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Matching Theoretic Beam Selection in Millimeter-Wave Multi-User MIMO Systems

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ABSTRACT Beamspace multiple-input multiple-output (MIMO) with beam selection offers an attractive solution to reduce the number of radio-frequency chains in a high-dimensional millimeter-wave MIMO system. Considering a multi-user MIMO system in which an access point, having an N element antenna array, communicates with K users ($K < N$), each having a single antenna, we address the problem of selecting K beams with sum rate as the performance metric. We model beam selection as a two-sided *matching* between the two sets of players (users and beams) and consider two different ways of modeling players preferences. When the interdependences between the players' preferences are considered, it becomes *matching with externalities*. The proposed algorithm, which finds a stable matching in such a case, outperforms the existing beam selection algorithms.

INDEX TERMS Millimeter waves, beamforming, sum-rate, matching theory, stable matching, precoding.

I. INTRODUCTION

To meet the exponentially growing demand for high data rate wireless services, it is essential to have larger operating bandwidths. Current wireless cellular networks operate at carrier frequencies below 6 GHz, where the spectrum is crowded. Communication at millimeter wave (mmWave) frequencies offer a potential solution to the problem of wireless spectrum shortage and is expected to be a key enabler in the emerging 5G wireless networks [1], [2]. Millimeter waves occupy the spectrum from 30 GHz to 300 GHz, which is underutilized as of now; It is expected that bandwidths of about 2 GHz can be made available for wireless services at mmWaves [3]. Apart from huge bandwidths, mmWaves have another distinct advantage: shorter wavelengths allow us to pack more antenna elements into a given antenna aperture, enabling a *massive*, or, *high-dimensional*, multiple-input multiple-output (MIMO) operation [4]. A massive MIMO system can achieve a significant amount of beamforming gain or spatial multiplexing gain [5].

However, the need for a separate RF chain to drive each antenna element restricts the practical realization of mmWave massive MIMO systems. Each RF chain consumes about 250 mW of power at mmWave frequencies [6], which is significantly higher than that of when operated at below 6 GHz. While analog beamforming requires only one RF chain, it supports only single-stream transmission. Spatial modulation, as well as antenna selection, activate only few antennas at any given time and thus require less number of RF chains; However, these techniques fail to perform when the underlying MIMO channel becomes correlated, which is the case with mmWave channels, due to the nature of highly directional propagation of mmWaves [6].

The highly directional nature of mmWave propagation can be used to our advantage. As the mmWaves occupy only a small range of directions during their propagation in the space, the mmWave channels are *sparse* in the angular domain. The beamspace MIMO (B-MIMO) concept, originally proposed in [7] and [8], and adopted to mmWaves in [9], provides a potential solution for reducing the RF hardware complexity: As each beam in B-MIMO corresponds to an RF chain, transforming the channel from spatial domain to angular domain (i.e., beamspace) and selecting few beams can possibly lead to mmWave massive MIMO systems with low RF hardware complexity. While the channel can be transformed to the angular domain using a discrete lens antenna, *what beams to select* is the main challenge.

The maximum magnitude beam selection [10] selects beams to maximize the received signal strength at the users, but the number of simultaneously active RF chains

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is a variable depending on the channel realization. The interference-aware (IA) beam selection [11] assigns a single beam to each user and achieves near optimal performance by minimizing the multi-user interference. Beam selection algorithms for multiple performance metrics have been developed in [12]. An iterative beam selection algorithm based on QR decomposition has been proposed in [13].

In this paper we formulate the problem of beam selection as the problem of finding a *stable matching* between the set of users and the set of beams. *Matching theory*, originally developed in the field of Economics, provides an elegant mathematical framework for the combinatorial problem of matching two distinct set of players, taking care of their individual preferences [14], [15]. It has been successfully employed for solving real-world problems in macro-economics and Social sciences, such as, devising better mechanisms for admissions into New York and Boston public schools, selecting medical residents and matching compatible kidney donors and recipients [16]. In recent times, matching theory has attracted the attention of wireless researchers as a powerful framework for solving resource management problems [17]. When the players preferences become interdependent, which is the case in a multi-user wireless network because of coupling between the users through interference, finding a stable matching is a challenging problem [18].

Following are the main contributions of this paper:

- With users' preferences depending only on the desired signal strength, we propose a simple, low complexity matching algorithm that achieves sum-rate performance close to that of the near optimal IA beam selection.
- Considering both signal and interference powers in determining the user's preferences, we model beam selection as *matching with externalities* and develop an iterative algorithm to obtain a stable matching; It outperforms existing algorithms, including the one that uses all the beams with a zero-forcing precoder.

Rest of the paper is organized as follows. After introducing system model in Section II, we present the matching theoretic beam selection algorithms in Section III. Linear precoder is discussed in Section IV. After presenting the simulation results in Section V, we conclude the paper in Section VI.

Notation: Matrices and vectors are denoted by boldface uppercase and lowercase letters, respectively. a_{ij} and x_i denote $(ij)^{\text{th}}$ element of matrix $\mathbf{A}$ and i^{th} element of vector $\mathbf{x}$, respectively. $\|\mathbf{a}\|$ denotes the Frobenius norm of a. $\mathbf{I}_N$ is $N \times N$ identity matrix. Superscripts $-1, T, H$ indicate inverse, transpose and conjugate transpose, respectively. $\mathcal{A} \setminus \mathcal{B}$ and $\mathcal{A} \times \mathcal{B}$ denote the difference and Cartesian product of sets $\mathcal{A}$ and $\mathcal{B}$, respectively. $E[\cdot]$ denotes the expectation operator.

II. SYSTEM MODEL

Consider an access point (AP) equipped with a uniform linear array (ULA) of N antenna elements with spacing between successive elements equal to $\frac{\lambda}{2}$ meters, where λ is the carrier wavelength in meters, communicating to K mobile users, each having a user equipment (UE) with a single antenna. Let $\mathbf{s} \in \mathbb{C}^{K\times 1}$ is the symbol vector (with symbol s_k intended for UE k), and $E[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_K$. With $\mathbf{P} \in \mathbb{C}^{N\times K}$ representing the precoder for canceling the multi-user interference, the discrete-time input-output relation, in *spatial domain*, of such a multi-user MIMO system can be expressed as

$$\mathbf{y} = \mathbf{H}^H\mathbf{P}\mathbf{s} + \mathbf{w}, \tag{1}$$

where $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K] \in \mathbb{C}^{N\times K}$ is the channel matrix and $\mathbf{h}_k \in \mathbb{C}^{N\times 1}$, k^{th} UE's channel vector, contains the channel gains from N elements of the ULA to the k^{th} UE. $\mathbf{w}$ denotes the additive white Gaussian noise vector with $\mathbf{w} \sim \mathcal{CN}(0, \sigma^2\mathbf{I}_K)$.

Using the Saleh-Valenzuela channel model [19], we model the downlink channel (i.e., from AP to users) in the *spatial domain*, as follows. Let $\phi_k^{(0)} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ be the physical angle of arrival of the Line-of-Sight (LoS) component at k^{th} UE and $\theta_k^{(0)} = 0.5\sin\phi_k^{(0)}$ be the spatial frequency induced by ϕ_k^0. Similarly, let the physical angle of arrival of and the corresponding spatial frequency of ℓ^{th} non line-of-sight (NLoS) component at k^{th} UE is given by $\phi_k^{(\ell)}$ and $\theta_k^{(\ell)}$, respectively. The array response vector corresponding to the LoS component is given by,

$$\mathbf{a}\left(\theta_k^{(0)}\right) = \frac{1}{\sqrt{N}}\left[\exp(-\jmath 2\pi\theta_k^{(0)} i)\right]_{i\in\mathcal{I}(N)}, \tag{2}$$

where $\mathcal{I}(N) = \{i - (N-1)/2, i = 0, 1, \ldots, N-1\}$ is an index set. The array response vector corresponding to the ℓ^{th} NLoS component, $\mathbf{a}\left(\theta_k^{(\ell)}\right)$, can be obtained in a similar

TABLE 1. Variables used and their physical meaning.

Variable	Physical Meaning
$\mathbf{P}$	Precoder in spatial domain
$\mathbf{H}$	Channel in spatial doamin
$\mathbf{U}$	Spatial discrete Fourier transform
$\mathbf{H}_b$	Beamspace channel
$\tilde{\mathbf{H}}_b$	$K \times K$ dimensional channel matrix after beam selection
$\tilde{\mathbf{P}}_b$	$K \times K$ dimensional precoder after beam selection
$\mathbf{h}_k$	k^{th} UE's channel vector
$\mathbf{s}$	Symbol vector
$\mathbf{w}$	Additive white Gaussian noise vector
K	Number of UEs
N	Number of beams
$\mathcal{K}$	Set of UEs
$\mathcal{N}$	Set of beams
$\phi_k^{(0)}$	Physical angle of arrival of LoS component at k^{th} UE
$\phi_k^{(\ell)}$	Physical angle of arrival of ℓ^{th}
$\theta_k^{(0)}$	Spatial frequency induced by ϕ_k^0
$\theta_k^{(\ell)}$	Spatial frequency induced by ϕ_k^ℓ
$\beta_k^{(0)}$	Path gain of LoS component at k^{th} UE
$\beta_k^{(\ell)}$	Path gain of the ℓ^{th} NLoS component at k^{th} UE
μ	A matching (one-to-one mapping from $\mathcal{K}$ to $\mathcal{N}$)
$\mu(k)$	Beam assigned to k^{th} UE under matching μ
$\mu(n)$	UE assigned to n^{th} beam under matching μ
μ_{DA}	Matching obtained by Deferred Acceptance (Algorithm 1)
$\mu_{\text{ext}}^{k,n}$	Matching derived from μ_{ext} by assigning beam n to UE k
μ_{ext}^*	Matching computed by Matching with Externalities (Algorithm 2)
$\mathcal{L}_\mu$	Set of all assigned beams under matching μ
$U_{i,j}(\mu)$	Utility for UE $i \in \mathcal{K}$ matched to beam $j \in \mathcal{N}$ under matching μ
$I_{i,j}(\mu)$	Interference power at UE i under matching μ
$R_{\text{sum}}(\mu)$	Sum-rate (in b/s/Hz) achieved by matching μ
$R_{\text{sum}}^{\text{THP}}$	Sum-rate (in b/s/Hz) achieved by Tomlinson-Harashima Precoder

manner using $\theta_k^{(\ell)}$ and $\phi_k^{(\ell)}$. UE k's channel vector is given by

$$\mathbf{h}_k = \sqrt{\frac{N}{L+1}} \sum_{\ell=0}^{L} \beta_k^{(\ell)} \mathbf{a}\left(\theta_k^{(\ell)}\right), \tag{3}$$

where $\beta_k^{(0)}$ and $\beta_k^{(\ell)}$ denote the complex-valued path gain of the LoS component, and ℓ^{th} NLoS component, respectively.

Due to the highly directional nature of propagation at mmWave frequencies, the angular spread of mmWaves is considerably low and beamspace domain (i.e., angular domain), captures the inherent sparsity in such channels. The spatial channel model can be transformed into the beamspace domain by employing a discrete lens array (DLA) at the transmitter [9]. DLA plays the role of a *spatial discrete Fourier transform*, which can be represented by the matrix $\mathbf{U} \in \mathbb{C}^{N\times N}$, with its i^{th} column given by

$$\mathbf{u}_i = \mathbf{a}\left(\theta_i = \frac{i}{N}\right). \tag{4}$$

Thus, columns of $\mathbf{U}$ are array response vectors corresponding to N orthogonal predefined directions covering the entire angular space, and, thus, to N fixed spatial frequencies, given by $\theta_i = \frac{i}{N}$, $i \in \mathcal{I}(N)$ [9].

The beamspace representation of the system is given by,

$$\mathbf{y} = \mathbf{H}_b^H \mathbf{P}_b \mathbf{s} + \mathbf{w}, \tag{5}$$

where $\mathbf{P}_b = \mathbf{U}^H \mathbf{P}$ and $\mathbf{H}_b = \mathbf{U}^H \mathbf{H}$ is the beamspace channel. Each $\mathbf{h}_{b,k} = \mathbf{U}^H \mathbf{h}_k$, $k = 1, \ldots, K$, will have few dominant entries, significantly less than N, around $\theta_k^{(0)}$.

Columns of $\mathbf{H}_b^H$ represent the beams available in the system. The concept of beam selection, in which we select few columns (out of N) from $\mathbf{H}_b^H$, has been introduced in [10]. We address the problem of selecting K beams, out of N, for the K UEs without causing significant loss to the sum-rate R_{sum}, where

$$R_{\text{sum}} = \sum_k R_k = \sum_k \log(1 + SINR_k). \tag{6}$$

R_k is the data-rate achieved by UE k and $SINR_k$ is the signal-to-interference plus noise ratio at UE k. With K beams, the number of required RF chains reduces from N to K, resulting in a significant saving in cost and energy. The system equation *after* beam selection can be expressed as

$$\mathbf{y} = \tilde{\mathbf{H}}_b^H \tilde{\mathbf{P}}_b \mathbf{s} + \mathbf{w}, \tag{7}$$

where, $\tilde{\mathbf{H}}_b^H \in \mathbb{C}^{K\times K}$ is the (low-dimensional) channel matrix and $\tilde{\mathbf{P}}_b$ is the corresponding (low-dimensional) precoding matrix.

For beam selection and precoding, discussed in subsequent sections, we assume that the access point has complete knowledge of the channel $\mathbf{H}_b$.

III. MATCHING THEORETIC BEAM SELECTION

Selecting, or, assigning K beams, out of N, to K users has a strong analogy to the problem of *stable matching* between two sets of players, studied in matching theory [15], [18].

Definition 1: Let $\mathcal{K} = \{1, \ldots, K\}$ and $\mathcal{N} = \{1, \ldots, N\}$ denote, respectively, the set of UEs and the set of beams. Each $k \in \mathcal{K}$ has a strict linear ordering $\succ_k$ over $\mathcal{N}$, where $n \succ_k n'$ denotes that k prefers n over n'. This ordering $\succ_k$ is known as the preference relation of k. Similarly each $n \in \mathcal{N}$ has a strict linear ordering $\succ_n$ over $\mathcal{K}$. These components together define a *one-to-one matching game* $(\mathcal{K}, \mathcal{N}, (\succ_i)_{i\in\mathcal{K}\cup\mathcal{N}})$, with $\mathcal{K}$ and $\mathcal{N}$ as the two sets of players.

Definition 2: The outcome of a matching game is a *matching* μ. A matching μ is a one-to-one mapping from $\mathcal{K}$ to $\mathcal{N}$ such that:

i. $\mu(k) \in \mathcal{N}, \ \forall k \in \mathcal{K}$
ii. $\mu(n) \in \mathcal{K}, \ \forall n \in \mathcal{L}_\mu$
iii. $\mu(\mu(i)) = i, \ \forall i \in \mathcal{K} \cup \mathcal{L}_\mu$

Here, $\mu(k)$ indicates the beam *matched*, or, equivalently, *assigned*, to user $k \in \mathcal{K}$ and $\mathcal{L}_\mu = \{\mu(k) : \forall k \in \mathcal{K}\}$ is the set of all *matched* (equivalently, *assigned*) beams. Thus $\mathcal{N} \setminus \mathcal{L}_\mu$ is the set of all unassigned (or, unmatched) beams.

Definition 3: A matching μ is said to have a blocking pair $(k, n) \in \mathcal{K} \times \mathcal{N}$ if $n \succ_k \mu(k)$, $k \succ_n \mu(n)$ and $\mu(k) \neq n$; i.e., both k and n prefer to be matched with each other over their current partner in μ.

Definition 4: A matching μ is said to be *pairwise stable* if it does not have any blocking pair.

A. DEFERRED ACCEPTANCE (DA) ALGORITHM FOR BEAM SELECTION

Gale and Shapely [14] have shown the existence of a pairwise stable matching for every matching game when the preferences are strict, and, also, provided an algorithm called *deferred acceptance*, to find such a matching. We now present our first algorithm for beam selection, based on deferred acceptance algorithm.

The DA algorithm is an iterative algorithm. In the first round, each UE proposes to its most preferred beam and then each beam *provisionally* accepts the proposal of the UE it prefers the most and rejects all other proposals. In the subsequent rounds, each unassigned UE proposes to its most preferred beam to whom it has not yet proposed; Each beam, then, provisionally accepts the proposal of the most preferred beam *if* it is *not* currently assigned or *if* it *prefers* the UE over its current provisional partner (in which case, the beam rejects its current provisional UE who becomes unassigned). Thus, a beam *defers* its decision in *accepting* a UE as its partner, until it gets the best possible partner as per its preference list.

The preference relation $\succ_k$ for each UE $k \in \mathcal{K}$ is determined based on the channel gain provided by the beams to UE k, such that, for any two beams $n, n' \in \mathcal{N}$,

$$n \succ_k n' \iff |h_{k,n}| > |h_{k,n'}| \tag{8}$$

Similarly, the preference relation $\succ_n$ for each beam $n \in \mathcal{N}$ is determined as follows:

$$k \succ_n k' \iff |h_{k,n}| > |h_{k',n}| \tag{9}$$

The idea is to maximize the channel gain for each of the UE. Based on the obtained preference lists, we apply Algorithm 1 to get the matching μ_{DA}.

Note that the above algorithm does *not* require a UE to know the preferences of other UEs. It was shown that the convergence of deferred acceptance algorithm to a stable matching does not depend on the order in which players participate and it does not require synchronization among the players [15]–[17]. Thus, Algorithm 1, DA for beam selection, can be implemented in a distributed manner. Further, DA enjoys a low computational complexity, given by $O(KN)$ [15].

Algorithm 1 Deferred Acceptance for Beam Selection

Phase I: Finding preference relations
Find $\succ_i, \ \forall i \in \mathcal{K} \cup \mathcal{N}$, using (8) and (9)
Phase II: Matching
while Some UE $k \in \mathcal{K}$ is unmatched **do**
  k proposes its most preferred beam $n \in \mathcal{N}$ to which it has *not* yet proposed
  if n is unmatched **then**
    n *accepts* proposal from k
  else if n prefers k to its current match k' **then**
    n accepts proposal from k *and*
    k' becomes unmatched
  else
    n rejects k
  end if
end while

The data-rate of a UE depends *not* only on the desired signal strength but also on the strength of interfering signals it receives from the beams transmitting to other UEs. As the preference relations (8) and (9) does *not* take interference into account, the sum-rate performance of DA for beam selection is relatively low, as will be shown in Section V.

B. THE CONCEPT OF EXTERNALITIES

When both signal and interference powers are considered for computing the preference relations, the players preferences become *interdependent*; When beam n gets matched to user k, the preference relations of some (or, all) of the users may get changed due to the interference created by the beam n at other users. Thus, preferences become a function of the matching and the matching problem exhibits *externalities* [18].

There is no general result showing the existence of a stable matching with externalities. We now propose a heuristic algorithm to find a stable matching, considering externalities. The algorithm starts with an *arbitrary* (or, equivalently, *randomly chosen*) matching μ_{a} between users and beams, and continuously updates the preferences and the matching in an iterative fashion.

C. MATCHING WITH EXTERNALITIES FOR BEAM SELECTION

Let $U_{i,j}(\mu)$ is the utility function for UE $i \in \mathcal{K}$ matched to beam $j \in \mathcal{N}$ under a matching μ, where

$$U_{i,j}(\mu) = \frac{|h_{i,j}|^2}{\sigma^2 + I_{i,j}(\mu)}. \tag{10}$$

Here, $I_{i,j}(\mu) = \sum_{n \in \mathcal{L}_\mu \setminus j} |h_{i,n}|^2$ is the interference power at UE i due to other UE-beam links under matching μ.

Let μ and μ' are two *different* matchings such that $\mu(i) = n, \mu'(i) = n'$ for UE $i \in \mathcal{K}$ and beams $n, n' \in \mathcal{N}, \ n \neq n'$. Then, the preference relation for UE $i \in \mathcal{K}$ is given by

$$(n, \mu) \succ_i (n', \mu') \iff U_{i,n}(\mu) > U_{i,n'}(\mu') \tag{11}$$

The preference relation for a beam $j \in \mathcal{N}$, UEs $k, k' \in \mathcal{K}$ and matchings μ, μ' such that $\mu(k) = j, \mu'(k') = j$ is given by

$$(k, \mu) \succ_j (k', \mu') \iff R_{\mathrm{sum}}(\mu) > R_{\mathrm{sum}}(\mu'), \tag{12}$$

where $R_{\mathrm{sum}}(\mu)$ represents the sum-rate achieved by matching μ. Observe that the preference relations, given by (11) and (12), are *dependent on the matching*, and enable us to incorporate the externalities into the matching problem. A matching μ is considered to be better than matching μ' if $R_{\mathrm{sum}}(\mu) > R_{\mathrm{sum}}(\mu')$.

Algorithm 2 updates the matching μ_{a}, iteratively, to obtain a stable matching μ^*_{ext}. In Phase I, we first compute the utilities $U_{k,n}\left(\mu^{k,n}_{\mathrm{ext}}\right)$, $\forall k \in \mathcal{K}$ and $\forall n \in \mathcal{N}$, where $\mu^{k,n}_{\mathrm{ext}}$ is the matching derived from μ_{ext} by assigning (or, linking) n to k (i.e., by forcing a matching between beam n and UE k). There arises three possible cases while computing $\mu^{k,n}_{\mathrm{ext}}$:

1) $n \notin \mathcal{L}_{\mu_{\mathrm{ext}}}$: n is not matched to any UE. We add the link (k, n) to μ_{ext} and remove the link $(k, \mu_{\mathrm{ext}}(k))$ from μ_{ext} to get $\mu^{k,n}_{\mathrm{ext}}$; Thus, $\mu^{k,n}_{\mathrm{ext}} = \mu_{\mathrm{ext}} \cup \ (k, n) \setminus (k, \mu_{\mathrm{ext}}(k))$.
2) $n \in \mathcal{L}_{\mu_{\mathrm{ext}}} \setminus \mu_{\mathrm{ext}}(k)$: n is matched to some other UE, i.e., $\mu_{\mathrm{ext}}(n) \neq k$. We obtain $\mu^{k,n}_{\mathrm{ext}}$ as follows:

$$\mu^{k,n}_{\mathrm{ext}} = \mu_{\mathrm{ext}} \cup \{(k, n), (\mu_{\mathrm{ext}}(n), n')\} \setminus \{(k, \mu_{\mathrm{ext}}(k)), (\mu_{\mathrm{ext}}(n), n)\} \tag{13}$$

Note that, we match $\mu_{\mathrm{ext}}(n)$ to a beam $n' \in \mathcal{N} \setminus \mathcal{L}_{\mu_{\mathrm{ext}}}$ (i.e., n' is from the set of unmatched beams) which causes least interference at k. The idea is to get maximum value of utility for UE k when linked to beam n.
3) $n = \mu_{\mathrm{ext}}(k)$: In this case, simply $\mu^{k,n}_{\mathrm{ext}} = \mu_{\mathrm{ext}}$.

Using the utilities $U_{k,n}\left(\mu^{k,n}_{\mathrm{ext}}\right)$, we obtain the preference relations for UEs according to (11).

In Phase II, each UE $k \in \mathcal{K}$ proposes to its most preferred beam n (if k is not already matched to the beam n), based on the updated preference relations computed in phase I. Beam n will accept the proposal only if the link (k, n) leads to an increase in the sum-rate. This is ensured by comparing the sum-rates achieved by μ_{ext} and μ_{new}, where μ_{new} is derived by adding the link (k, n) to μ_{ext}. Two cases can arise while obtaining μ_{new} from μ_{ext}.

Algorithm 2 Matching With Externalities for Beam Selection

Input: A matching μ_a obtained by an arbitrary one-to-one matching between users and beams.
Initialize: $\mu_{ext} = \mu_a$
repeat
  Phase I: Utility and preference relation computation
  - $\forall k \in \mathcal{K}$ and $\forall n \in \mathcal{N}$, compute $U_{k,n}\left(\mu_{ext}^{k,n}\right)$
  - Generate $\succ_k$, $\forall k \in \mathcal{K}$, according to (11)
  - Set $\mu_{previous} = \mu_{ext}$

  Phase II: UE proposal evaluation
  For each $k \in \mathcal{K}$
  repeat
    Propose to the most preferred beam n which has *not* rejected k at an earlier step
    if $\mu_{ext}(k) = n$ **then**
      Continue with the next user
    else
      Compute μ_{new}
      if $R_{sum}(\mu_{new}) > R_{sum}(\mu_{ext})$ **then**
        Accept the proposal
        $\mu_{ext} = \mu_{new}$
      else
        Reject the proposal
      end if
    end if
  until proposal is accepted
until $\mu_{ext} = \mu_{previous}$
$\mu^*_{ext} = \mu_{ext}$

1) $n \notin \mathcal{L}_{\mu_{ext}}$: n is not matched to any UE in μ_{ext}. We obtain μ_{new} as $\mu_{new} = \mu_{ext} \cup (k, n) \setminus (k, \mu_{ext}(k))$.
2) $n \in \mathcal{L}_{\mu_{ext}} \setminus \{\mu_{ext}(k)\}$: n is matched to some UE other than k; i.e., $\mu_{ext}(n) \neq k$. μ_{new} is obtained as follows:

$$\mu_{new} = \mu_{ext} \cup \{(k, n), (\mu_{ext}(n), n')\} \setminus \{(k, \mu_{ext}(k)), (\mu_{ext}(n), n)\}$$

Beam n' is the most preferred beam for user $\mu_{ext}(n)$ which is *not* linked to any user.

Thus, in Phase II, μ_{ext} gets updated *only* if there is a matching that achieves a higher sum-rate. Hence, with every iteration of Phase I and II, μ_{ext} improves in terms of sum-rate.

While computing $R_{sum}(\mu)$ in (12), we compute sum-rate for the system given by (7), where $\tilde{\mathbf{H}}_b$ is the channel matrix corresponding to matching μ and $\tilde{\mathbf{P}}_b$ is the corresponding precoder, computed according to either (14) or (16) (discussed in Section IV), to cancel the multi-user interference. It is worth noting that Algorithm 2 can be employed with *any* precoder and need not be restricted to ZF or the precoder discussed in the next section.

To summarize, Phase I computes the preferences under μ_{ext}, which are used by Phase II to update μ_{ext}. The updated μ_{ext} is passed to Phase I to update the preferences; This cycle continues until Phase II returns the same μ_{ext} that it receives.

Lemma 1: Algorithm 2 will converge.

Proof: There are two aspects to be understood. First, since the number of UEs and beams are finite, the number of matchings will also be finite. Second, for every iteration of the algorithm we *strictly* attain a better matching. Thus, the algorithm will converge to a matching μ^*_{ext}. ■

Lemma 2: Upon convergence of Algorithm 2, a stable matching μ^*_{ext} will be obtained.

Proof: Suppose, Algorithm 2 converges to a matching μ^*_{ext} which means that μ^*_{ext} cannot be further updated. Let us assume that μ^*_{ext} has a blocking pair (k, n) which means that $n \neq \mu^*_{ext}(k)$, $n \succ_k \mu^*_{ext}(k)$ and $k \neq \mu^*_{ext}(n)$, $k \succ_n \mu^*_{ext}(n)$. Consider another iteration of Algorithm 2 with μ^*_{ext} as the input. In Phase I, for UE k we get $n \succ_k \mu^*_{ext}(k)$. Thus in Phase II, k will propose beam n before $\mu^*_{ext}(k)$. Also, beam n will accept the proposal as $k \succ_n \mu^*_{ext}(n)$. Therefore, $\mu^*_{ext}(k)$ will be updated with a (k, n) link contradicting our assumption of convergence of Algorithm 2 at $\mu^*_{ext}(k)$. Therefore, it means that the Algorithm 2 will never converge to a matching having a blocking pair. Thus upon convergence of Algorithm 2, a stable matching will be obtained. ■

IV. LINEAR PRECODING

The matching μ determined by either of the matching theoretic beam selection algorithms gives us $\tilde{\mathbf{H}}_b^H$ and (7) specifies the system after beam selection. The multi-user interference present in the effective channel $\tilde{\mathbf{H}}_b^H$ can be cancelled by employing a precoder $\tilde{\mathbf{P}}_b \in \mathbb{C}^{K\times K}$ at the transmitter.

Zero-forcing (ZF) precoder, given by

$$\tilde{\mathbf{P}}_b = \frac{\left(\tilde{\mathbf{H}}_b^H\right)^{-1}}{\left\|\left(\tilde{\mathbf{H}}_b^H\right)^{-1}\right\|}, \tag{14}$$

completely cancels the interference but results in poor performance as it reduces the received signal power [10]. The normalization by $\left\|\left(\tilde{\mathbf{H}}_b^H\right)^{-1}\right\|$ is to avoid increase in the transmit power due to precoding. Over Gaussian MIMO channels (where each element of the channel matrix is a complex Gaussian random variable with zero mean and unit variance), loss in sum-rate due to the normalization factor in (14) has been analyzed in [21].

We propose to employ a *linear* precoder inspired by the well-known *non-linear* Tomlison-Harashima precoder (THP) [22], [23]. QR-decomposition of $\tilde{\mathbf{H}}_b$ yields a unitary matrix $\mathbf{Q} \in \mathbb{C}^{K\times K}$ and an upper-triangular matrix $\mathbf{R} \in \mathbb{C}^{K\times K}$, such that $\tilde{\mathbf{H}}_b^H = \mathbf{R}^H\mathbf{Q}^H$ [20] and (7) can be written as

$$\mathbf{y} = \mathbf{L}\mathbf{Q}^H\tilde{\mathbf{P}}_b\mathbf{s} + \mathbf{w}, \tag{15}$$

where $\mathbf{L} = \mathbf{R}^H$ is a lower-triangular matrix. We propose to employ the *linear* precoder given by

$$\tilde{\mathbf{P}}_b = \frac{\mathbf{Q}\mathbf{L}^{-1}\mathbf{L}_D}{\alpha}, \tag{16}$$

where $\alpha = \|\mathbf{Q}\mathbf{L}^{-1}\mathbf{L}_D\|$ and $\mathbf{L}_D$ is a *diagonal* matrix constructed with the diagonal elements of $\mathbf{L}$. In (16), scaling by α is to ensure that $\|\tilde{\mathbf{P}}_b\mathbf{s}\| = \|\mathbf{s}\|$. It can easily be verified that the above precoder *diagonalizes* the channel and the received (*interference free*) symbols are given by

$$y_i = \frac{1}{\alpha} l_{ii} s_i + w_i, \quad i = 1, \ldots, K, \tag{17}$$

where l_{ii} is the i^{th} diagonal element of $\mathbf{L}$.

For the MU-MIMO downlink, characterized by (15), THP is a non-linear precoding scheme for canceling the multi-user interference at the transmitter [22], [23]. By making $\tilde{\mathbf{P}}_b = \mathbf{Q}$, the effective channel becomes equal to $\mathbf{L}$. Let the signaling constellation $\mathcal{A}$ is an M-QAM, (i.e., $s_k \in \mathcal{A}, k = 1, \ldots, K$), where M is a square number and $\mathcal{A} = \{a_I + \jmath a_Q | a_I, a_Q \in \{\pm 1, \pm 3, \ldots, \pm(\sqrt{M}-1)\}\}$. A symbol vector $\mathbf{x}$ is transmitted over the channel $\mathbf{L}$, where

$$x_i = \begin{cases} s_i, & i = 1, \\ s_i + e_i - \dfrac{\sum_{j=1}^{i-1} l_{ij} x_j}{l_{ii}}, & i = 2, \ldots, K, \end{cases} \tag{18}$$

where $e_i \in \{2\sqrt{M}(e_I + \jmath e_Q) | e_I, e_Q \in \mathbb{Z})\}$; Thus, integer multiples of $2\sqrt{M}$ are added to the real and imaginary parts of s_i. This *modulo* operation is performed to minimize the excess transmit power due to precoding (i.e., to minimize the difference $\|\mathbf{x}\| - \|\mathbf{s}\|$). The *effective* data symbols $d_i = s_i + e_i$ are from an expanded signal set $\mathcal{D} = \{d_I + \jmath d_Q | d_I, d_Q \in \{\pm 1, \pm 3, \ldots\}\}$ and the signal points that are separated in real and imaginary part by $2\sqrt{M}$ are *congruent* and represent the *same* message. The receiver takes care of the modulo congruence while decoding, and the *effective input-output relation*, with THP, becomes

$$y_i = l_{ii} s_i + w_i, \quad i = 1, \ldots, K. \tag{19}$$

It should be noted that THP still suffers from power loss (referred to as precoding loss), and is equal to $\frac{M}{M-1}$ for square QAM constellations with uniform probability of signaling, which becomes negligible even for moderate values of M. THP was shown to achieve significantly superior performance compared to ZF precoding [22].

Based on (17) and (19), sum-rate achieved by the linear precoder given by (16), denoted by $R_{\text{sum}}^{\text{P}}$, and the sum-rate achieved by THP, denoted by $R_{\text{sum}}^{\text{THP}}$, can be expressed as

$$R_{\text{sum}}^{\text{P}} = \sum_{k=1}^{K} \log_2 \left(1 + \frac{\left| l_{kk}^2 \right|}{\alpha} \right), \tag{20}$$

$$R_{\text{sum}}^{\text{THP}} = \sum_{k=1}^{K} \log_2 \left(1 + \left| l_{kk}^2 \right| \right). \tag{21}$$

Performance of the linear precoder given by (16) will be lower than that of the THP due to the normalization factor α, which is *essential* to avoid transmitting excess power with precoding; However, its linearity makes it more favorable for implementation. Also, from (20) and (21), we can observe that increasing the transmit power by α we can achieve the same sum-rate as that of THP. Further, simulation results (presented in the next section) show that the linear precoder given by (16) outperforms the ZF precoder. We do not have analytical results to quantify the performance difference between the linear precoder given by (16) and the ZF precoder. We will address this problem in our future work.

Note that the linear precoder proposed (and computed through an iterative algorithm) in [13] also results in the same input-output relation given by (17); Thus, the precoder that we propose to employ here (given by (16)), may be considered as equivalent to the one proposed in [13].

V. SIMULATION RESULTS

In this section, we present the sum-rate performance of the proposed matching theoretic beam selection algorithms, evaluated through Monte Carlo simulations. We consider an AP equipped with an 80 element array communicating with 40 UEs, each having a single antenna, i.e., $N = 80$ and $K = 40$. The channel from AP to each UE has one LoS component with $\beta_k^{(0)} \sim \mathcal{CN}(0, 1), \forall k$, and two NLoS components with $\beta_k^{(i)} \sim \mathcal{CN}(0, 0.01), \forall k$, and $i = 1, 2$.

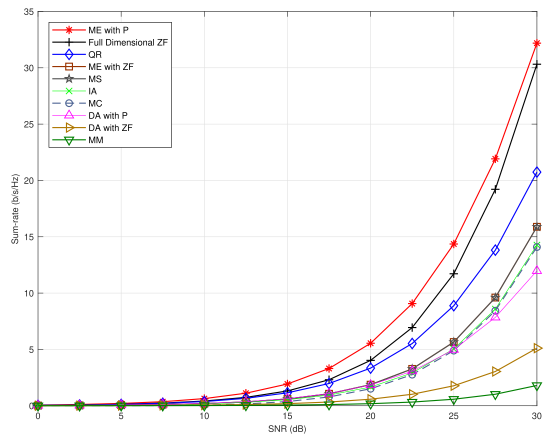

FIGURE 1. **Sum-rate performance of proposed matching theoretic beam selection algorithms in comparison with other algorithms.**

Fig. 1 shows how sum-rate varies with signal-to-noise ratio (SNR) for different beam selection algorithms. "DA with ZF" and "ME with ZF" denote Deferred Acceptance for beam selection (i.e., Algorithm 1) and Matching with Externalities for beam selection (i.e., Algorithm 2), respectively, with ZF precoder, given by (14), for canceling the interference. Similarly, "DA with P" and "ME with P" refers to Algorithm 1 and Algorithm 2, respectively, with precoder given by (16). "MM" refers to the maximum magnitude beam selection (proposed in [10] and modified in [12] to assign only one beam per user), "MC" is the maximizing capacity decremental beam selection[1] [12], "IA" refers to the interference-aware beam selection [11], "MS" is the maximizing SINR beam selection [12] and "QR" represents

[1]Difference in performance between MC decremental and incremental algorithms, proposed in [12], is negligible.

TABLE 2. Idealistic upper bound on sum-rate computed using Eqn. (13) in [10] and sum-rate achieved by ME with TH precoding.

SNR (dB)	Idealistic upper bound on sum-rate (b/s/Hz)	ME with THP sum-rate (b/s/Hz)
0	63.39	12.72
5	114.90	29.77
10	175.69	59.12
15	240.22	101
20	306.04	153.1
25	372.28	212.15
30	438.66	275.26

the QR decomposition based beam selection algorithm [13]. ‘‘Full Dimensional ZF’’ employs *all* the N beams along with ZF precoding and requires N RF chains [10]–[12].

As discussed previously, DA for beam selection neglects interference among users, and achieves better performance only compared to MM beam selection. Matching with externalities achieves a significantly higher sum-rate compared to DA. The performance of both DA and ME improve significantly when we employ the linear precoder given by (16). ‘‘ME with P’’ outperforms other beam selection algorithms, including the full dimensional ZF; Thus, with K beams (thereby requiring only K RF chains), matching with externalities along with the precoder given by (16) enables us to achieve higher sum-rate compared to a system that uses *all* the beams with a ZF precoder. Note that the performance of full dimensional ZF is *not* ‘‘the’’ upper bound on the sum-rate of a full dimensional system, as different precoders (such as matched filter and Wiener filter [10]), result in different upper bounds. Compared to the idealistic upper bound on the sum-rate given by [10, eq. (13)] (and computed in Table 2 for the system parameters that we consider here), the sum-rate achieved by ‘‘ME with P’’ is lower, justifying that we are not violating any fundamental limits. Table 2 also presents[2] the sum-rate performance of ’’ME with THP’’ (Algorithm 2 for beam selection followed by TH precoder); Comparing the simulation results for ‘‘ME with P’’ with ‘‘ME with THP’’ confirms that the performance difference between the linear precoder (given by (16)) and the THP corresponds exactly to the scaling factor α, as mentioned in the previous section. Though IA beam selection considers multi-user interference, it does so *only* for the interfering UEs (denoted as IUs in [11]); While assigning beams to the non-interfering UEs, IA algorithm does *not* consider the interference such a beam causes at other users, which results in a loss of performance. QR decomposition based beam selection [13], which employs a linear precoder that results in exactly the same effective channel as that of the linear precoder (given by (16)) that we employ here, also performs poorly compared to ‘‘ME with P’’. It is to be noted that, in QR based beam selection, once a beam is selected for a user, it’s effect on the beams selected *subsequently* for other users is *not* accounted for; In ME, after obtaining a matching the beam assignments get revised for *all* the users *if* there is a possibility for obtaining higher sum-rate with a new matching. At the same time, the this iterative process of modifying the beam assignment to all the users for achieving higher sum-rate may prove to be computationally costly.

Complexity of deferred acceptance algorithm is $O(NK)$. Thus, the complexity of Algorithm 1 is of the same order as that of the MM beam selection, but it has significantly better performance compared to MM algorithm. Complexity of Matching with Externalities (Algorithm 2), may be analyzed as follows:

(a) Complexity of Phase 1 is mainly determined by the complexity of computing the utility functions and which is equal to $O(K^2N)$.

(b) In Phase 2, the algorithm has to compare $R_{\text{sum}}(\mu_{\text{new}})$ with $R_{\text{sum}}(\mu_{\text{ext}})$ to accept or reject a UE’s proposal. Computing $R_{\text{sum}}(\mu)$ requires computing the precoding matrix for that particular matching through Eqn. (16) and then computing the sum-rate through (20). This complexity amounts to $O(K^3)$. In the worst case, a UE has to propose to N beams, resulting in a complexity of $(KN)O(K^3)$ for K users (It is worth noting that, during our simulations with $N = 80$, $K = 40$, we have observed that the average number of proposals that a UE has made is only 2.8, which is significantly less than 80).

(c) Thus, the worst case complexity of one iteration of Algorithm 2 is given by $(KN)O(K^3) + O(K^2N)$. The number of iterations required depends on the channel realization and the values of N and K. For the case of $N = 80$, $K = 40$, we have observed that the ME algorithm required an average of 3.75 iterations before computing a stable matching.

ME may become computationally complex, especially when it requires more iterations (containing Phase 1 and Phase 2) to find a stable matching. Thus, we pay a price in terms of complexity for the higher sum-rate achieved by ME.

VI. CONCLUSIONS

We have considered the problem of beam selection in a high-dimensional mmWave multi-user MIMO system. By modeling it as a problem of finding a *stable matching* between the set of users and the set of beams, we have proposed two beam selection algorithms. The first algorithm, deferred acceptance for beam selection, is a simple and low complexity one but has low sum-rate performance as it does *not* account for the multi-user interference. When both the interfering and the desired signal powers are considered in finding a stable matching, it becomes a problem of *matching with externalities*. We have proposed an iterative heuristic algorithm that solves this problem by finding a stable matching. Simulation results show that matching with externalities, combined with a linear precoder that was inspired by the Tomlinson-Harashima precoder, outperforms other beam selection methods.

[2]As the sum-rate of idealistic upper bound and ‘‘ME with THP’’ are much higher than the sum-rate achieved by other schemes, plotting them in Fig. 1 makes it difficult to distinguish performance of different schemes.

We hope that our work will serve as a starting point for developing more *efficient* beam selection algorithms based on matching theory. Exploring matching with *incomplete information* [24], [25], which would be useful when it is difficult to have complete channel knowledge, is another interesting avenue for future work.

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REFERENCES

[1] P. Wang, Y. Li, L. Song, and B. Vucetic, "Multi-gigabit millimeter wave wireless communications for 5G: From fixed access to cellular networks," *IEEE Commun. Mag.*, vol. 53, no. 1, pp. 168–178, Jan. 2015.

[2] T. S. Rappaport *et al.*, "Millimeter wave mobile communications for 5G cellular: It will work!" *IEEE Access*, vol. 1, pp. 335–349, May 2013.

[3] R. W. Heath, N. González-Prelcic, S. Rangan, W. Roh, and A. M. Sayeed, "An overview of signal processing techniques for millimeter wave MIMO systems," *IEEE J. Sel. Topics Signal Process.*, vol. 10, no. 3, pp. 436–453, Apr. 2016.

[4] S. Han, C.-L. I, Z. Xu, and C. Rowell, "Large-scale antenna systems with hybrid precoding analog and digital beamforming for millimeter wave 5G," *IEEE Commun. Mag.*, vol. 53, no. 1, pp. 186–194, Jan. 2015.

[5] S. Sun, T. S. Rappaport, R. W. Heath, Jr., A. Nix, and S. Rangan, "MIMO for millimeter-wave wireless communications: Beamforming, spatial multiplexing, or both?" *IEEE Commun. Mag.*, vol. 52, no. 12, pp. 110–121, Dec. 2014.

[6] X. Gao, L. Dai, and A. M. Sayeed, "Low RF-complexity technologies to enable millimeter-wave MIMO with large antenna array for 5G wireless communications," *IEEE Commun. Mag.*, vol. 56, no. 4, pp. 211–217, Apr. 2018.

[7] A. M. Sayeed, "Deconstructing multiantenna fading channels," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2563–2579, Oct. 2002.

[8] A. S. Y. Poon, R. W. Brodersen, and D. N. C. Tse, "Degrees of freedom in multiple-antenna channels: A signal space approach," *IEEE Trans. Inf. Theory*, vol. 51, no. 2, pp. 523–536, Feb. 2005.

[9] J. Brady, N. Behdad, and A. M. Sayeed, "Beamspace MIMO for millimeter-wave communications: System architecture, modeling, analysis, and measurements," *IEEE Trans. Antennas Propag.*, vol. 61, no. 7, pp. 3814–3827, Jul. 2013.

[10] A. Sayeed and J. Brady, "Beamspace MIMO for high-dimensional multiuser communication at millimeter-wave frequencies," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Atlanta, GA, USA, Dec. 2013, pp. 3679–3684.

[11] X. Gao, L. Dai, Z. Chen, Z. Wang, and Z. Zhang, "Near-optimal beam selection for beamspace mmWave massive MIMO systems," *IEEE Commun. Lett.*, vol. 20, no. 5, pp. 1054–1057, May 2016.

[12] P. V. Amadori and C. Masouros, "Low RF-complexity millimeter-wave beamspace-MIMO systems by beam selection," *IEEE Trans. Commun.*, vol. 63, no. 6, pp. 2212–2223, Jun. 2015.

[13] R. Pal, K. V. Srinivas, and A. K. Chaitanya, "A beam selection algorithm for millimeter-wave multi-user MIMO systems," *IEEE Commun. Lett.*, vol. 22, no. 4, pp. 852–855, Apr. 2018.

[14] D. Gale and L. S. Shapley, "College admissions and the stability of marriage," *Amer. Math. Monthly*, vol. 69, no. 1, pp. 9–15, Jan. 1962.

[15] A. E. Roth and M. A. O. Sotomayor, *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1990.

[16] A. E. Roth, "Deferred acceptance algorithms: History, theory, practice, and open questions," *Int. J. Game Theory*, vol. 36, pp. 537–569, Mar. 2008.

[17] Y. Gu, W. Saad, M. Bennis, M. Debbah, and Z. Han, "Matching theory for future wireless networks: Fundamentals and applications," *IEEE Commun. Mag.*, vol. 53, no. 5, pp. 52–59, May 2015.

[18] M. Pycia and M. B. Yenmez. *Matching With Externalities*. Accessed: Jan. 10, 2018. [Online]. Available: https://ssrn.com/abstract=2475468

[19] X. Gao, L. Dai, S. Han, I. Chih-Lin, and X. Wang, "Reliable beamspace channel estimation for millimeter-wave massive MIMO systems with lens antenna array," *IEEE Trans. Wireless Commun.*, vol. 16, no. 9, pp. 6010–6021, Sep. 2017.

[20] G. H. Golub and C. F. Van Loan, *Matrix Computations*, 3rd ed. Baltimore, MD, USA: Johns Hopkins University Press, 1996.

[21] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication—Part I: Channel inversion and regularization," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 195–202, Jan. 2005.

[22] C. Windpassinger, R. F. H. Fischer, T. Vencel, and J. B. Huber, "Precoding in multiantenna and multiuser communications," *IEEE Trans. Wireless Commun.*, vol. 3, no. 4, pp. 1305–1316, Jul. 2004.

[23] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge, U.K.: Cambridge Univ. Press, 2005.

[24] Q. Liu, G. J. Mailath, A. Postlewaite, and L. Samuelson, "Stable matching with incomplete information," *Econometrica*, vol. 82, no. 2, pp. 541–587, Mar. 2014.

[25] A. E. Roth, "Two-sided matching with incomplete information about others' preferences," *Games Econ. Behav.*, vol. 1, no. 2, pp. 191–209, Jun. 1989.

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