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PMU Placement for Maximum Observability of Power System under Different Contingencies

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Abstract

The demands of power supply increases day by day which increase the risk issue due to non-linearity of the power system. Sometimes, a small disturbance may create the chain of disturbances and results in a blackout. To avoid this possibility, intensely islanding is the key option in which some area of the power system is detached from the affected area. In this paper, an Integer Linear Programming (ILP) based optimal placement of Phasor Measurement Units (PMUs) has been proposed to provide the full observability of the system under two contingencies voltage stability based contingency and intensely islanding based contingency. Maximum observability of the power system is the additional objective of proposed method which increases the reliability of the power system. All the simulations have been tested on IEEE 14-bus, IEEE 30-bus, IEEE 118-bus, New England 39-bus test systems and Indian NRPG 246-bus system. To check the usefulness of proposed method, results have been compared with methods available in the literature.

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Keywords: Phasor Measurement Unit; Integer Linear Programming;Islanding; Maximum redundancy; Contingency

1. Introduction

The power industries have the major responsibility to full fill the demands of electrical supply where the demands of electric supply increase day by day. The main concern of power industries are reliability and stability of power system, however increasing demands of energy increases the risk factor due to non-linearity of the power system. Sometimes, a small disturbance may create the chain of disturbances and results in a blackout [1]. Therefore, to

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prevent this condition, intensely islanding is used in which some area of the power system have been isolated from affected area. One basic and most important reason behind the power system blackouts is deficient data. The installation of PMUs in power system prevail this drawback [2]. PMUs are highly accurate and advanced time synchronized technology which provides the voltage and current phasor and frequency information with global positioning system (GPS) receivers that allow the synchronization of the several readings taken at distance points. PDC collects the data from all the PMUs which are directly connected to that PDC through communication links.

In recent years, several methods have been employed to solve the common problem of finding the minimum number of PMUs for full observability of power system. These methods may be divided into two parts, first is conventional methods and second is heuristics methods. In conventional methods, optimal placement of PMU is expressed as Integer Linear Programming (ILP) problem [3-7] and MILP problem [8-9]. Heuristic methods such as Immunity Genetic Algorithm [10], PSO [11, 12], Tabu Search [13], GSA [14, 15] etc. have been developed. The authors in [3] use the ILP to optimize the PMU and addressed the issue of measurement redundancy. In [4], ILP is used to find the optimal PMU locations also update the neighbouring buses of zero injection buses (ZIB) to get the multiple results via ILP. The author approach in [5], is that of contingency problem, where ILP is used to determine the minimum number of PMUs. In [6], with the aim of maximum observability (MO), a new objective function is proposed using ILP. Reference [7] proposed the OPP with a limited number of channels and its results have been discussed with considering different contingencies. A multi-objective PMU placement methodology for MO has been developed in [8-9]. In [9], an additional objective function has also been incorporated which minimizes the cost of installation of PMUs by reducing the communication infrastructure. The authors in [10], proposed the Immunity Genetic Algorithm to optimize the number of PMUs in an electric power network. Also to increase the simulation speed, a new effect which prevents the familial reproduction is considered. A Binary Particle Swarm Optimization (BPSO) is proposed in [11] to minimize the number of PMUs and maximize the observability. In [12], a modified BPSO is used to determine the minimum number of PMUs. The author in [13] proposed the Tabu Search method to solve the optimal PMU problem and a priority list based on a heuristic rule is used to accelerate optimization. The authors in [14] and [15] proposed the new methodology, Gravitational Search Algorithm to optimize the number of PMUs.

Recently Lei Huang et al. [16] incorporated controlled islanding of power system in their formulation of PMU placement method. The PMU placement problem reported present paper had been formulated such that the entire system is observable to the maximum extent with a minimum number of PMUs in islanding case. The solutions have been obtained by ILP method. An additional formulation voltage stability ranking [17] based OPP has also been included in this paper. The results have also been compared with the methods reported in [16] and [18] which reveal that the proposed method required least number of PMUs with higher observability.

The paper is systemized as follows: Section II states the problem formulation of minimum PMU placement incorporating two different contingencies of power system. Section III explains the solution methodology of proposed method. Finally case study and test results are given in Section IV, and Section V concludes the paper.

2. Formulation of the proposed method

In this paper, a single objective function has been used to optimize the number of PMUs and provide the maximum observability. The proposed objective function can be written as follows:

$$\text{Minimize} \sum_{i=1}^{n} w_i z_i = \sum_{i=1}^{n} \frac{z_i}{(c_{cl} * NC_i) + c_p} \tag{1}$$

$$\text{Subject to:} \quad f = A.Z \geq 1 \tag{2}$$

Connectivity matrix (A) defines the interconnection of system buses by transmission lines. The entries in A are defined as follows:

$$A_{ij} = \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

wherez_i is the elements of vector Z, which represents the status of the installation of a PMU at bus i. if $z_i = 1$, it means PMU is installedat bus$i$, otherwise $z_i = 0$. c_{cl} and c_p is the cost of a channel and cost of a single PMU. NC_i is the number of channel at bus i. p is the total number of PMUs. In previous references [6,9,11,16,19], authors used the multiobjective function to optimize the number of PMUs having maximum observability. In this paper, Equ.(1) provides both objectives in a single objective function which is the advantage over previous methods. In Equ.(1), w_i is the weight factor and element of column vector W, which represents the inverse of the cost of PMU with respect tonumber of channels (branches) connected to bus i. $w_i$is defined as follows:

$$w_i = \frac{1}{(c_{cl} * NC_i) + c_p} \tag{4}$$

If W is unity vector matrix, it means weights of all the buses aresame, and this value of W has been used for without considering maximum observability. In case of intensely islanding, results have been included without and with zero injection bus (ZIB) concept. The presence of ZIB, may reduce the optimal number of PMUs. To inject the concept of ZIB, modify the Equ.(2) as given in [19]:

$$f_i = \sum_{j=I} a_{ij} z_j + \sum_{j=I} a_{ij} u_j y_{ij} \quad \forall \ i \in I \tag{5}$$

$$\sum_{i \in I} a_{ij} y_{ij} = u_j \quad \forall \ j \in I \tag{6}$$

whereu_j is equal to 1 if bus j is zero injection bus; otherwise 0. In Equation (6), z_j is 1 for ZIB, it means only one auxiliary binary variable is equal to one. y_{ij} is the auxiliary binary variable to handle the ZIB. If y_{ij} is equal to 1 it means, bus i is observable due to bus j that is zero injection bus.

All the buses must be observed at least twice by PMU in the case of single PMU outage. Hence, Equation (2) can be modified to [19];

$$f_i + \sum_{j \in I} a_{ij} y_{ij} \geq 2 \quad \forall \ j \in I \tag{7}$$

From Equation (7), it can be seen that a single PMU outage cannot affect the full power system observability. If *ZI* buses are also present in the system, the value of $\sum_{j \in I} a_{ij} y_{ij} = 1$. If *ZI* buses had not been considered in Equation (7), then $\sum_{j \in I} a_{ij} y_{ij} = 0$. Accordingly, $f_i \geq 2$ indicates that all the buses are observed at least twice by a PMU. Finally, the system observability can be computed as [4],

$$Obs. = \sum_{k=1}^{p} A_L(k) \tag{8}$$

Where L is the location of PMUs at the power system buses.

3. Solution Methodology

In his paper, two contingencies intensely islanding and voltage stability based contingency ranking [17] have been considered to optimize the PMUs in power system. The methodologies for both the contingencies are as follows:

3.1. Intensely Islanding based OPP:

This paper proposes an algorithm for optimal PMU placement with maximum observability, whereas, the system is normal or islanded. Maximum observability is the additional advantage over [16]. For intensely islanding the system, details of open lines of the entire test systems are given in Table 1, [16]. To optimize the number of PMUs, the elements of connectivity matrix have been updated according to opened lines. If the opened lines between the bus i-j, then a_{ij}=a_{ji}=0. Following two case studies have been considered in this paper:

Case 1: Normal case without and with ZIB

Case 2: Single PMU outage without and with ZIB

Flow chart of proposed method is shown in Fig.1. The overall process is summarized as follows.

Step 1: Determine the connectivity matrix (A) and Zero Injection Buses (ZIB).

Step 2: Update the connectivity matrix using open lines.
Step 3: Find the w_i from Equ. (4), where i= 1, 2, …p (number of PMUs)
Step 4: Determine the set of constraints using Equ. (2) and (5-7).
Step 5: For case (1), optimal PMU locations with maximum observability are obtained using Equ. (1), (2) and (5).Then go to step (7)
Step 6: For case (2), modify Equ. (2) as Equ. (7). Then solve the Equ. (1).
Step 7: Check system is fully observable are not. If not then go to step (4).
Step 8: Save the results.
Step 9: Stop

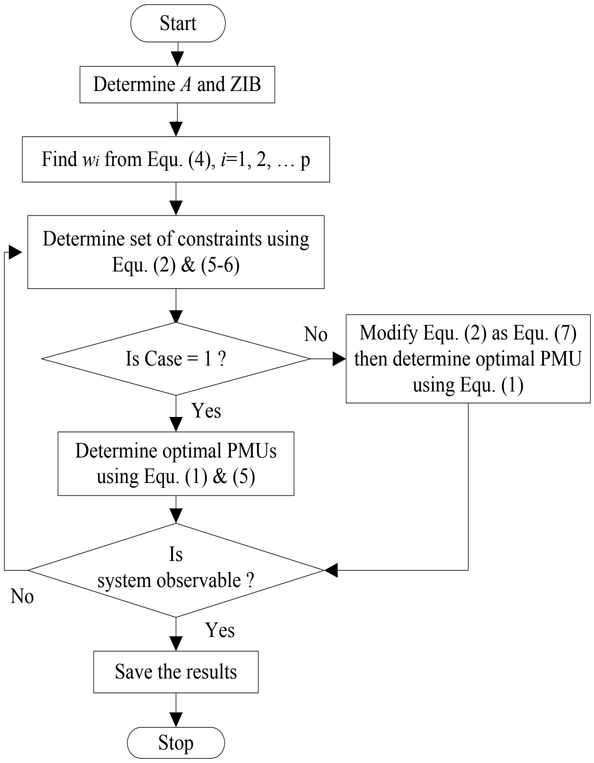

Fig. 1. Flow chart of proposed method in intensely islanding study

3.2. Voltage Stability based contingency ranking in OPP:

In this contingency, Reactive Violation Index (*RVI*) and nose point estimation [17] have been used to find out the rank of lines which is based on voltage stability. The *RVI* is described with the help of Fig. 2 where, λ denotes the system loading factor. In Fig. 2, Operating point A represents the nose point of the intact system and second point B represents the post-contingency operating point with the similar loading parameter value as in case of point A. The *RVI* for a contingency-i is defined as,

$$RVI_i = \sum_{i=1}^{Nq} (Q_{ji}^{no} - Q_j^{max}) \tag{9}$$

where,
Q_j^{max} = Maximum limit on reactive power output of source-j.
Q_{ji}^{no}= Reactive generation of the j^{th} Q-source with its Q limit open, following a contingency-i at a point B in Fig. 2 (with the same loading parameter value as in case of point A).
Nq = Number of reactive power sources violating their Q limit.

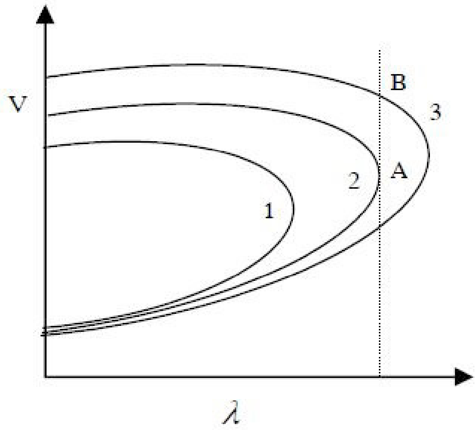

Fig. 2. V-λ curves at critical load bus for Q_{ji}^{no} calculation. [17]

Curve 1 shows the post contingency nose curve, curve 2 shows the pre contingency nose curve and curve 3 shows the post contingency nose curve with Q-limits open. The detail description of RVI has been given in [17]. In this contingency, without ZIB systems have been considered.

4. Case Study and results

This paper has been proposed a method for optimal placement of PMUs considering maximum observability for both the normal and islanding cases and also for the voltage stability ranking based system. The solution of this problem has been obtained by ILP. Optimal PMU and maximum observability are the main objectives of this paper. The performance of proposed method has been tested on IEEE 14-bus, IEEE 30-bus, IEEE 118-bus, New England 39-bus test systems and Indian NRPG 246-bus real system. The data of IEEE test systems and NE 39-bus test system have been taken from [21] and [22] respectively and data of NRPG 246-bus has been taken from [23]. Results of proposed method have also been compared with existing methods to check the effectiveness of proposed method.

4.1 Results based on Intensely Islanding:

Table 1 shows the number of zero injection buses, their locations for the test systems and details of open lines to create the intensely islanding. Table 2 shows the results for without considering ZIB of case 1. The optimal number of PMUs and their locations for both the conditions, without considering maximum observability and considering maximum observability have been given this table. Maximum observability (MO) and percentage of required PMUs for each test system have been given in Table 2. Formulation for MO has been taken from [3], which is the summation of individual bus observability. Similarly, Table 3 shows the results of case 1including ZIB. It is clear from this table the number of PMUs further reduces in the presence of ZIB in the system. In both the Tables 2 & 3 of case 1, the percentage of required PMUs and maximum observability are similar to [16]. Because author in [16], also proposed the OPP for maximum observability.

Table 1.Specifications and Islanding scheme of the test systems

Test system	No. of ZIB	Location of ZIB	No. of Islands	No. of opened lines	Opened lines
IEEE 14-bus	1	7	2	5	[1-2], [2-5], [4-5], [10-11], [13-14]
IEEE 30-bus	6	6, 9, 22, 25, 27, 28	2	7	[2-5],[2-6],[4-6],[10-17],[10-20],[22-24],[24-25]
NE 39-bus	12	1, 2, 5, 6, 9-11, 13, 14, 17, 19, 22	3	5	[8-9], [3-4], [3-18], [17-27], [1-2]
IEEE 118-bus	10	5, 9, 30, 37, 38, 63, 64, 68, 71, 81	3	9	[15-33], [19-34], [30-38], [23-24], [77-82], [96-97], [80-96], [98-100], [80-99]

In case 2, each bus should be observable at least two times by the PMUs. Therefore, numbers of PMUs are increases in this case. Table 4 and Table 5 show the results of case 2 for without and with ZIB respectively. In both the tables, results of considering MO have the advantage of observability over without considering MO. Comparisons of results of case 2 have been given in Table 6. For without ZIB, the number of PMUs and their observability are same in this table. While considering the ZIB in the system, all the test systems have less number of PMUs as compared to [16] except IEEE 14-bus.

Table 2. Results of proposed method under intensely islanding in case 1 (without ZIB)

Test System	No of PMUs	PMU %	CPU time (s)	Without considering MO		Considering MO	
				Location of PMUs	MO	Location of PMUs	MO
IEEE 14-bus	5	35.71	0.06	1, 4, 6, 8, 9	21	4, 5, 6, 7, 9	25
IEEE 30-bus	11	36.67	0.06	1,5,8,10,11,12,17,19,24,26,29	39	1,5,6,9,10,12,17,19,24,25,27	50
NE 39-bus	15	38.46	0.07	2,8,12,14,17,22,23,27,29,31,32, 33,34,37,39	48	2,5,6,10,13,16,17,19,20,22,23, 25,26,29,39	63
IEEE 118-bus	33	27.97	0.09	2,5,10,12,15,17,21,25,29,34,37,41, 45,49,53,56,62,64,72,73,75,77,80, 85,87,91,92,96,100,105, 110,114,116	164	3,5,9,12,15,17,21,25,28,34,37,40, 45,49,52,56,62,64,68,70,71,75,77, 80,85,86,89,92,96,100,105,110,114	179

TABLE 3. Results of proposed method under intensely islanding in case 1 (with ZIB)

Test System	No of PMUs	PMU %	CPU time (s)	Without considering MO		Considering MO	
				Location of PMUs	MO	Location of PMUs	MO
IEEE 14-bus	4	28.57	0.08	1,4,6,9	19	4,5,6,9	21
IEEE 30-bus	8	26.67	0.08	1,5,10,12,17,19,24,29	32	1,5,10,12,17,19,24,27	34
NE 39-bus	10	25.64	0.09	6,9,12,16,23,27,29,30,34,37	34	2,4,6,9,16,20,23,25,26,29	43
IEEE 118-bus	29	24.58	0.28	2,10,11,12,17,20,23,28,34,37,41, 45,49,53,56, 62,72,75,77,80,85,87, 91,92,96,100,105,110,114	150	3,8,11,12,19,22,27,31,32,34,37, 40,45,49,53,56, 62,72,75,77,80, 85,86,89,92,96,100,105,110	160

TABLE 4. Results of proposed method under intensely islanding in case 2 (without ZIB)

Test System	No of PMUs	PMU %	CPU time (s)	Without considering MO		Considering MO	
				Location of PMUs	MO	Location of PMUs	MO
IEEE 14-bus	12	85.71	0.06	1-3, 5-11, 13, 14	45	1, 2, 4-11,13,14	48
IEEE 30-bus	24	80	0.06	1,2,3,5,6,7,8,9,10,11,12,13,15-17,19,20,22- 26,29,30	89	2-7,9-13,15-17,19, 20,22-28,30	94
NE 39-bus	32	82.05	0.08	1-3,5-7,9-11,13,14, 16-20,22, 23,25-27,29-39	108	1-3,5,6,8-11,13,14,16-20, 22,23,25-27,29-39	109
IEEE 118-bus	72	61.02	0.13	1,2,5,7,9-12,15,17,19,21,22, 25,26,28,29,32-35,37,40,41, 43,45,46,49,50,52, 53,56,58, 59,62,63,65,67,68,70-73,75-77,79,80,83,85-87,89,91 ,92,94,96-101,105,107,109-112,114-117	305	2,3,5,7,9-12,15,17,19,21,22,24, 25,27,29-35,37,40,42,43,45,46, 49,50,51,53,54,56,59,62,64,65, 66,68,70,71,73,75,76,77,79,80, 83,85-87,89,91,92,94,96-101, 105,106,109-112,114,116,117	321

TABLE 5. Results of proposed method under intensely islanding in case 2 (with ZIB)

Test System	No of PMUs	PMU %	CPU time	Without considering MO		Considering MO	
				Location of PMUs	MO	Location of PMUs	MO
IEEE 14-bus	10	71.43	0.09 s	1,3-6,9-11,13,14	40	1, 2, 4- 6, 9-11,13,14	42
IEEE 30-bus	17	56.67	0.09 s	1-3,5-7,10,12,13,15-17,19,20, 23,24,30	67	1,2,4,5,6,7,10,12,13,15,16,17, 19,20,23,24,27	71
NE 39-bus	18	46.15	0.12 s	1,2,5,6,10,16,17,20,23,25- 27, 29,34-38	64	1,2,5,6,11,16,17,20,22,23,25, 26,27,29,34,36,37,38	66
IEEE 118-bus	63	53.39	0.37 s	2, 3,7,9,11,12,15,17,19,21,22, 24,26-29, 32,34,36,37,40, 41,44-46,49,52,53,56-59,62,66, 70,73,75,77,79,80,83,85-87, 89,90,92,94,96-101,105,107, 109-112,114,117,118	277	2,3,7,9,11,12,15,17,19,21,22, 24,26-28,31,32,34,36,37,40, 42,44-46,49,51,52,54,56,57,59, 62,66,70,71,75,77,79,80,83,85-87,89,90,92,94,96-101,105, 106,109-112,114,117,118	286

TABLE 6. Results comparison of proposed method under intensely islanding in single PMU Outage (case 2)

ZIB	Method	IEEE 14-bus			IEEE 30-bus			NE 39-bus			IEEE 118-bus		
		No. of PMU	PMU %	Obs.	No. of PMU	PMU %	Obs.	No. of PMU	PMU %	Obs.	No. of PMU	PMU %	Obs.
Without	Proposed	12	85.71	48	24	80	94	32	82.05	109	72	61.02	321
	Ref. [16]	12	85.71	48	24	80	94	32	82.05	109	72	61.02	321
With	Proposed	10	71.43	42	17	56.67	71	18	46.15	66	63	53.39	286
	Ref. [16]	10	71.43	42	18	60	NR	21	53.84	NR	65	55.08	NR

NR: Not required due to more number of PMUs than proposed method

4.2 Results based on Voltage Stability Ranking:

Table 7 shows the results of RVI [17] which provides the ranking of the serious contingencies for the IEEE 14-bus, NE 39-bus and Indian NRPG 246-bus systems. Table 8 provides the proposed results with the comparison of Ref. [18]. It is clear from this table that the results of proposed method are better as compared to Ref. [18].

Table 7.Results of the ranking of critical contingencies from Ref. [17]

	IEEE 14-bus	NE 39-bus	NRPG 26-bus
Rank 1	1-2	16-19	105-245
Rank 2	2-3	6-31	75-91
Rank 3	5-6	19-20	116-229
Rank 4	-	-	166-173
Rank 5	-	-	188-190
Rank 6	-	-	121-122

Table 8. Results of voltage stability ranking based OPP and their comparison (Without ZIB)

System	Case	Proposed			Ref. [18]		
		No of PMUs	Locations	MO	No of PMUs	Locations	MO
IEEE 14-bus	Intact	4	4 5 6 9	19	4	2 6 8 9	17
	1-2	5	4 5 6 7 9	25	5	4 5 6 8 9	23
	2-3	5	4 5 6 7 9	25	5	4 5 6 8 9	23
	5-6	4	2 6 7 9	18	5	4 5 6 8 9	23
NE 39-bus	Intact	13	2,6,9,10,13,14,17, 19,20,22,23,25,29	52	13	2,6,9,10,13,14,17, 19,20,22,23,25,29	52
	16-19	13	2,6,9,10,13,14,17, 19,20,22,23,25,29	51	13	2,6,9,10,13,14,17 ,19,20,22,23,25,29	51
	6-31	13	2,6,9,10,13,14,17, 19,20,22,23,25,29	52	14	1,2,8,10,11,14,17,19, 20,22,23,25,29,31	52
	19-20	13	2,6,9,10,13,14,17, 19,20,22,23,25,29	50	14	1,2,8,10,11,14,17, 19,20,22,23,25,29,31	52
NRPG 246-bus	Intact	70	-	357	70	-	330
	121-122	70	-	356	71	-	329

5. Conclusion

In this paper, two contingencies have been considered to fulfill two objectives with help of a single multi-objective function. The first objective is to minimize the number of PMUs for full observability of power system and the second objective is to maximize the system observability. In intensely islanding, the results have been obtained for both with and without considering ZIB. Whereas, in second contingency only without ZIB system have been considered. A single PMU outage case has also been included in intensely islanding contingency. Besides, the results of maximum observability having included in the objective function and not included have also been reported. The simulation results indicate that the proposed method is more effective than the existing methods.

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