FRICTION FACTOR FOR NEWTONIAN AND NON-NEWTONIAN FLUID FLOW IN CURVED PIPES

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The present work deals with the study of laminar and turbulent flow of Newtonian and non-Newtonian fluids flowing through helical and Archimedian spiral coils. By using a suitable viscosity expression corresponding to the shear stress prevailing at the wall, Newtonian correlations are shown to be applicable to a non-Newtonian fluid also. Relationships for spiral coils are developed from correlations for helical coils and are successfully compared with the data of Newtonian and non-Newtonian fluids.

Introduction

In curved pipes there exists a secondary flow by the action of centrifugal force which has attracted the attention of many investigators. Most of the earlier studies were confined to laminar flow and turbulent flow of Newtonian fluids through helical coils (Dean[2], White[20], Ito[3], Kubair and Kuloor[6]) and through spiral coils (Kubair and Kuloor[7], Ali and Sheshadri[16], Mori and Nakayama[12], Noble[13]). The theoretical analysis of curved pipe problems for a non-Newtonian visco-inelastic fluid was first considered by Jones[4] and later Thomas and Walter[17], Jones[5], Austin and Seeder[1] and Topakonglu[19] also presented their investigations of curved pipes. Rajshekharan[14,15] *et al.* and Gupta and Mishra[10,11] obtained experimental data for non-Newtonian fluids in helical and spiral coils and observed a pronounced effect of the non-Newtonian character of the fluid on the rate of momentum transfer.

The present work is taken up to analyse Newtonian and non-Newtonian flow through spiral and helical coils. This aim is achieved by considering the correlations for a helical coil of a constant curvature ratio and by integrating it for a spiral coil over its length, having regular variation in curvature.

1. Experimental

A thick-walled smooth plastic pressure tube of uniform cross section with inside diameter of 1.19 cm and length 410 cm was used for making spiral and helical coils. The tube was wound round a cylindrical frame of known diameter to form a helical coil. The coil diameter could be varied easily by varying the diameter of the cylinder. The spiral coils were made on a horizontal wooden surface with the help of span clips. Fluid circulation was provided with the help of a three-stage centrifugal pump suitable for pumping the fluid from a tank of 200 liters capacity and equipped with cooling coils to maintain uniform temperature throughout the experiment. The pressure drop was measured by connecting the pressure taps with manometers containing CCl_4 for lower pressure drop and Hg for higher pressure drop. Flow rate was measured by collecting the fluids for a known interval of time.

The above data were taken with different fluids on different helical and spiral coils. The rheological properties of each fluid used in the present work were determined with the help of a capillary tube viscometer consisting of a uniform-bore glass capillary tube of diameter 1 mm and length-to-diameter ratio 300. Rheological constants K' and n were determined from the pseudoshear diagram. Details of coils and rheological properties of fluids used in the present experimental work are summarized in **Table 1**. 2% starch solution was found to behave like a Newtonian fluid. 2% and 3%, CMC solutions and 4% starch solutions were found to behave like a pseudoplastic fluid.

2. Results and Discussion

In a curved pipe, at a particular radius of curvature, the shear stress is expected to vary with wall position at the same cross-section. However, a mean wall shear stress τ_w can be defined by considering overall force balance. For pseudoplastic laminar flow through a helical coil, Mishra and Gupta have suggested an appropriate viscosity expression corresponding to the average shear stress prevailing at the wall:

$$\mu_2 = K'(\tau_w/K')^{(n'-1)/n'} \tag{1}$$

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Table 1 Range of variables

Sl. No.	Particulars	Coil dia. [cm] D_{C_1}	Coil dia. [cm] D_{C_2}	Pitch [cm]	Sl. No.	Fluids	n	K'	ρ
1	Helical Coil I	15.6	—	1.91	1	Water	1.000	0.0079	1.000
2	Helical Coil II	27.6	—	1.91	2	2% starch solution	1.000	0.0210	1.002
3	Spiral Coil I	14.6	32	1.91	3	2% CMC solution	0.930	0.1090	1.020
4	Spiral Coil II	14.6	46	5.00	4	3% CMC solution	0.827	0.1560	1.025
5	Spiral Coil III	14.6	65	10.00	5	4% starch solution	0.860	0.1020	1.005
6	Spiral Coil IV	14.6	100	24.50		(Length of tube L=410 cm	D_t=1.19 cm)		

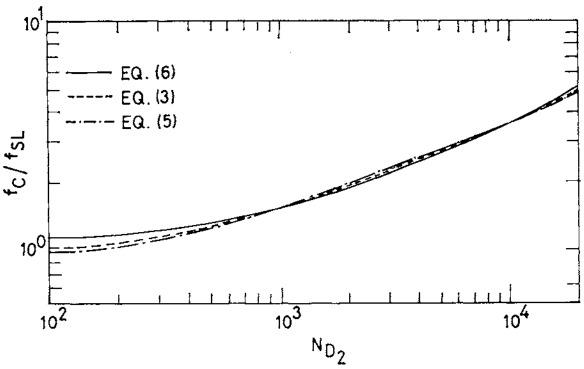

Fig. 1 Comparison of various correlations for helical coil

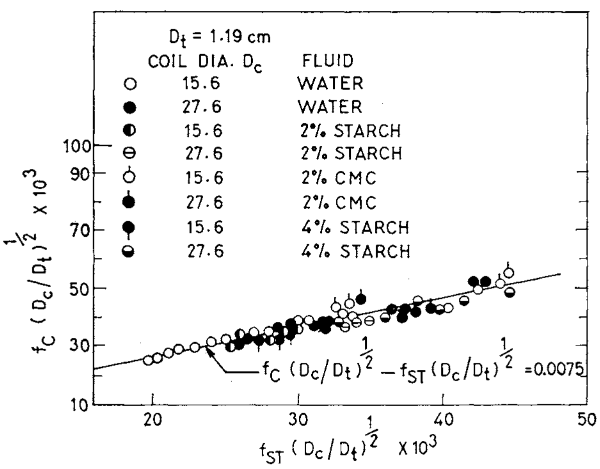

Fig. 2 Turbulent flow friction factor correlation for helical coils

in the Reynolds number:

$$N_{Re_2}=D_t U \rho/\mu_2 \tag{2}$$

Their investigations of various non-Newtonian fluids flowing through helical coils show that the variation of f vs. N_{Re_2} is independent of flow behaviour index. Present data on water, 2% starch, 2% CMC and 4% starch flowing through two helical coils of 15.6 cm and 27.6 cm coil diameter were found to be in excellent agreement with the laminar flow correlation of Mishra and Gupta[10,11]

$$(f_C/f_{SL})-1=0.33[\log N_D]^{4.0} \tag{3}$$

where

$$f_{SL}=16/N_{Re_2} \tag{4}$$

White's correlation

$$f_C/f_{SL}=[1-\{1-(11.6/N_D)^{0.45}\}^{2.2}]^{-1} \tag{5}$$

and the Mishra and Gupta[11] correlation Eq. (3) are compared in **Fig. 1**. Equations (3) and (5) almost coincide with each other in N_{D_2} range of 10^2–3×10^3. However, a simpler correlation of a form suitable for minimizing the analytical work for spiral coil flow analysis is proposed as

$$(f_C/f_{SL})-1=0.021\,N_D^{0.7} \tag{6}$$

This correlation satisfies the data equally well and is in excellent agreement with Eq. (3), as shown by comparison in Fig. 1.

With a view to interpret present experimental results for turbulent flow through helical coils in terms of a resistance formula deduced from the 1/7th power law, described by Ito for Newtonian fluids, turbulent flow of a power law fluid through straight pipe must be considered first. Mishra and Tripathi[9] have shown the applicability of a single-valued turbulent flow correlation for both Newtonian and non-Newtonian fluid by using differential viscosity in the Reynolds number. So, the Blasius resistance formula for a power law fluid flowing through straight tubes takes the following form:

$$f_{ST}=0.079/(N_{Re,d})^{1/4} \tag{7}$$

where

$$N_{Re,d}=D_t U \rho/\mu_d$$

and

$$\mu_d=nK(\tau_w/K)^{(n-1)/n} \tag{8}$$

Mishra and Gupta[11] correlated their Newtonian and non-Newtonian turbulent flow data by

$$f_C-f_{ST}=0.0075(D_t/D_c)^{1/2} \tag{9}$$

Figure 2 shows the plot $f_C(D_c/D_t)^{1/2}$ against $f_{ST}(D_c/D_t)^{1/2}$ for both Newtonian and non-Newtonian fluids. The value of f_{ST} was evaluated from Eq. (7) at Reynolds number $N_{Re,d}$ in the coil. It is observed that the turbulent flow data are in good agreement with Eq. (9).

3. Spiral Coils

Figure 3 shows the data for 3% CMC solution flowing through spirals of various diameters and pitches on a pseudo-shear plot. The deviation from the straight-pipe data are seen to be similar to that observed in helical coils. In this set of experiments the inner diameter of the spiral was kept constant at 14.6 cm and pitch was varied from 1.91 cm to 24.5 cm. **Figures 4** and **5** are friction coefficient vs. Reynolds number plots for water and 4% starch solution flowing through spiral coils.

In a spiral coil, radius of curvature varies with the length of the pipe, which causes variation in pressure gradient and intensity of secondary circulation along the length of the pipe. However, an approximation can be made by considering a spiral coil made of a large number of differential sections of helical coils of varying radius of curvature. Assuming that the intensity of the secondary circulation in a section of spiral coil is the same as that in a helical coil of the same curvature, the rate of change of friction factor in a spiral coil can be estimated from the rate of change of curvature of the spiral coil.

Kubair and Kuloor[8] used the integral

$$f_{CS}=\int_{D_{C1}}^{D_{C2}}(f_C/D_C^2)dD_C \qquad (10)$$

to obtain the following integrated friction factor expression for a spiral coil.

$$f_{CS}=16.1N_{Re}^{-0.5}[\exp 3.554(D_t/D_{C_1})-\exp 3.544(D_t/D_{C_2})] \qquad (11)$$

It is to be noted that Eq. (11) gives $f_{CS}=0$, for $D_{C_1}\rightarrow D_{C_2}$ i. e., a case of helical coil. The value of f_{CS} given by Eq. (11) seems to be a difference between friction factors of helical coils of dia. D_{C_1} and D_{C_2}. The friction factor f_{CS} in Eq. (11), therefore, does not represent the friction factor in the spiral coil. Thus the expression given by Kubair and Kuloor seems to be invalid.

If f_C is the friction factor at a position length x of a spiral coil of total length L, the average friction coefficient may be defined as

$$f_{CS}=(1/L)\int_0^L f_C dL \qquad (12)$$

The polar equation of an Archimedian spiral is given by

$$r=a_s\theta$$

where $a_s=P/2\pi$ and P=pitch of the spiral coil. The arc element dL and the radius of curvature R at any point $P(r, \theta)$ of the curve are given by the following relationships:

$$dL=(1/a_s)\sqrt{(1+a_s^2/r^2)}rdr \qquad (13)$$

and

$$R=r(1+a_s^2/r^2)^{3/2}/(1+2a_s^2/r^2) \qquad (14)$$

respectively.

Now, let us consider the magnitude of $(a_s/r)^2$ compared to unity. For 1/4th, 1/2 and one full turns of the spiral coil from the origin, i.e. for θ values of $\pi/2$, π and 2π, the values of $(a_s/r)^2$ are calculated to be 0.404, 0.101, and 0.0253 respectively. We find that as θ increases $(a_s/r)^2$ decreases and beyond half turn of the

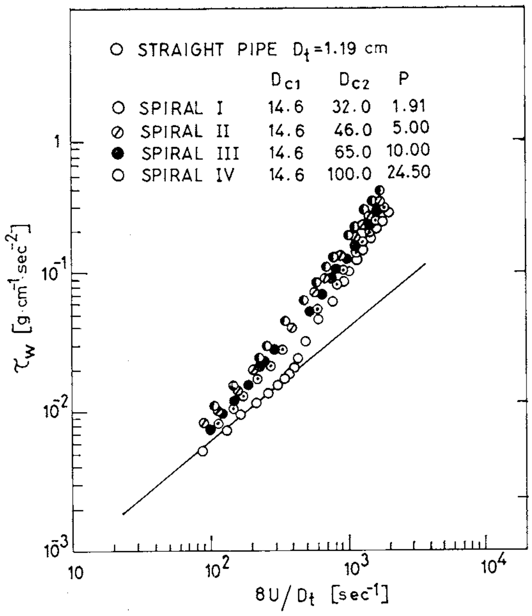

Fig. 3 Flow diagram for 3% CMC solution in spiral coil

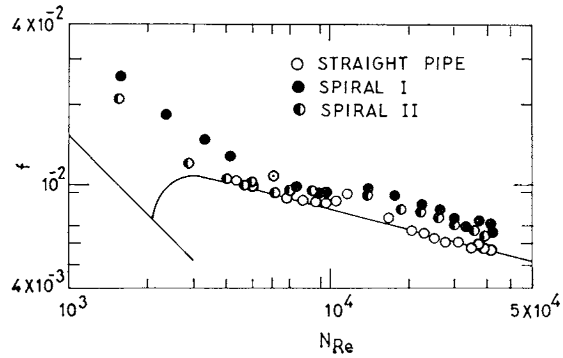

Fig. 4 Friction factor vs. Reynolds number for water

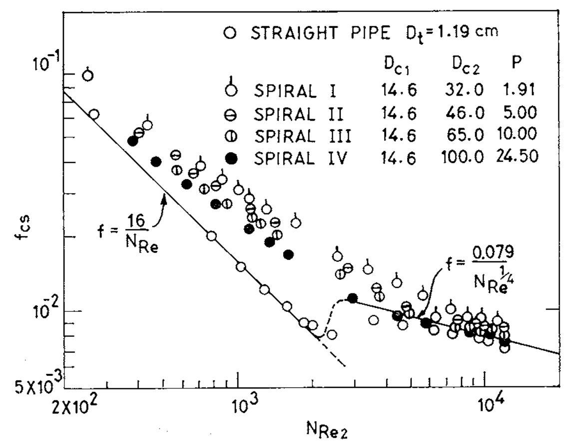

Fig. 5 Friction factor vs. Reynolds number plot for 4% starch solution

spiral its values become negligible compared to unity. Therefore, for spiral coils where minimum radius r_1 is such that $(a_s/r)^2 \ll 1$, Eqs. (13) and (14) simplify to $dL \approx (r/a_s)dr$ and $R \approx r$ respectively.

Substituting the above approximated value of dL in Eq. (12), we get

$$f_{CS} = 2\pi/PL \int_{r_1}^{r_2} f_C r dr \tag{15}$$

Equation (15) is a general expression for average friction coefficient in a spiral coil. Integration of the differential polar equation of an Archimedian spiral gives

$$PL/\pi = r_2^2 - r_1^2 \tag{16}$$

Evaluation of f_{CS} for laminar flow can be made by substituting the value of f_C from Eq. (6) in Eq. (15).

$$f_{CS} = 2\pi/PL \int_{r_1}^{r_2} f_{SL}[1 + aN_{Re}^b(D_t/2R)^{b/2}]rdr \tag{17}$$

where a and b are constants, 0.021 and 0.7 respectively in Eq. (6).

For a Newtonian fluid the integration term in Eq. (17) can easily be evaluated, as Reynolds number and f_{SL} are constants and radius of curvature r is the only variable. In a helical coil the radius of curvature remains constant. Therefore, wall shear stress and viscosity of a non-Newtonian fluid do not change with length of coil. In a spiral coil the radius of curvature varies with the length of the pipe due to which the wall shear stress and hence the viscosity of a non-Newtonian fluid change with position. Thus, we find that it is difficult to find a local value of N_{Re} and the evaluation of the integral term in Eq. (17) is not so simple for non-Newtonian fluids. To obtain a simple relationship, however, a viscosity term will be evaluated from an overall mean wall shear stress obtained from total pressure drop and length of the coil. If N_{Re} is defined, using this mean value of viscosity, and f_{SL} is evaluated at this Reynolds number, Eq. (17) can be simplified by assuming N_{Re} having a constant mean value. The resulting expression for average friction coefficient in a spiral coil under laminar flow condition is obtained as

$$\frac{f_{CS}}{f_{SL}} - 1 = \left[\frac{2a}{2-(b/2)} N_{D1}^b (r_1/r_2)^{b/2}\right]\left[\frac{1-(r_1/r_2)^{2-(b/2)}}{1-(r_1/r_2)^2}\right] \tag{18}$$

Mean friction coefficient for turbulent flow in a spiral coil can also be estimated by substituting Eq. (9) in Eq. (15). The integration and simplification results in

$$f_{CS} - f_{ST} = \left(\frac{4}{3}\right) A \left(\frac{D_t}{D_{C_2}}\right)^{1/2}\left[\frac{1-(r_1/r_2)^{3/2}}{1-(r_1/r_2)^2}\right] \tag{19}$$

where $A = 0.0075$.

Here also, f_{ST} the turbulent friction coefficient in a straight pipe is evaluated at Reynolds number using the differential viscosity at mean wall shear stress across the total length of the coil.

The critical Reynolds number relationship

$$N_{Re,C} = 2 \times 10^4 (D_t/2R)^{0.32} \tag{20}$$

was proposed by Ito[3] and later verified by Gupta and Mishra[11] using their own data on 60 helical coils of various diameters and pitches. Further, it was also pointed out that the critical Reynolds number for moderate non-Newtonian behaviour could be approximated from the above relationship. From Eq. (20) it is seen that the critical Reynolds number is a function of the curvature ratio, i.e. the higher the curvature ratio $(D_t/2R)$ the higher the critical Reynolds number. Based upon this observation, transition in a spiral coil will start at the point where the curvature ratio is minimum or the radius of curvature is maximum. With increased flow rate, the transition point will move from higher radius of curvature towards lower radius of curvature. Let the minimum and maximum radii of curvature of a spiral coil be r_1 and r_2 respectively and the corresponding critical Reynolds number, evaluated from Eq. (20), be N_{Re}^*. The flow remains laminar throughout the coil if the Reynolds number is less than N_{Re2}^*, and the flow becomes turbulent throughout the coil if the Reynolds number is more than N_{Re1}. However, both laminar and turbulent flow prevails if the Reynolds number is between N_{R1} and N_{Re2}. In such a case the average friction coefficient should be evaluated as

$$f_{CS} = 2\pi/PL\left[\int_{r_1}^{r^*} f_{CL} r dr + \int_{r^*}^{r_2} f_{CT} r dr\right] \tag{21}$$

where r^* corresponds to the radius of curvature in a spiral coil where transition from laminar to turbulent takes place, and it can be evaluated from Eq. (20) at a known value of Reynolds number. Evaluation of the integral in Eq. (21) results in

$$f_{CS} = \frac{16}{N_{Re}}\left[1 + \frac{2a}{2-(b/2)} N_{Re}^b \left(\frac{r_1}{r^*}\right)^{b/2}\right]\left[\frac{1-(r_1/r^*)^{(2-b/2)}}{1-(r_1/r^*)^2}\right] + \frac{0.079}{N_{Re,d}^{1/4}} + \frac{1}{4} A \left(\frac{r_1}{r_2}\right)^{1/2}\left[\frac{1-(r^*/r_2)^{3/2}}{1-(r^*/r_2)^2}\right] \tag{22}$$

Figure 6 shows the plot of f_{CS}/f_{SL} against N_{D1} for laminar flow of water and 2% starch solution representing Newtonian behaviour through various spiral coils. Solid lines in this figure represent the semiempirical correlation

$$\left(\frac{f_{CS}}{f_{SL}}\right) - 1 = \left[0.0254 N_{D1}^{0.612}\left(\frac{r_1}{r_2}\right)^{0.35}\right]\left[\frac{1-(r_1/r_2)^{1.65}}{1-(r_1/r_2)^2}\right] \tag{23}$$

$$\text{for} \quad N_{Re} < N_{Re}^*$$

Similar plots for 3% CMC and 4% starch solution having pseudo-plastic behaviour show reasonably

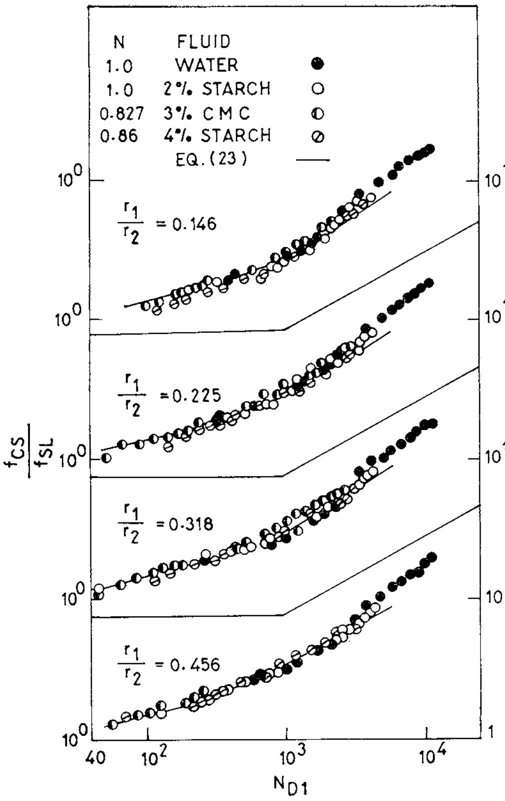

Fig. 6 Correlation of laminar flow friction coefficient in spiral coils

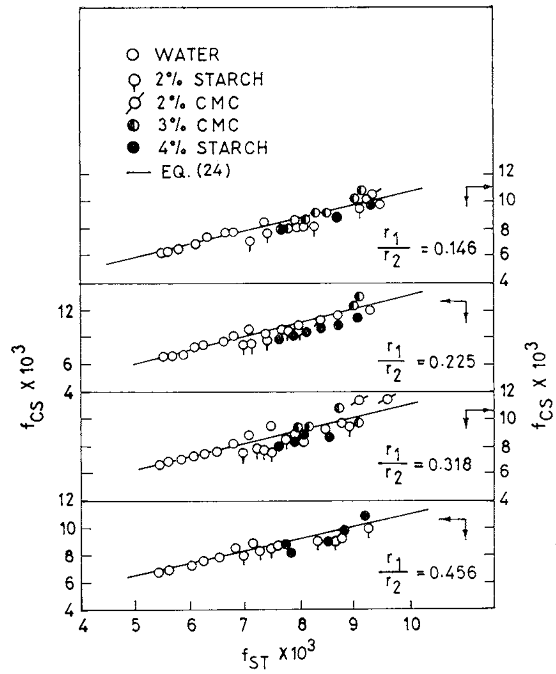

Fig. 7 Correlation of turbulent flow friction coefficient in spiral coils

good agreement between the data and Eq. (23).

Turbulent flow data ($N_{Re}>N^*_{Re1}$) are shown in **Fig. 7** where f_{CS} is plotted against f_{ST} for both Newtonian and non-Newtonian fluids flowing through spiral coils (radius ratio r_1/r_2 ranging from 0.146 to 0.456). Curves shown in this figure represent the semitheoretical correlation for turbulent flow through a spiral coil.

$$f_{CS}-f_{ST}=0.01\left(\frac{D_t}{D_C}\right)^{1/2}\left[\frac{1-(r_1/r_2)^{1.5}}{1-(r_1/r_2)^2}\right] \tag{24}$$

Turbulent flow data are also seen to be in good agreement with Eq. (24).

Conclusion

Behaviour of non-Newtonian fluids in both spiral and helical coils is seen to be identical to that of Newtonian fluids. Shear stress at the wall in laminar flow is more than that in a straight pipe. Therefore, the use of viscosity, evaluated at the mean shear stress at the wall, in the Reynolds number, is more appropriate for correlating non-Newtonian data. Integrated friction coefficient obtained from correlations for helical coils has been found to be in good agreement with the data on spiral coils in both the regions of laminar and of turbulent flow.

Nomenclature

A, a, b	= constants	[—]
D_C	= $2R$	[m]
D_{C1}	= $2r_1$	[m]
D_{C2}	= $2r_2$	[m]
D_t	= inner diameter of straight and coil tube	[m]
f	= Fanning friction factor	[—]
f_C	= friction factor in helical coil	[—]
f_{CL}	= laminar flow friction factor in helical coils	[—]
f_{CS}	= friction factor in spiral coil	[—]
f_{CT}	= turbulent flow friction factor in helical coils	[—]
f_{SL}	= laminar friction factor in straight tube	[—]
f_{ST}	= turbulent flow friction factor in straight pipe	[—]
K	= consistency index	[kg·s^{n-2}/m]
K'	= consistency index	[kg·s$^{n'-2}$/m]
L	= length of straight pipe or coil	[m]
N_D	= Dean number $N_{Re}(D_t/2R)^{1/2}$	[—]
N_{Re}	= Reynolds number $D_tU\rho/\mu$	[—]
N^*_{Re1}	= upper critical Reynolds number	[—]
N^*_{Re2}	= lower critical Reynolds number	[—]
$N_{Re,d}$	= Reynolds number based on differential viscosity $D_tU\rho/\mu_d$	[—]
n	= flow behaviour index	
P	= pitch in spiral coils	[m]
ΔP	= pressure drop	[Pa]
R	= radius of curvature	[m]
r	= radial position (= R)	[m]
r_1	= inner radius of spiral coil	[m]
r_2	= outer radius of spiral coil	[m]
r^*	= critical radius	[m]
U	= average velocity	[m/s]
μ	= viscosity	[kg/m·s]
μ_2	= effective pseudoshear viscosity in curved pipe	[kg/m·s]
μ_d	= differential viscosity	[kg/m·s]

ρ	= density		[kg/m^3]
τ_w	= shear stress at wall		[Pa]
θ	= angle at any point in spiral		[rad]

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