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Mathematical modelling to predict mechanical properties of Copper (C101) feedstock in continuous extrusion

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Abstract. The continuous extrusion process is a process for producing endless profiles with a high degree of accuracy. The two important and vital process parameters are extrusion wheel velocity and product diameter which affect the mechanical and metallurgical properties of the material. In this work, continuous extrusion of pure copper (C101) rod has been carried out on commercial setup TBJ 350 for feedstock diameter of 9.5 mm at different extrusion wheel velocities and product diameters using different die sizes. The yield strength and percent elongation of extruded feedstock at different wheel velocities and product diameters has been estimated by carrying out the tensile test. The continuous extrusion process has been carried out based on two factor three-level central composite rotatable design using response surface methodology. A mathematical model has been developed to predict the mechanical properties of the extruded feedstock through continuous extrusion process and its validation has been carried out using ANOVA technique.

1. Introduction

The continuous extrusion forming process uses the frictional force between a circular driving wheel and material and can produce significantly long continuous products of a variety of sectional shapes which are hard in the classical forming processes owing to their methodological limitation. Because its superiority and impact on the current forming technology are evident, demand from industry has been growing rapidly. Basic experimental and analytical studies have been underway continuously since the early 1970s [1, 2]. However, theoretical and numerical studies are still insufficient to analyze accurately the complicated process characteristics. Furthermore, a study on the effects of major process parameters such as wheel velocity, relative die opening width, and flash gap size is essential for the optimal design of continuous extrusion process.



Recently, [3] investigated the use of extrusion-cutting as a material test method operating at severe conditions of strain, strain-rate and temperature, such as in machining. Many researchers have tried to study the mechanical properties of pure copper in the extrusion process [4]–[6]. The extrusion wheel angular velocity of CONFORM process for Aluminium alloy has been studied by Kim et al. [7] and Cho et al. [8] by using the two-dimensional finite element technology. Wu et al. [9] carried out by three-dimensional finite element technology, the detailed distributions of the velocity field, stress field, strain field, thermal field and damage field. Zhao et al. [10] carried out the effect of deformation speed on the microstructure and mechanical properties of AA6063 during the continuous extrusion process. Their results indicated that the variations in extrusion wheel velocity directly influence material deformation and significantly influence the maximum extrusion temperature. They concluded that there exists an optimum extrusion wheel velocity which generates products with good mechanical and metallurgical properties. Palanivel et al. [11] carried out a systematic approach to develop the mathematical model through response surface methodology (RSM) for predicting the yield strength (YS), and percent elongation of AA6351 aluminium alloy which is widely used in automotive, aircraft and defence industries by incorporating friction stir welding (FSW) process parameters such as tool rotational speed, welding speed and axial force. Balasubramaniam et al. [12] developed a mathematical model using RSM to predict tensile properties of pulsed current gas tungsten arc welded Ti-6Al-4V alloy. Niranjana et al. [13] carried out numerical modelling of aluminium sheets formability using response surface methodology. Sinha et al. [14] fabrication of actual set-up and did virtual design using computer-aided design (CAD) software.

In the present work, continuous extrusion forming of the pure copper (C101) rod has been carried out on commercially available setup TBJ 350 at different extrusion wheel velocities and product diameters. A mathematical model through RSM is developed to analyse the influence of wheel velocity and product diameter on mechanical properties such as YS and percent elongation. An optimum value of the extrusion wheel velocity and product diameter has been determined to predict the best mechanical properties of the continuous extrusion forming products at different wheel velocities and product diameters. The adequacy of the model has also been tested by the analysis of variance test (ANOVA).

2. Experimental

For the continuous extrusion forming, pure copper (C101) rod available commercially of 9.5 mm diameter feedstock has been used as the raw material. The commercially available continuous extrusion set up TBJ350 has been used to carry out experimentation at different wheel velocity and Product Diameter. The pure copper (C101) rod has been subjected to continuous extrusion under the extrusion wheel velocities and different product diameters. The continuous extrusion products have not been subjected to any artificial ageing and treatments. The tensile testing of the samples has been performed on the INSTRON machine. The YS and percent elongation of the broken sample after tensile testing has been obtained and recorded for mathematical modelling.

3. Mathematical model

For finding out the relationship between the process parameters and the mechanical properties, second order polynomial response surface mathematical models can be considered [12, 13]

$$Y_u = b_0 + \sum b_i x_{iu} + \sum b_{ii} x_{iu}^2 + \sum b_{ij} x_{iu} x_{ju} \quad (1)$$

where Y_u is the corresponding response; x_{iu} is the coded values of the i^{th} continuous extrusion parameters for the u^{th} experiments; and b_i , b_{ii} , b_{ij} are the second order regression coefficients. The second term under the summation sign of this polynomial equation corresponds to linear effect, while the third term denotes to the higher order effect. The fourth term of the equation includes the interactive effects of the process parameters.

The two factors with three levels and a central composite rotatable design matrix has been chosen to optimize the experimental conditions. The main aim of the factorial experiments is to depict the relationship between the response as a dependent variable and the parameter levels. This approach helps to understand how the change in levels of parameters affects the response. The combination of different levels of the parameters leads to certain optimum response. The present investigation studied the effects of factors such as extrusion wheel velocity and product diameter. The response parameters are YS and percent elongation of the extruded C101 feedstock during the continuous extrusion process. A 2^k factorial with central composite second order design has been used (here $k = 2$).

The coded and actual values of the parameters used in the present work have been listed in Table 1. Experiments were conducted and the output values of YS and percent elongation along with the design matrix is tabulated in Table 2. For the analysis of the data, the fitness of the model is necessary and well required. For checking the accuracy of the model includes a test for the significance of the regression model, the test for significance of model coefficient and test for lack of fit. For this purpose, analysis of variance (ANOVA) is performed.

Table 1. Experimental parameter and levels

| Factor | Level | | |
|------------------|-------|---|---|
| | -1 | 0 | 1 |
| Wheel velocity | 4 | 6 | 8 |
| Product diameter | 6 | 7 | 8 |

3.1 Mathematical model for yield strength

The test of significance of YS has been carried out using the quadratic model. The results of the quadratic model for YS given in Table 4. The value of R^2 and adjusted R^2 are 95.96 % and 93.07 %. This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is less than 0.05 (i.e., $\alpha = 0.05$ or 95 % confidence level), indicating that the model is considered to be statistically significant. It is also seen from Table 3 that from the p values, the main effect of X_2 and second-order effect of X_1 and X_2 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05. Figure 1 depicts the normal probability of residuals for YS. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 3, the derived model is shown as:

$$\sigma = 34.5255 - 0.9350X_1 - 1.4483X_2 - 3.6547X_1^2 - 8.0953X_2^2 + 1.5700X_1X_2 \quad (2)$$

3.2 Mathematical model for percent elongation

The test of significance of percent elongation has been carried out using the quadratic model. The results of the quadratic model for percent elongation are given in Table 4. The value of R^2 and adjusted R^2 are 97.76 % and 96.15 %. This means that the regression model provides a complete relationship between the dependent and independent variables. The associated p value for the model is less than 0.05 (i.e., $\alpha = 0.05$ or 95 % confidence level), indicating that the model is considered to be statistically significant. It is also seen from Table 3 that from the p values, the main effect of X_2 and second-order effect of X_1 and X_2 is much more significant. The other model terms can be regarded as insignificant due to their probabilities values being more than 0.05. Figure 2 depicts the normal probability of residuals for percent elongation. It is observed from the plot that residuals values are distributed normally and in a straight line and hence the model is adequate.

Using the results presented in Table 3, the derived model is shown as:

$$L = 46.3973 - 0.2500X_1 + 1.0000 X_2 - 5.8200 X_1^2 - 3.2575 X_2^2 + 3.0000 X_1X_2 \quad (3)$$

The adequacy of both YS and percent elongation models has also been tested through analysis of variance (ANOVA). The results of the analysis justify the closeness of the fit of the derived mathematical model. It has been concluded that the evolved mathematical models given by Eqs 2 and 3 are quite adequate at 95 % confidence level.

Table 2. Experimental plan and result based on central composite second order rotatable design

| Expt. No. | Wheel velocity | | Product diameter | | Yield strength (M Pa) | Percent elongation |
|-----------|----------------|--------|------------------|--------|--------------------------|--------------------|
| | (RPM) | | (mm) | | | |
| | Coded | Actual | Coded | Actual | | |
| 1 | -2 | 1 | 0 | 7 | 18.000 | 25.500 |
| 2 | 1 | 10 | 1 | 8 | 51.500 | 43.000 |
| 3 | -1 | 4 | 1 | 8 | 42.750 | 34.000 |
| 4 | 0 | 7 | 0 | 7 | 33.000 | 46.500 |
| 5 | 0 | 7 | -2 | 5 | 65.000 | 32.000 |
| 6 | 0 | 7 | 0 | 7 | 33.000 | 46.500 |
| 7 | 2 | 13 | 0 | 7 | 18.000 | 21.000 |
| 8 | -1 | 4 | -1 | 6 | 37.200 | 37.000 |
| 9 | 0 | 7 | 0 | 7 | 33.000 | 46.500 |
| 10 | 1 | 10 | -1 | 6 | 39.670 | 34.000 |
| 11 | 0 | 7 | 2 | 9 | 65.000 | 35.000 |
| 12 | 0 | 7 | 0 | 7 | 33.000 | 46.500 |
| 13 | 0 | 7 | 0 | 7 | 33.000 | 46.500 |

Table 3. Test result of ANOVA for yield strength and percent elongation

| Source | DF | Yield strength | | | | Percent elongation | | | |
|----------------|----|----------------|---------------------|---------|---------|--------------------|---------------------|---------|---------|
| | | Sum of squares | Mean sum of squares | F value | P value | Sum of squares | Mean sum of squares | F value | P value |
| Regression | 5 | 2430.48 | 486.10 | 21.86 | 0.000 | 889.097 | 177.819 | 61.01 | 0.000 |
| Linear | 2 | 35.66 | 35.66 | 0.80 | 0.486 | 12.750 | 6.375 | 2.19 | 0.183 |
| Square | 2 | 2384.96 | 1192.48 | 53.62 | 0.000 | 840.347 | 420.173 | 144.16 | 0.000 |
| Interaction | 1 | 9.86 | 9.86 | 0.44 | 0.527 | 36.000 | 36.000 | 12.35 | 0.010 |
| Residual error | 7 | 155.69 | 22.24 | | | 20.403 | 2.915 | | |
| Lack of fit | 3 | 155.69 | 51.90 | | | 20.403 | 6.801 | | |
| Pure error | 4 | 0.00 | 0.00 | 0.00 | | 0.000 | 0.000 | | |
| Total | 12 | 2586.17 | | | | 909.500 | | | |

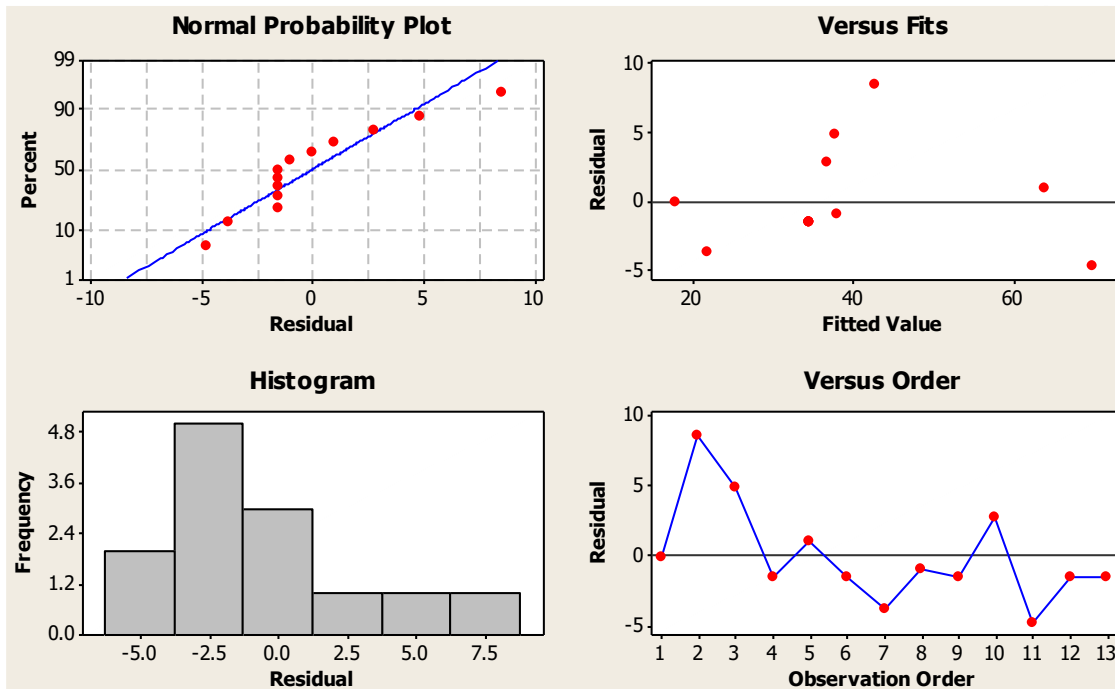


Figure 1. Residual plots for yield strength

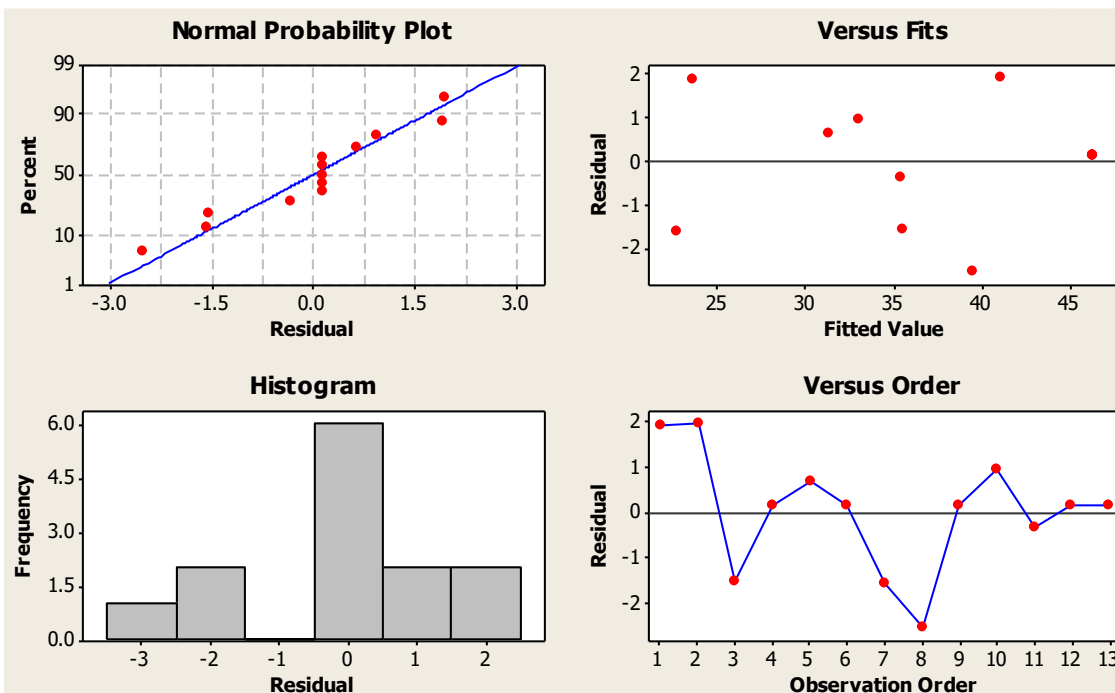


Figure 2. Residual plots for percent elongation

4. Results and Discussion

The effects of extrusion wheel velocity and product diameter on YS and percent elongation are shown in Figures 3 to 6 by contour and surface plots respectively. It can be inferred from Figures 3 and 4 that as the extrusion wheel velocity and product diameter increases, the YS of the extruded feedstock through continuous extrusion process increases and reaches to a maximum value and then further

increase in the value of extrusion wheel velocity and product diameter results in decrease in the value of YS. The percent elongation value of the extruded feedstock also follows the same trend like that of Yield Strength with the variation in extrusion wheel velocity and product diameter. It can be inferred from Figures 5 and 6 With the increase in extrusion wheel velocity and product diameter increases the percent elongation of the extruded product to a certain maximum value and then decreases with further increase in the value of extrusion wheel velocity and product diameter.

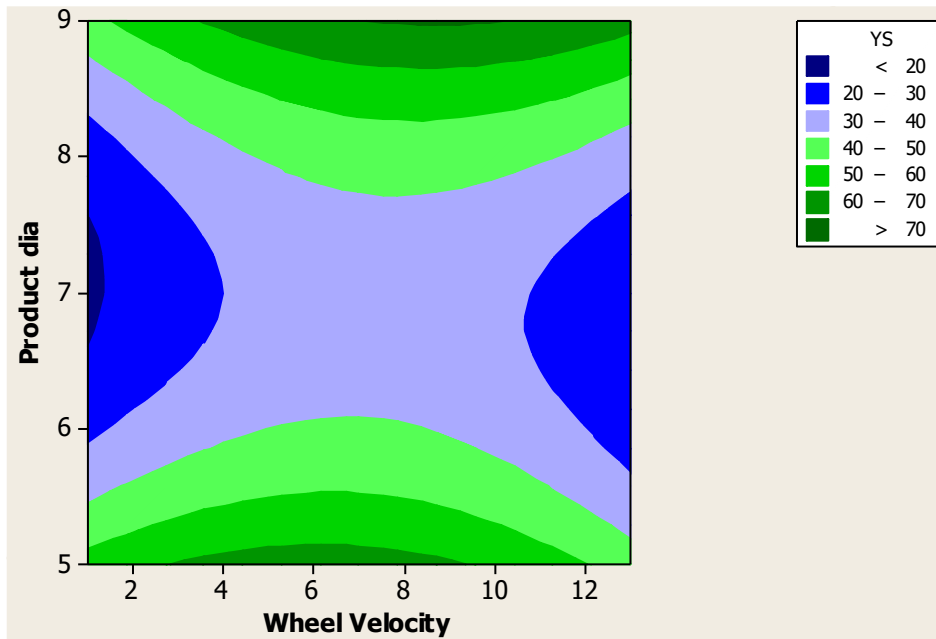


Figure 3. Contour plot of yield strength with variation in wheel velocity and product diameter

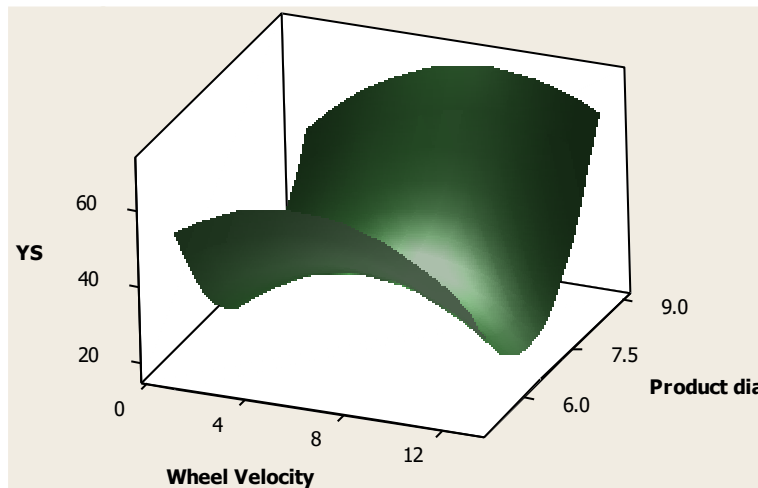


Figure 4. Surface plot of yield strength with variation in wheel velocity and product diameter

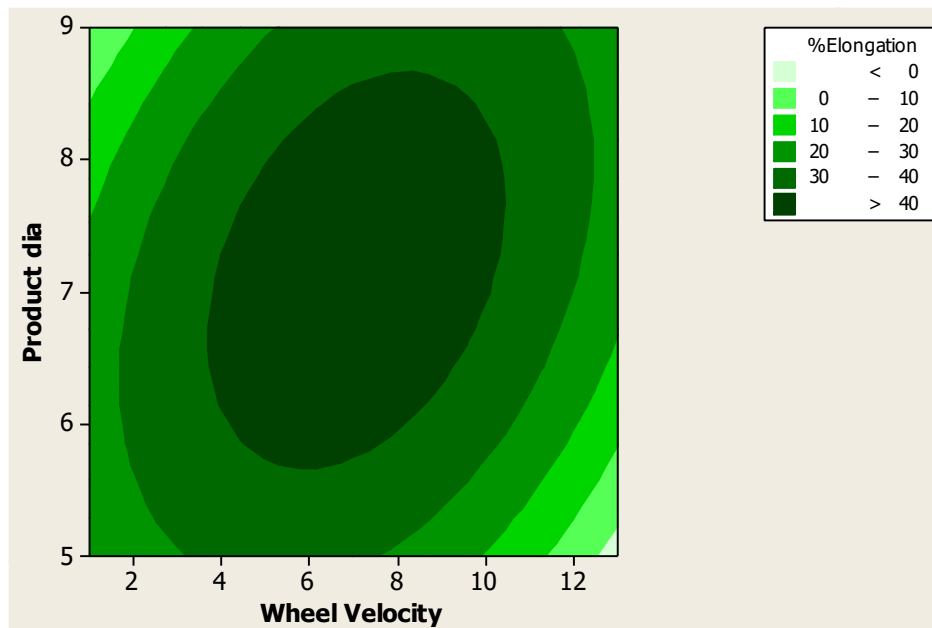


Figure 5. Contour plot of percent elongation with variation in wheel velocity and product diameter

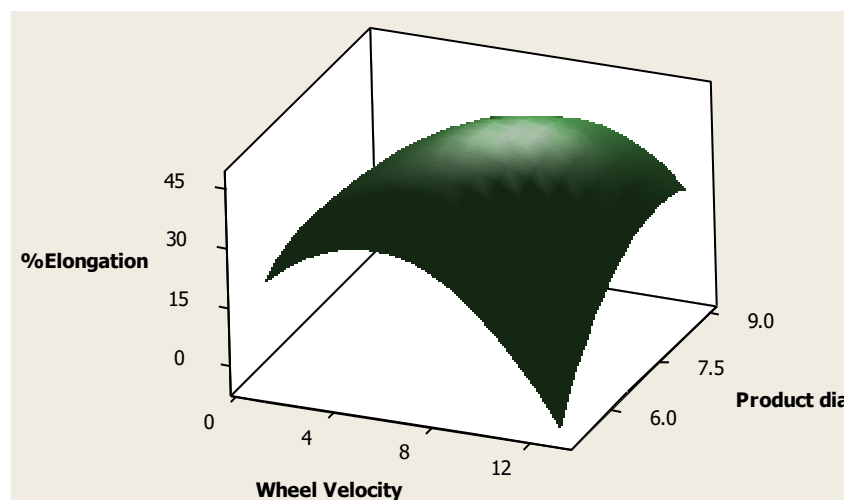


Figure 6. Surface plot of percent elongation with variation in wheel velocity and product diameter

5. Optimization of process parameters

Based on the developed second-order response surface equations and correlating the various continuous extrusion process parameters affecting YS and percent elongation of the extruded product, the optimal value of extrusion wheel velocity and product diameter has been determined. An analysis for the optimization of process parameter has been carried out using RSM optimization technique, similar to the technique used by Vettivel et al. [15]. The optimum values of the input process parameters in continuous extrusion for extrusion wheel velocity and product diameter are 8.63 RPM and 9.0 mm respectively for the optimal value of YS 70.93 MPa as shown in Figure 7. The optimum values of the input process parameters in continuous extrusion for extrusion wheel velocity and product diameter are 7.06 RPM and 7.18 mm respectively for the optimal value of percent elongation 46.46 as shown in Figure 8.

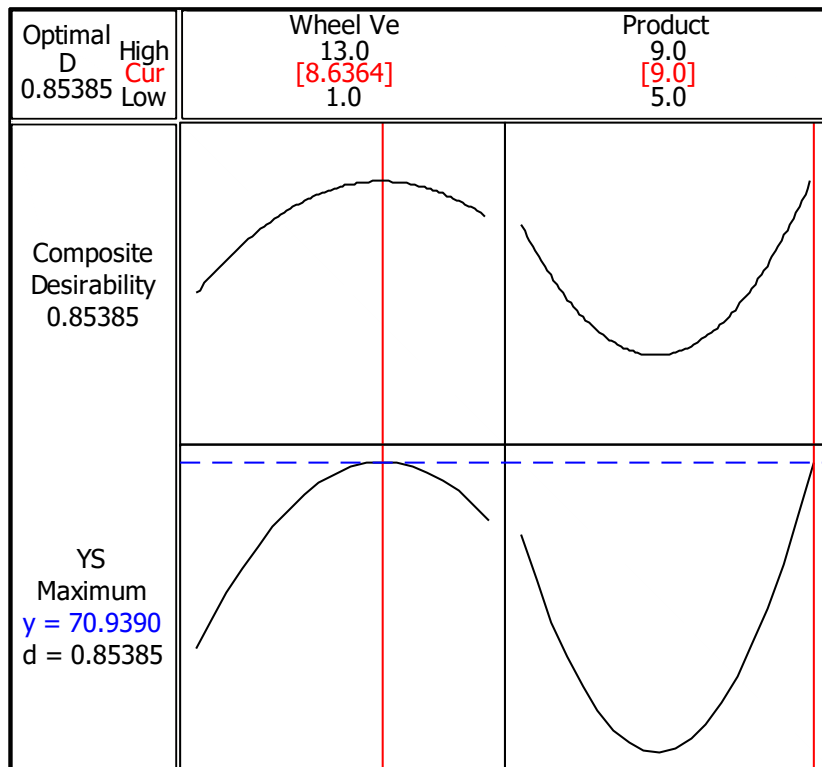


Figure 7. Optimization plot for yield strength with respect to wheel velocity and product diameter

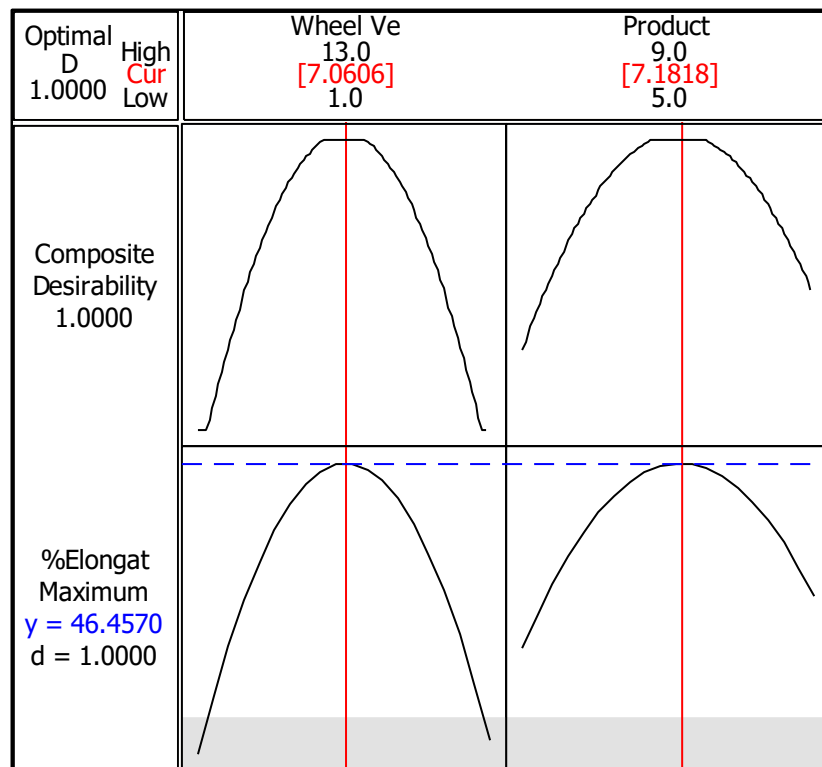


Figure 8. Optimization plot for percent elongation with respect to wheel velocity and product diameter

Table 4. Test for significance of yield strength and percent elongation

| Term | Coefficient | <i>t</i> | <i>P</i> | Coefficient | <i>t</i> | <i>P</i> |
|-------------------------------------|-------------|----------|----------|-------------|----------|----------|
| Constant | 34.5255 | 17.631 | 0.000 | 46.3793 | 65.424 | 0.000 |
| Wheel Velocity | 0.9350 | 0.687 | 0.514 | -0.2500 | -0.507 | 0.628 |
| Product diameter | 1.4483 | 1.064 | 0.323 | 1.0000 | 2.029 | 0.082 |
| Wheel velocity × Wheel velocity | -3.6547 | -3.710 | 0.008 | -5.8200 | -16.32 | 0.000 |
| Product diameter × Product diameter | 8.0953 | 8.217 | 0.000 | -3.2575 | -9.134 | 0.000 |
| Wheel velocity × Product diameter | 1.5700 | 0.666 | 0.527 | 3.0000 | 3.514 | 0.010 |

6. Conclusions

Continuous extrusion of pure copper (C101) feedstock has been successfully done. A mathematical model has been developed using the RSM approach. The developed RSM based mathematical modelling has the potential to evaluate YS and percent elongation under various process parameter settings. The adequacy of the developed mathematical model has also been tested through the analysis of variance (ANOVA). Following conclusions can be drawn from this study:

1. The YS increases with the increase of extrusion wheel velocity and is maximum at a particular value of extrusion wheel velocity and then decrease with further increase in the value of extrusion wheel velocity.
2. The percent elongation increases with the increase of wheel velocity and is maximum at a particular value of extrusion wheel velocity and then decrease with further increase in the value of extrusion wheel velocity.
3. The optimal input factor for YS was found as 8.63 RPM and 9.0 mm respectively for the optimal value of Yield Strength 70.93 MPa.
4. The optimal input parameter for percent elongation was found as 7.06 RPM extrusion wheel velocity and 7.18 mm to be product diameter for the maximum percent elongation value of 46.46.

References

- [1] Green D 1972 Continuous extrusion of wire sections. *J. Inst. Met.* **100** 296–300.
- [2] Green D 1974 Conform – A new concept for continuous extrusion forming of metals. *J. Manuf. Sci. Eng.* **96** 893–900.
- [3] Segalina F, De Chiffre L 2017 Material Testing of Copper by Extrusion-cutting. *Procedia CIRP* **58** 375–380.
- [4] Kral P, Kvapilova M, Dvorak J, Ponizil P, Sedivy O, Helisova K. *et al.* 2014 Quantitative characteristics of inhomogeneous microstructure in UFG copper. in *IOP Conf. Ser.: Mater. Sci. Eng.* **63** 012137.
- [5] Sheppard T, Yiu H L 1984 Microstructure and properties of some extruded copper-phosphorus alloys. *Metal Sci.* **18** 439–447.
- [6] Bazaz B, Zarei-Hanzaki A, Fatemi-Varzaneh S M 2013 Hardness and microstructure homogeneity of pure copper processed by accumulative back extrusion. *Mater. Sci. Eng. A* **559** 595–600.
- [7] Kim Y H, Cho J, Jeong H S, Kim K S, Yoon S 1998 A study on optimal design for CONFORM process. *J. Mater. Process. Technol.* **81** 671–675.
- [8] Cho J R, Jeong H S 2001 Parametric investigation on the curling phenomenon in CONFORM process by three-dimensional finite element analysis. *J. Mater. Process. Technol.* **110** 53–60.

- [9] Wu P, Xie S, Li H, Yan M, Huan G, Cheng L 2007 Effect of extrusion wheel angular velocity on continuous extrusion forming process of copper concave bus bar. *Trans. Nonferrous Met. Soc. China* **17** 280–286.
- [10] Zhao Y, Song B, Pei J, Jia C, Li B, Linlin G 2013 Effect of deformation speed on the microstructure and mechanical properties of AA6063 during continuous extrusion process. *J. Mater. Process. Technol.* **213** 1855–1863.
- [11] Palanivel R, Mathews P K, Murugan N 2012 Development of mathematical model to predict the ultimate tensile strength of friction stir welded dissimilar aluminum alloy. *Mechanika* **18** 517–523.
- [12] Balasubramanian M 2016 Statistical analysis of tensile strength and elongation of pulse TIG welded titanium alloy joints using Weibull distribution. *Cogent Eng.* **3** 1–9.
- [13] Niranjan K, Lakshminarayanan P R 2013 Optimization of process parameters for in situ casting of Al/TiB₂ composites through response surface methodology. *Trans. Nonferrous Met. Soc. China* **23** 1269–1274.
- [14] Sinha D K, Kumar S, Kumar A 2012 Virtual Design And Fabrication Of A Continuous Extrusion Setup With Process Analysis. *Int. J. Eng. Res. Technol.* **1** 1–7.
- [15] Vettivel S C, Selvakumar N, Leema N, Lenin A H 2014 Electrical resistivity, wear map and modeling of extruded tungsten reinforced copper composite. *Mater. Des.* **56** 791–806.
- [16] Seeman M, Ganesan G, Karthikeyan R, Velayudham A 2010 Study on tool wear and surface roughness in machining of particulate aluminum metal matrix composite-response surface methodology approach. *Int. J. Adv. Manuf. Technol.* **48** 613–624.