CHAPTER-7

Summary of the thesis and scope for future work

7.1 Summary of the thesis

The present thesis is aimed at analyzing various aspects of the recently proposed thermoelasticity theories by investigating some unsolved problems involving thermoelstic interactions inside a medium due to various types of thermo-mechanical loads. We have investigated the behavior of all the physical fields in the contexts of different thermoelasticity theories under different boundary and initial conditions. This main contribution of thesis is divided into following four parts:

7.1.1 Part-I: Behavior of field variables due to thermoelastic interactions inside a plate

This part is divided into two Chapters $(2-3)$ in which we have considered a homogeneous, isotropic and infinitely extended thick plate which is in undisturbed state and initially at uniform reference temperature. However, both the upper and lower surfaces of the plate have been assumed to be traction free and are subjected to an axi-symmetric temperature distribution. Mathematical formulation of the problem leads to a coupled system of partial differential equations. Helmholtz decomposition technique is used to decouple the problem and Laplace and Hankel transform techniques have been employed to solve the problem.

In Chapter-2, we have studied the problem under different models namely, Green-Naghdi models of type-I and type-II $(1991, 1993)$, dual phase-lag (1995) model and Green-Lindsay (1972) model. We present both analytical as well numerical results. We note that the distributions of each physical fields consist of two different parts: one is elastic and the other one is thermal in nature. It is found out that the elastic wave as well as thermal wave propagate with finite speed under GN-II model. However, these waves propagate without any attenuation. Thermal waves reach first the middle plane of plate as compared to the elastic waves in the case of all three models: GN-II model, Green-Lindsay model and dual phase-lag model. Thermal waves are not appeared in the case of GN-I model due to the presence of damping term in the heat conduction equation. However, we observe from the numerical values of physical fields that the behavior of all the fields in the case of GN-I model and Green-Lindsay model is almost similar in nature, although the Green-Lindsay model shows a small difference in the results near the boundary and at lower time as compared to GN-I model and dual phase-lag model. However, the behavior of all the physical fields predicted by GN-II model is significantly different as compared to the behavior of all the field variables predicted by other three models at any time. Further, this difference increases with the increase of time for all the field variables.

In Chapter-3, we investigated the behavior of physical fields under thermoelastic models: GN-III model (Green and Naghdi (1992)), Quintanilla models of type-I and II (Quintanilla (2011)) and LS model (Lord and Shulman(1967)). Here, we find that our analytical and numerical results for the solutions of different field variables predicted by the Quintanilla models are very much similar in nature with the corresponding results under GN-III model. However, a prominent difference is observed in the results predicted by LS-model as compared to these three models and on contrary to LS-model, the thermal wave does not propagate with finite speed in the context of other three models.

7.1.2 Part-II: The behavior of field variables due to thermoelastic interactions using memory dependent time derivatives

In this part, we have studied the thermoelasticity theory with one relaxation parameter (Lord and Shulman (1967)) using memory dependent derivatives. In Chapter-4, we have considered a homogeneous, isotropic and infinitely extended thick plate subjected to axi-symmetric temperature distribution applied at the boundary planes. We have compared our results with the corresponding results predicted by LS model in absence of memory dependent derivatives. In order to consider the memory effects, we have taken four different kernel functions and we have concluded that the trend of variation of all the physical field variables are similar in nature in the context of both the theories. However, there is a prominent difference in predictions of values of different physical fields by the theory with memory dependent derivative as compared to the predictions by the generalized theory without memory dependent derivatives. When the delay time parameter is considered to be very small, then each physical field variable shows almost the same value corresponding to all four different kernel functions. However, there is always a significant difference in the results between the cases when we consider the theory with memory dependent derivative and the theory without memory dependent derivatives for each kernel function.

7.1.3 Part-III: The behavior of field variables due to thermoelastic interactions under stochastic thermo-mechanical loads at the boundary

In this part (Chapter-5), we have investigated the responses of stochastic type thermo-mechanical distributions at the boundary of an elastic medium. We consider a one dimensional problem of half space and assume that the bounding surface of the half space is traction free. The thermoelasticity theories in the contexts of Green-Naghdi (1993) and Green-Lidsay theory (1972) have been considered for the investigation. In section 5.1, we have considered the case when the stress free boundary of the half space is subjected to two types of time dependent temperature distributions which are stochastic in nature. In section 5.2, we have taken into account the effects of the presence of some noise or uncertainty in the applied load in the boundary of elastic half space and represented it as a stochastic load. In both the sections, the corresponding deterministic type boundary conditions are also considered in order to investigate the effects of stochastic conditions in more detailed way. It has been shown analytically and numerically that in both the cases that the mean of the stochastic solutions of all the physical fields coincides with the respective deterministic solution. It is observed that the deviation of the stochastic solution from its mean decreases with the distance from the bounding plane which is the source of noise. Our analytical results as well as numerical results clearly indicate that all the physical fields vanish after a certain distance from the boundary in the stochastic case as well as in the deterministic case. This ascertains the fact that the thermoelasticity without energy dissipation theory and thermoelasticity with two relaxation parameters theory predicts finite speed of thermal waves.

7.1.4 Part-IV: The behavior of field variables near Mode-I crack in thermoelastic medium

This part (Chapter-6) aims at investigating a two dimensional Griffith crack problem represented by a line segment. We have considered a two dimensional dynamical problem of an infinite space with a finite linear Mode-I crack. The thermoelastic medium is taken to be homogeneous and isotropic. However, the boundary of the crack is subjected to a prescribed temperature and stress distributions. Special attention has been paid to understand the thermoelastic behavior inside the medium in the neighborhood of the crack in which we have employed the thermoelasticity theory given by Quintanilla (2011), namely Quintanilla-I and Quintanilla-II models and compared the results with the corresponding results of type-III thermoelasticity theory of Green and Naghdi. We have formulated the problem in such way that all the three models (new model-I, II and GN-III model) can be written in a unified way. We observe the behavior of all the physical fields in the vicinity of the crack and concluded that under all models, each physical field shows the same nature throughout the domain which vanishes after some distance from the end edge of the crack. However, the value of each physical field decreases with the increase of the vertical distance from the end of the crack region under each thermoelasticity theory. The results under different models differ significantly, although the Quintanilla model-II and GN-III model predict more similar results as compared to new Quintanilla model-I. This implies that there is a significant effect of single delay time parameter for the present crack problem.

7.2 Future scope for research work

In recent years, a serious attention is being paid on coupled thermoelasticity due to its wide applications in science and technology. Various generalizations are proposed to overcome the limitations/drawbacks in uncoupled thermoleasticity and the classical theory of thermoelasticity. The present thesis analyzed the predictions of some recently proposed theories and highlighted the drawbacks in some models. There is a need of further research in this respect to understand to the coupling effects of thermal and mechanical fields. The best suitable model need to be identified through further research in theoretical as well as in experimental direction. Although, some works are reported in this direction but they are very few in number to consider the fully coupled system of equations. Therefore, it is worth pursuing research in this area.

In recent years, the concept of stochastic simulation techniques are being used for the analysis of heat conduction and thermoelastic problem due to the fact that stochastic thermal stress analysis, instead of deterministic thermal stress analysis, is more necessary to maintain the reliability and safety gain in the design of high-temperature apparatuses or heat resistant structures. The stochastic thermal stress involves uncertainties in the apparatus structures and in thermal environments like, external temperatures, heat transfer coefficients etc. Therefore, it will also be possible to understand more realistic behavior of physical fields in the thermoelastic media when the stochastic noise is applied in the boundary of the medium.

Now days, heat conduction with fractional order derivatives and memory dependent derivatives have also been introduced in thermoleasticity theory to address more realistic results related to thermoelastic interactions. Also problems on cracks and failures in solids have the wide applications in the industry, particularly in the fabrication of electronic components, geophysics, earthquake engineering, etc. In the analysis of thermal stresses in elements the actual properties of the material cannot be described precisely without taking the following properties: (i) temperature dependency of material properties (ii) time dependent strain and stress response of materials (iii) considering the decrease in yield stress of the material with rising temperature. Hence, it is believed that further research is required to study various models of thermoelasticity by considering these mentioned properties. The thesis is therefore being completed with a great expectation that the above mentioned points will be taken under consideration in future.