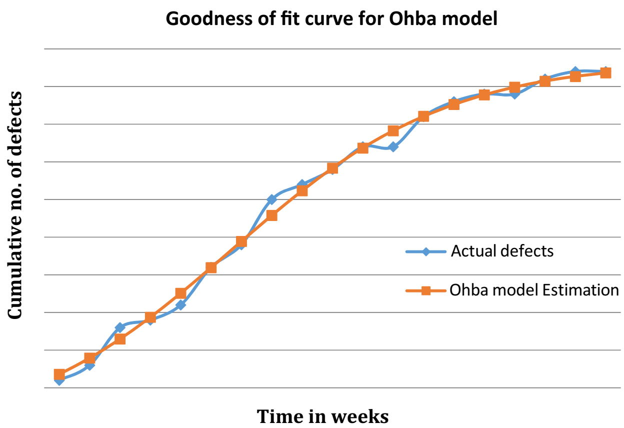

Figure 9a Goodness of fit of Ohba model on DS-IV.

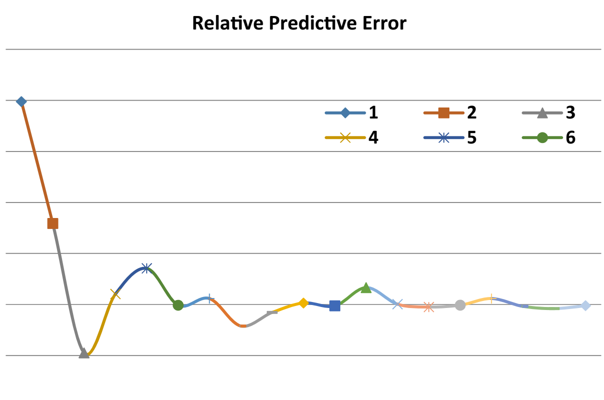

Figure 9b RPE of Ohba model on DS-IV.

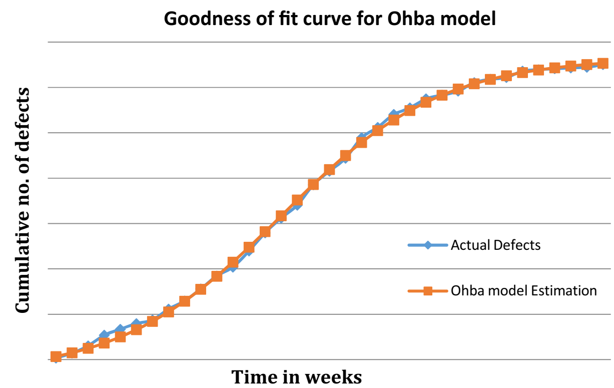

Figure 10a Goodness of fit of Ohba model on DS-V.

which 120 faults were discovered), Release-3 (The software was tested for 12 weeks in which 61 faults were discovered), and Release-4 (The software was tested for 19 weeks in which 42 faults were discovered) respectively. The Parameter Estimation results for the developed SRGM are given in Table 1.

3.3. DS-V

These data are cited from [20], and fault data set is for a radar system of size 124 KLOC (Kilo Line of Code) tested for 35 weeks in which 1301 faults were removed. The Parameter estimation results for the developed SRGM are given in Table 1.

4. Data analysis and model comparison

4.1. Yamada delay S-shaped software reliability growth model

Fig. 1a is based on DS-I, in which cumulative number of defects and test time in weeks are denoted by *Y*-axis and

Table 10 Ohba model estimation on DS-IV.

Test weeks	Execution hrs/efforts	Actual defects	Ohba model estimation	Estimated defects–actual defects	(Estimated defects–actual defects)^2	MSE	R^2	RPE	RPE for curve
1	254	1	1.795337903	0.795337903	0.63256238	0.425454	0.995	0.002143	0.795337903
2	788	3	3.952718601	0.952718601	0.907672732				0.317572867
3	1054	8	6.483455937	−1.516544063	2.299905894				−0.189568008
4	1393	9	9.369627136	0.369627136	0.13662422				0.041069682
5	2216	11	12.55719435	1.557194346	2.424854233				0.141563122
6	2880	16	15.95523708	−0.044762918	0.002003719				−0.002797682
7	3593	19	19.4437047	0.443704699	0.19687386				0.023352879
8	4281	25	22.88918515	−2.110814851	4.455539336				−0.084432594
9	5180	27	26.16468153	−0.835318467	0.697756941				−0.030937721
10	6003	29	29.16743982	0.16743982	0.028036093				0.005773787
11	7621	32	31.82983271	−0.170167291	0.028956907				−0.005317728
12	8783	32	34.12149684	2.121496845	4.500748863				0.066296776
13	9604	36	36.04426381	0.04426381	0.001959285				0.00122955
14	10,064	38	37.62321043	−0.376789568	0.141970379				−0.009915515
15	10,560	39	38.89708866	−0.102911338	0.010590744				−0.002638752
16	11,008	39	39.91025656	0.910256564	0.828567013				0.023339912
17	11,237	41	40.70694971	−0.293050294	0.085878475				−0.007147568
18	11,243	42	41.32783086	−0.672169143	0.451811356				−0.016004027
19	11,305	42	41.80832661	−0.191673391	0.036738689				−0.004563652
		489	490.0478384		17.86905112				

Table 11 Ohba model estimation on DS-V.

Test weeks	Execution hrs/efforts	Actual defects	Ohba model estimation	Estimated defects–actual defects	(Estimated defects–actual defects)^2	MSE	R^2	RPE	RPE for curve
1	7.25	7	13.82504733	6.825047331	46.58127107	5.48234037	0.999	−0.002869663	0.975006762
2	10.42	29	30.3358062	1.3358062	1.784378205				0.046062283
3	17.5	61	49.96034745	−11.03965255	121.8739285				−0.180977911
4	24.83	108	73.15432167	−34.84567833	1214.221298				−0.32264517
5	32.08	134	100.3843839	−33.6156161	1130.009646				−0.250862807
6	44.66	159	132.1031917	−26.89680829	723.4382961				−0.169162316
7	64.58	175	168.7149892	−6.285010776	39.50136045				−0.035914347
8	117.08	223	210.5319952	−12.46800479	155.4511436				−0.055910335
9	164.26	259	257.7237728	−1.276227199	1.628755864				−0.004927518
10	259.36	312	310.264375	−1.735625047	3.012394306				−0.005562901
11	315.11	369	367.8848828	−1.115117193	1.243486354				−0.003021998
12	374.36	408	430.0411146	22.04111462	485.8107339				0.05402234
13	417.94	479	495.9066282	16.90662816	285.8340758				0.035295675
14	462.69	559	564.3986632	5.39866319	29.14556424				0.009657716
15	505.02	624	634.2391365	10.23913646	104.8399154				0.016408873
16	580.02	681	704.0452437	23.04524374	531.0832589				0.033840299
17	642.85	771	772.4368871	1.436887092	2.064644514				0.001863667
18	716.43	831	838.1436593	7.143659287	51.03186801				0.008596461
19	759.18	888	900.094238	12.09423804	146.2705937				0.013619637
20	799.85	978	957.4757219	−20.52427805	421.2459896				−0.020985969
21	896.6	1024	1009.75785	−14.24214956	202.8388241				−0.013908349
22	985.18	1081	1056.684553	−24.31544706	591.2409655				−0.022493476
23	1041.93	1110	1098.24067	−11.75933049	138.2818536				−0.010593991
24	1121.18	1150	1134.604009	−15.3959914	237.0365512				−0.013387819
25	1194.68	1166	1166.092446	0.092446035	0.008546269				7.92848E−05
26	1260.01	1184	1193.113551	9.113550668	83.05680578				0.007697256
27	1327.84	1221	1216.121392	−4.878607877	23.80081481				−0.003995584
28	1444.76	1236	1235.582595	−0.417405326	0.174227206				−0.000337707
29	1532.84	1244	1251.951779	7.951779008	63.23078939				0.006392105
30	1610.92	1272	1265.655363	−6.344636988	40.25441851				−0.004987922
31	1648.84	1278	1277.082153	−0.917846588	0.84244236				−0.00071819
32	1689.92	1283	1286.579059	3.579059246	12.80966508				0.002789602
33	1744.42	1286	1294.450411	8.450411297	71.40945108				0.006571082
34	1807.42	1289	1300.959639	11.95963875	143.032959				0.00927823
35	1846.92	1301	1306.332345	5.332345268	28.43390606				0.004098651
		26,180	26104.87222		7132.524822				

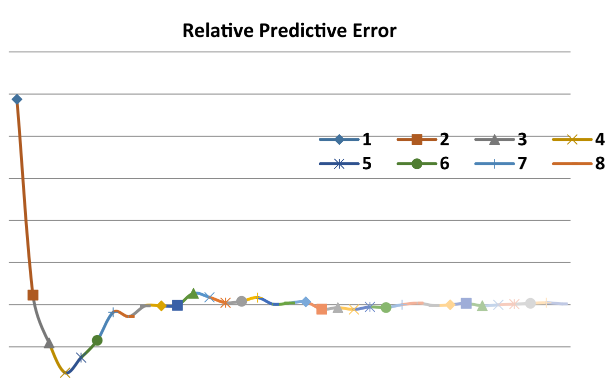

Figure 10b RPE of Ohba model on DS-V.

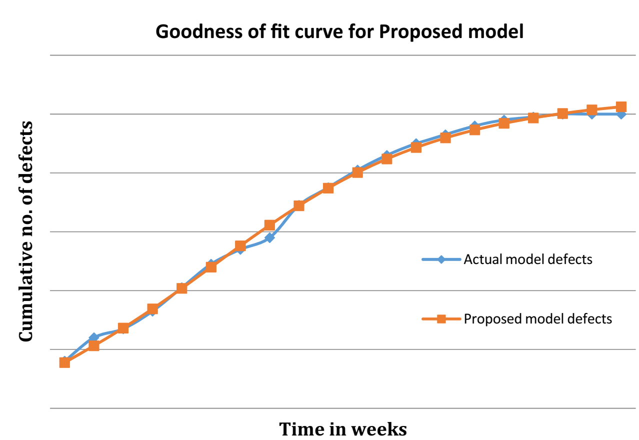

Figure 11a Goodness of fit of Developed model on DS-I.

X-axis respectively. In this *figure* Blue line represents actual defects and Red line represents Yamada model estimation for the removal of the actual defects. The estimation results are shown in Table 2. In the starting actual defects were found (16) from the data set and by the Yamada model (3.073942722) defects have been removed. The fitting of the model is

Table 12 Developed model estimation on DS-I.

Test weeks	Execution hrs/efforts	Actual defects	Developed model estimation	Proposed estimation–actual estimation	(Proposed estimation–actual estimation)^2	MSE	R^2	RPE	RPE for curve
1	519	16	15.52661312	−0.473386877	0.224095136	0.462765944	0.997	−0.000889851	−0.02958668
2	968	24	21.23806167	−2.761938328	7.628303326				−0.115080764
3	1430	27	27.26033885	0.260338848	0.067776316				0.00964218
4	1893	33	33.79128984	0.791289841	0.626139613				0.02397848
5	2490	41	40.76079899	−0.239201013	0.057217125				−0.005834171
6	3058	49	47.98830869	−1.011691307	1.023519302				−0.020646761
7	3625	54	55.23866753	1.238667526	1.53429724				0.022938288
8	4422	58	62.26709772	4.267097718	18.20812293				0.07357065
9	5218	69	68.85832382	−0.141676178	0.020072139				−0.002053278
10	5823	75	74.85368987	−0.146310134	0.021406655				−0.001950802
11	6539	81	80.16221283	−0.83778717	0.701887343				−0.010343051
12	7083	86	84.75685215	−1.243147846	1.545416567				−0.014455208
13	7487	90	88.66118073	−1.338819272	1.792437042				−0.01487577
14	7846	93	91.93251506	−1.067484942	1.139524102				−0.011478333
15	8205	96	94.64612497	−1.353875027	1.832977588				−0.014102865
16	8564	98	96.88293302	−1.11706698	1.247838639				−0.011398643
17	8923	99	98.72126737	−0.278732632	0.07769188				−0.002815481
18	9282	100	100.2321535	0.232153517	0.053895256				0.002321535
19	9641	100	101.4772253	1.47722529	2.182194557				0.014772253
20	10,000	100	102.5083424	2.508342403	6.291781609				0.025083424
		1389	1387.763997		46.27659436				

Table 13 Developed model estimation on DS-II.

Test weeks	Execution hrs/efforts	Actual defects	Developed model estimation	Proposed estimation–actual estimation	(Proposed estimation–actual estimation)^2	MSE	R^2	RPE	RPE for curve
1	384	13	12.08273502	−0.917264978	0.841375039	0.435366631	0.998	−0.000378905	−0.070558844
2	1186	18	18.03741627	0.037416271	0.001399977				0.002078682
3	1471	26	24.899993	−1.100007003	1.210015406				−0.042307962
4	2236	34	32.9051793	−1.0948207	1.198632365				−0.032200609
5	2772	40	41.96026552	1.96026552	3.842640908				0.049006638
6	2967	48	51.75374228	3.753742283	14.09058113				0.078202964
7	3812	61	61.8197989	0.819798896	0.672070229				0.013439326
8	4880	75	71.63697571	−3.363024286	11.30993235				−0.044840324
9	6104	84	80.74035245	−3.25964755	10.62530215				−0.038805328
10	6634	89	88.8055617	−0.194438297	0.037806251				−0.0021847
11	7229	95	95.67865775	0.67865775	0.460576342				0.007143766
12	8072	100	101.3558397	1.35583967	1.838301212				0.013558397
13	8484	104	105.9363773	1.93637729	3.749557008				0.018619012
14	8847	110	109.5727418	−0.427258204	0.182549573				−0.003884165
15	9253	112	112.4318857	0.431885657	0.186525221				0.003856122
16	9712	114	114.6712533	0.671253284	0.450580971				0.005888187
17	10,083	117	116.4271662	−0.572833816	0.32813858				−0.004896016
18	10,174	118	117.8114454	−0.188554643	0.035552853				−0.001597921
19	10,272	120	118.9125911	−1.087408946	1.182458216				−0.009061741
		1478	1477.439978		52.24399578				

illustrated graphically in Figs. 1a and 1b and shows the relative predictive error to check the validity of the model.

Fig. 2a is based on DS-II, in which cumulative number of defects and test time in weeks are denoted by Y-axis and X-axis respectively. In this *figure* Blue line represents actual defects and Red line represents Yamada model estimation for the removal of the actual defects. The estimation results are shown in Table 3. In the starting actual defects were found (13) from the data set and by the Yamada model (3.172376338) defects have been removed. The fitting of the model is illustrated graphically in Figs. 2a and 2b and shows the relative predictive error to check the validity of the model.

Fig. 3a is based on DS-I, in which cumulative number of defects and test time in weeks are denoted by Y-axis and X-axis respectively. In this *Figure* Blue line represents actual defects and Red line represents Yamada model estimation

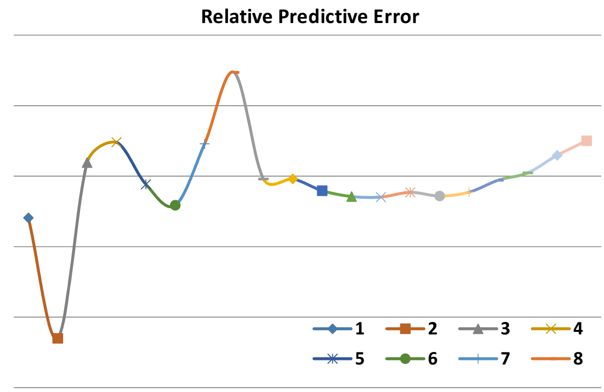

Figure 11b RPE of Developed model on DS-I.

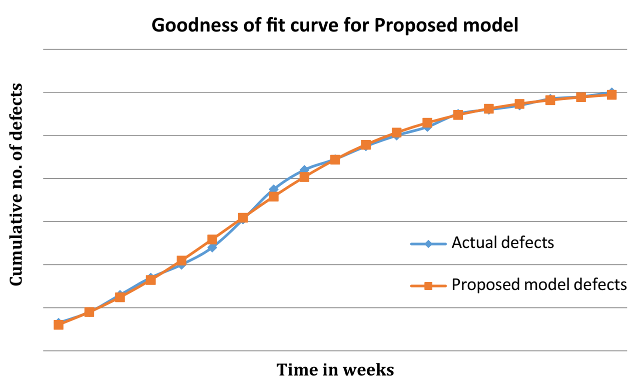

Figure 12a Goodness of fit of Developed model on DS-II.

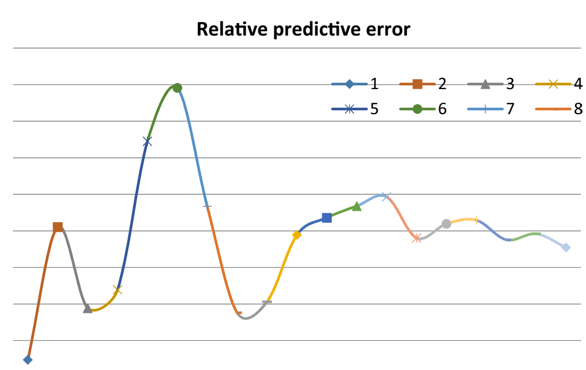

Figure 12b RPE of Developed model on DS-II.

for the removal of the actual defects. The estimation results are shown in Table 4. In the starting actual defects were found (6) from the data set and by the Yamada model (2.570151037) defects have been removed. The fitting of the model is illustrated graphically in Figs. 3a and 3b and shows the relative predictive error to check the validity of the model.

Fig. 4a is based on DS-I, in which cumulative number of defects and test time in weeks are denoted by *Y*-axis and *X*-axis respectively. In this *figure* blue line represents actual defects and Red line represents Yamada model estimation for the removal of the actual defects. The estimation results are shown in Table 5. In the starting actual defects were found

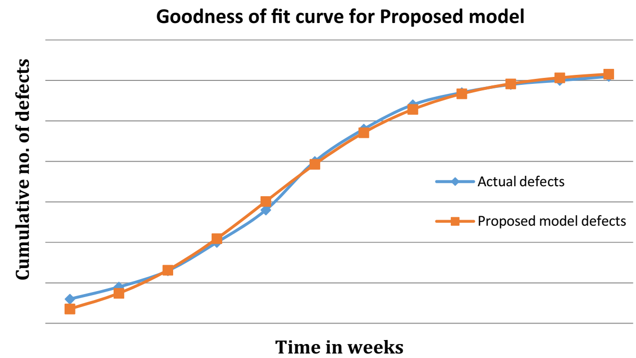

Figure 13a Goodness of fit of Developed model on DS-III.

(1) from the data set and by the Yamada model (0.088268485) defects have been removed. The fitting of the model is illustrated graphically in Figs. 4a and 4b and shows the relative predictive error to check the validity of the model.

Fig. 5a is based on DS-I, in which cumulative number of defects and test time in weeks are denoted by *Y*-axis and *X*-axis respectively. In this *figure* Blue line represents actual defects and Red line represents Yamada model estimation for the removal of the actual defects. The estimation results are shown in Table 6. In the starting actual defects were found (7) from the data set and by the Yamada model (6.42387355) defects have been removed. The fitting of the model is illustrated graphically in Figs. 5a and 5b and shows the relative predictive error to check the validity of the model.

4.2. Inflection S-shaped SRGM observation

Fig. 6a is based on DS-I, in which cumulative number of defects and test time in weeks are denoted by *Y*-axis and *X*-axis respectively. In this *figure* Blue line represents actual defects and Red line represents Ohba model estimation for the removal of the actual defects. The estimation results are shown in Table 7. In the starting actual defects were found (16) from the data set and by the Ohba model (8.697695695) defects have been removed. The fitting of the model is illustrated graphically in Figs. 6a and 6b and shows the relative predictive error to check the validity of the model.

Fig. 7a is based on DS-II, in which cumulative number of defects and test time in weeks are denoted by *Y*-axis and *X*-axis respectively. In this *figure* Blue line represents actual defects and Red line represents Ohba model estimation for the removal of the actual defects. The estimation results are shown in Table 7. In the starting actual defects were found (13) from the data set and by the Ohba model (7.086181539) defects have been removed. The fitting of the model is illustrated graphically in Figs. 7a and 7b and shows the relative predictive error to check the validity of the model.

Fig. 8a is based on DS-III, in which cumulative number of defects and test time in weeks are denoted by *Y*-axis and *X*-axis respectively. In this *figure* Blue line represents actual defects and Red line represents Ohba model estimation for the removal of the actual defects. The estimation results are shown in Table 8. In the starting actual defects were found (6) from the data set and by the Ohba model (2.6590445723) defects have been removed. The fitting of the model is illustrated

Table 14 Developed model estimation on DS-III.

Test weeks	Execution hrs/efforts	Actual defects	Developed model estimation	Proposed estimation–actual estimation	(Proposed estimation–actual estimation)^2	MSE	R^2	RPE	RPE for curve
1	162	6	3.561997647	−2.438002353	5.943855474	0.28248012	0.997	−0.005534319	−0.406333726
2	499	9	7.42494498	−1.57505502	2.480798315				−0.175006113
3	715	13	13.14310125	0.143101247	0.020477967				0.011007788
4	1137	20	20.92306401	0.923064005	0.852047158				0.0461532
5	1799	28	30.11218135	2.112181355	4.461310076				0.075435048
6	2438	40	39.29736578	−0.70263422	0.493694846				−0.017565855
7	2818	48	47.08888253	−0.911117473	0.83013505				−0.018981614
8	3574	54	52.8525886	−1.147411405	1.316552932				−0.021248359
9	4234	57	56.71778182	−0.282218175	0.079647098				−0.004951196
10	4680	59	59.15703631	0.157036308	0.024660402				0.002661632
11	4955	60	60.64929927	0.649299273	0.421589546				0.010821655
12	5053	61	61.55364112	0.553641116	0.306518486				0.009076084
		455	452.4818847		17.23128735				

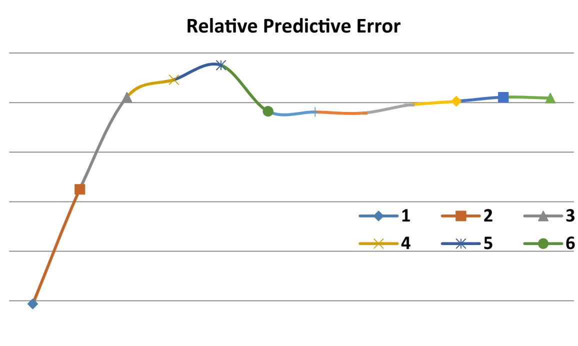

Figure 13b RPE of Developed model on DS-III.

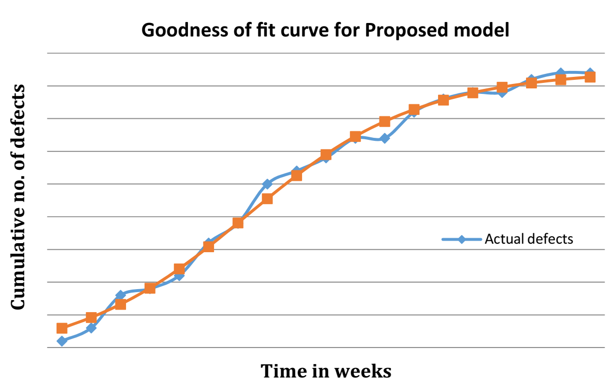

Figure 14a Goodness of fit of Developed model on DS-IV.

graphically in Figs. 8a and 8b and shows the relative predictive error to check the validity of the model.

Fig. 9a is based on DS-IV, in which cumulative number of defects and test time in weeks are denoted by Y-axis and X-axis respectively. In this *figure* blue line represents actual defects and Red line represents Ohba model estimation for the removal of the actual defects. The estimation results are shown in Table 9. In the starting actual defects were found (1) from the data set and by the Ohba model (1.795337903) defects have been removed. The fitting of the model is illustrated graphically in Figs. 9a and 9b and shows the relative predictive error to check the validity of the model.

Fig. 10a is based on DS-V, in which cumulative number of defects and test time in weeks are denoted by Y-axis and X-axis respectively. In this *figure* blue line represents actual defects and Red line represents Ohba model estimation for the removal of the actual defects. The estimation results are shown in Table 10. In the starting actual defects were found (7) from the data set and by the Ohba model (13.82504733) defects have been removed. The fitting of the model is illustrated graphically in Figs. 10a and 10b and shows the relative predictive error to check the validity of the model.

4.3. Developed model observation

Fig. 11a is based on DS-I, in which cumulative number of defects and test time in weeks are denoted by Y-axis and X-axis respectively. In this *figure* Blue line represents actual defects and Red line represents Proposed model estimation for the removal of the actual defects. The estimation results are shown in Table 12. In the starting (First Week) actual defects were found (16) from the data set (DS-I) and Developed model has removed (15.52661312) defects, while in Table 2 and Fig. 1a Yamada model removed (3.073942722) defects only. From Table 7 and Fig. 6a, Ohba model has removed (8.697695695) defects only. Similarly in the Second week of the same data set (24) defects were found. Yamada model removed (10.36422371) defects, while Ohba model removed (17.40593238) defects. For same data set developed model has removed (21.23806167) defects. Similarly all defect removal has been represented in Tables 3, 8 and 13. Graphical illustrations have also been represented in Figs. 1a, 6a, 11a respectively. Thus the performance of Developed model for the removal of software fault defects is better than the other two models namely: Yamada Delay S-shaped SRGM and Ohba inflection S-shaped SRGM. The best fitting of the model is illustrated graphically in Figs. 11a and 11b and shows relative predictive error to check the validity of the developed model.

Fig. 12a is based on DS-II, in which cumulative number of defects and test time in weeks are denoted by Y-axis and X-axis

Table 15 Developed model estimation on DS-IV.

Test weeks	Execution hrs/efforts	Actual defects	Developed model estimation	Proposed estimation–actual estimation	(Proposed estimation–actual estimation)^2	MSE	R^2	RPE	RPE for curve
1	254	1	2.956096201	1.956096201	3.826312348	0.590927427	0.993	0.003957993	1.956096201
2	788	3	4.601670069	1.601670069	2.565347011				0.533890023
3	1054	8	6.60778134	−1.39221866	1.938272796				−0.174027332
4	1393	9	9.081384065	0.081384065	0.006623366				0.009042674
5	2216	11	12.03852498	1.038524975	1.078534125				0.094411361
6	2880	16	15.41236939	−0.587630605	0.345309728				−0.036726913
7	3593	19	19.05397627	0.053976271	0.002913438				0.002840856
8	4281	25	22.75647865	−2.243521354	5.033388064				−0.089740854
9	5180	27	26.30190459	−0.698095408	0.487337199				−0.025855385
10	6003	29	29.51146252	0.511462524	0.261593913				0.017636639
11	7621	32	32.27708215	0.277082147	0.076774516				0.008658817
12	8783	32	34.56520699	2.565206991	6.580286908				0.080162718
13	9604	36	36.39983653	0.399836531	0.159869252				0.01110657
14	10,064	38	37.83834142	−0.161658579	0.026133496				−0.004254173
15	10,560	39	38.9503976	−0.049602403	0.002460398				−0.001271856
16	11,008	39	39.8041208	0.8041208	0.646610261				0.020618482
17	11,237	41	40.45902229	−0.540977711	0.292656883				−0.013194578
18	11,243	42	40.96369095	−1.036309046	1.073936438				−0.024674025
19	11,305	42	41.35611198	−0.64388802	0.414591783				−0.015330667
		489	490.9354588		24.81895192				

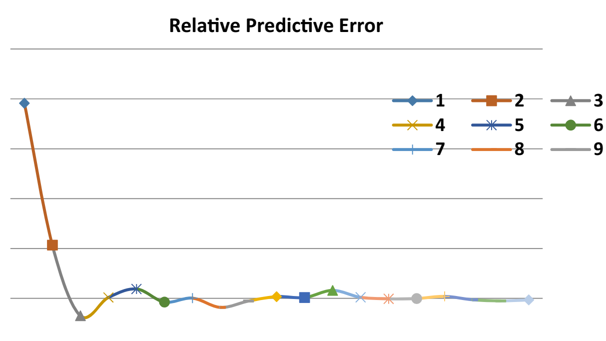

Figure 14b RPE of Developed model on DS-IV.

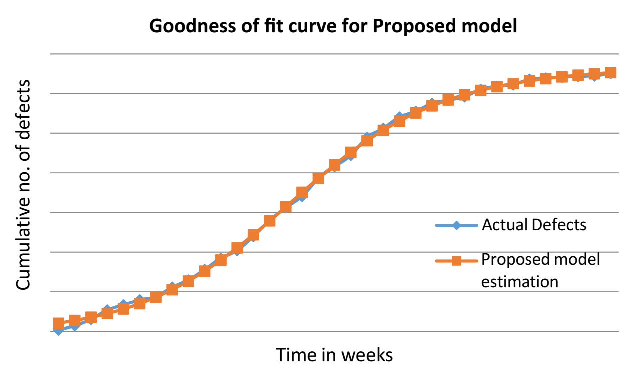

Figure 15a Goodness of fit of Developed model on DS-V.

respectively. In this *figure* Blue line represents actual defects and Red line represents Proposed model estimation for the removal of the actual defects. The estimation results are shown in Table 13. In the starting (First Week) actual defects were found (13) from the data set (DS-II) and Developed model has removed (12.08273502) defects, while in Table 3 and Fig. 2a Yamada model removed (3.172376338) defects only. From Table 8 and Fig. 7a, Ohba model has removed (7.086181539) defects only. Similarly in the Second week of the same data set (18) defects were found. Yamada model removed (10.85509886) defects, while Ohba model removed (15.10839391) defects. For same data set developed model has removed (18.03741627) defects. Similarly all defect removal has been represented in Tables 3, 8 and 13. Graphical illustrations have also been represented in Figs. 2a, 7a, and 12a respectively. Thus the performance of Developed model for the removal of software fault defects is better than the other two models namely: Yamada Delay S-shaped SRGM and Ohba inflection S-shaped SRGM. The best fitting of the model is illustrated graphically in Figs. 12a and 12b and shows relative predictive error to check the validity of the developed model.

Fig. 13a is based on DS-III, in which cumulative number of defects and test time in weeks are denoted by *Y*-axis and *X*-axis respectively. In this *figure* Blue line represents actual defects and Red line represents Proposed model estimation for the removal of the actual defects. The estimation results are shown in Table 14. In the starting (First Week) actual defects were found (6) from the data set (DS-III) and Developed model has removed (3.561997647) defects, while in Table 4 and Fig. 3a Yamada model removed (2.570151037) defects only. From Table 9 and Fig. 8a, Ohba model has removed (2.659044572) defects only. Similarly in the Second week of the same data set (9) defects were found. Yamada model removed (8.542716316) defects, while Ohba model removed (6.840980399) defects. For same data set developed model has removed (7.42494498) defects. Similarly all defect removal

Table 16 Developed model estimation on DS-V.

Test weeks	Execution hrs/efforts	Actual defects	Developed model estimation	Proposed estimation–actual estimation	(Proposed estimation–actual estimation)^2	MSE	R^2	RPE	RPE for curve
1	7.25	7	41.28357825	34.28357825	1175.363738	4.485572945	0.999	0.000423207	4.897654036
2	10.42	29	55.50168818	26.50168818	702.3394763				0.913851316
3	17.5	61	71.55666017	10.55666017	111.443074				0.173060003
4	24.83	108	90.58657142	−17.41342858	303.2274948				−0.16123545
5	32.08	134	113.3453604	−20.65463962	426.6141378				−0.154139102
6	44.66	159	140.5070449	−18.49295508	341.9893874				−0.116307894
7	64.58	175	172.6965546	−2.303445418	5.305860794				−0.013162545
8	117.08	223	210.4584906	−12.54150941	157.2894584				−0.056239953
9	164.26	259	254.1974739	−4.802526084	23.06425679				−0.018542572
10	259.36	312	304.1027665	−7.897233516	62.36629721				−0.025311646
11	315.11	369	360.0697595	−8.930240501	79.7491954				−0.024201194
12	374.36	408	421.6348937	13.63489371	185.9103264				0.033418857
13	417.94	479	487.9432922	8.943292173	79.9824749				0.018670756
14	462.69	559	557.7663028	−1.233697151	1.522008661				−0.002206972
15	505.02	624	629.5773727	5.577372711	31.10708636				0.008938097
16	580.02	681	701.6803622	20.68036218	427.6773797				0.030367639
17	642.85	771	772.3691981	1.369198069	1.874703351				0.001775873
18	716.43	831	840.0879641	9.087964082	82.59109115				0.010936178
19	759.18	888	903.5606231	15.5606231	242.1329913				0.017523224
20	799.85	978	961.8694277	−16.13057232	260.1953633				−0.016493428
21	896.6	1024	1014.476076	−9.523923866	90.70512581				−0.009300707
22	985.18	1081	1061.193585	−19.80641475	392.2940653				−0.018322308
23	1041.93	1110	1102.125233	−7.874766538	62.01194802				−0.007094384
24	1121.18	1150	1137.588719	−12.4112807	154.0398887				−0.010792418
25	1194.68	1166	1168.040565	2.04056492	4.163905192				0.001750056
26	1260.01	1184	1194.010483	10.01048349	100.2097797				0.0084548
27	1327.84	1221	1216.050195	−4.949805425	24.50057374				−0.004053895
28	1444.76	1236	1234.697245	−1.302754613	1.697169582				−0.001054009
29	1532.84	1244	1250.452065	6.452065293	41.62914654				0.005186548
30	1610.92	1272	1263.765485	−8.234514967	67.80723674				−0.006473675
31	1648.84	1278	1275.033841	−2.966158839	8.798098258				−0.002320938
32	1689.92	1283	1284.599147	1.599147048	2.557271281				0.001246412
33	1744.42	1286	1292.752349	6.752348952	45.59421636				0.00525066
34	1807.42	1289	1299.738226	10.7382259	115.3094955				0.008330664
35	1846.92	1301	1305.760954	4.760953519	22.66667841				0.003659457
		26,180	26191.07955		5835.730401				

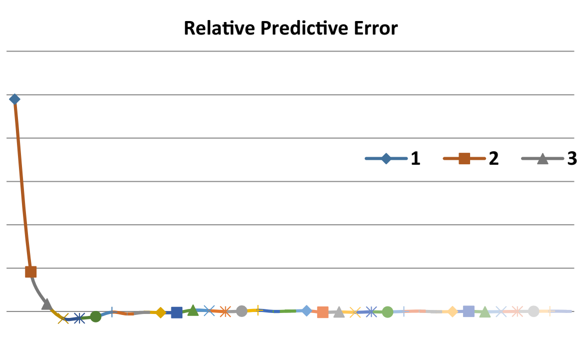

Figure 15b RPE of Developed model on DS-V.

has been represented in Tables 4, 9 and 14. Graphical illustrations have also been represented in Figs. 3a, 8a, and 13a respectively. Thus the performance of Developed model for the removal of software fault defects is better than the other two models namely: Yamada Delay S-shaped SRGM and Ohba inflection S-shaped SRGM. The best fitting of the model is illustrated graphically in Figs. 13a and 13b and shows relative predictive error to check the validity of the developed model.

Fig. 14a is based on DS-IV, in which cumulative number of defects and test time in weeks are denoted by Y-axis and X-axis respectively. In this *figure* Blue line represents actual defects and Red line represents Proposed model estimation for the removal of the actual defects. The estimation results are shown in Table 15. In the starting (First Week) actual defects were found (1) from the data set (DS-IV) and Developed model has removed (2.956096201) defects, while in Table 5 and Fig. 4a Yamada model removed (0.88268485) defects only. From Table 10 and Fig. 9a, Ohba model has removed (1.795337903) defects only. Similarly in the Second week of the same data set (3) defects were found. Yamada model removed (3.086805743) defects, while Ohba model removed (3.952718601) defects. For same data set developed model has removed (4.601670069) defects. Similarly all defect removal has been represented in Tables 5, 10 and 15. Graphical illustrations have also been represented in Figs. 4a, 9a, 14a respectively. Thus the performance of Developed model for the removal of software fault defects is

Table 17 Comparison among Yamada delay S-shaped, Ohba inflection S-shaped and proposed model.

Test data set observed	Yamada delay S-shaped model Comparison criteria			Ohba inflection S-shaped model			Proposed model		
	MSE	R^2	RPE	MSE	R^2	RPE	MSE	R^2	RPE
Data set-I (Tandem Release-1)	5.051276	0.969	−0.02043	1.795843	0.989	−0.006267571	**0.462765944**	**0.997**	**−0.000889851**
Data set-II (Tandem Release-2)	2.080806	0.990	−0.01099	0.950217	0.995	−0.00437	**0.435366631**	**0.998**	**−0.000378905**
Data set-III (Tandem Release-3)	1.577445	0.981	0.00034	0.412171	0.995	−0.0086	**0.28248012**	**0.997**	**−0.005534319**
Data set-IV (Tandem Release-4)	0.443288	0.995	−3.95954E−05	0.425454	0.995	0.002143	**0.590927427**	**0.993**	**0.003957993**
Data set-V (Brooks and Motley)	72.9920357	0.987	0.007861599	5.48234037	0.999	−0.002869663	**4.485572945**	**0.999**	**0.000423207**

better than the other two models namely: Yamada Delay S-shaped SRGM and Ohba inflection S-shaped SRGM. The best fitting of the model is illustrated graphically in Figs. 14a and 14b and shows relative predictive error to check the validity of the developed model.

Fig. 15a is based on DS-V, in which cumulative number of defects and test time in weeks are denoted by Y-axis and X-axis respectively. In this *figure* Blue line represents actual defects and Red line represents Proposed model estimation for the removal of the actual defects. The estimation results are shown in Table 16. In the starting (First Week) actual defects were found (7) from the data set (DS-V) and Developed model has removed (41.28357825) defects, while in Table 6 and Fig. 5a Yamada model removed (6.42387355) defects only. From Table 11 and Fig. 10a, Ohba model has removed (13.82504733) defects only. Similarly in the Second week of the same data set (29) defects were found. Yamada model removed (24.21799158) defects, while Ohba model removed (30.3358062) defects. For same data set developed model has removed (55.50168818) defects. Similarly all defect removal has been represented in Tables 6, 11 and 16. Graphical illustrations have also been represented in Figs. 5a, 10a, 15a respectively. Thus the performance of Developed model for the removal of software fault defects is better than the other two models namely: Yamada Delay S-shaped SRGM and Ohba inflection S-shaped SRGM. The best fitting of the model is illustrated graphically in Figs. 15a and 15b and shows relative predictive error to check the validity of the developed model.

4.4. Comparison criteria

4.4.1. The Mean Square Fitting Error (MSE)

The model under comparison is used to simulate the fault data, and the difference between the expected values $\hat{m}(t_i)$ and observed data x_i is measured by MSE as follows:

$$\text{MSE} = \sum_{i=1}^{k} \frac{(\hat{m}(t_i) - x_i)^2}{k}$$

where k is the number of observations. The lower MSE indicates less fitting error, thus better goodness of fit.

4.4.2. Coefficient of Multiple Determinations (R^2)

This goodness of fit measure can be used to investigate whether a significant trend exists in the observed failure intensity. We define this coefficient as the ratio of the sum of squares resulting from the trend model to that from constant model subtracted from 1. That is

$$R^2 = 1 - \frac{residual\_SS}{corrected\_SS}$$

R^2 measures the percentage of the total variation about the mean accounted for the fitted curve. It ranges in value from 0 to 1. Small values indicate that the model does not fit the data well the larger the R^2, the better the model explains the variation in the data.

4.5. Predictive validity criterion

Relative Prediction Error (RPE) is described by the following expression:

$$\text{RPE} = \frac{(m(n_k) - x_k)}{x_k}$$

Predictive validity is defined as the capability of the SRGM to determine the future fault/failure behaviours from present and past fault/failure behaviour (i.e. data). This capability is significant only when failure behaviour is changing. The RPE ratio will approach 0 (zero). If the RPE value is negative/positive the model is said to underestimate/overestimate the future failure phenomenon. A value close to zero for RPE indicates more accurate prediction, thus more confidence in the model and better predictive validity. The value of RPE is said to be acceptable if it is within $\pm 10\%$.

5. Result and discussion

Obtained results and various discussions have been illustrated in this section with comparison of the developed software reliability model with Yamada delay S-shaped and Ohba inflection S-shaped models. Table 17 compares two models: Yamada delay S-shaped model and Ohba inflection S-shaped model with developed software reliability growth model. All these models have been used to remove the defects of software failures. Models have been applied on 5 different software failure data sets and observed estimated values of the software fault removals. To check the goodness of fit of the models, MSE and R^2 have been evaluated. RPE has also been calculated to check the confidence, capability and validity of the developed model. Performance of the models has been evaluated and various results have been included in Table 17. For data set-I evaluated MSE by Yamada, Ohba and developed model is 5.051276, 1.795843, and 0.462765944 respectively. Less MSE represents better goodness of fit. Numerical values of another comparison criterion (R^2) for the same data set-I by Yamada, Ohba and developed model are 0.969, 0.989, and 0.997 respectively. Higher values of the R^2 show the better goodness of fit. RPE has also been evaluated in Table 17 to verify the validity of the models. The closeness of the RPE values to Zero verifies the best validity of the developed model. So, on the basis of the above observations developed model has better goodness of fit and has valid model with comparison of the other two models. Similarly from Table 17, various observations of the developed software reliability growth model for different data sets declare that the developed model has better goodness of fit and proves that it is a valid software reliability growth model.

6. Conclusion

In this paper, new SRGM have been developed to use Weibull distribution with inflection S-shaped software reliability growth model and predicted estimation using SPSS software. The estimated values of developed model have been compared with two existing models: Yamada delay S-shaped model and Ohba inflection S-shaped model. Results estimated by developed models are far better than existing two models and very close to the actual defects. To judge the performance and reliability of the model, two types of compression criterion: Goodness of Fit Criterion (GFC) and Predictive Validity Criterion (PVC) have been used. From the numerical observations developed model provides considerably improved results with better predictability due to lower MSE, higher R^2, and near to zero RPE. The results obtained in Table 17 show better goodness of fit and wider applicability of the model to different types of failure data sets of the software.

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