PREFACE

The motion of a solid body is characterized by mutual interactions between deformation and temperature fields. The domain of science dealing with the mutual interactions of these two fields is called as thermoelasticity. Initially, the investigations in this area were based on the "uncoupled theory of thermoelasticity" with the simplifying assumption that the influence of the strain and stresses on the temperature field may be neglected. The absence of any elasticity term in the heat conduction equation for uncoupled thermoelasticity appears to be unrealistic, since due to the mechanical loading of an elastic body, the strain so produced causes variation in the temperature field. Moreover, the parabolic type classical heat conduction equation results in an infinite velocity of thermal wave propagation, which also contradicts the actual physical phenomena. Hence, in recent years a serious attention is being paid on coupled thermoelasticity due to its wide applications in science and technology. Introducing the strain-rate term in the uncoupled heat conduction equation, Biot (1956) extended the analysis to incorporate coupled thermoelasticity for the first time. However, although the first shortcoming was eliminated in this theory, there remained the parabolic type partial differential equation of heat conduction, which leads to the paradox of infinite velocity of the thermal wave. During last few years, various generalized models of thermoelasticity have come into existence. The main attempt of these modified theories is to eliminate the so-called paradox of infinite speed of thermal signal inherent in the classical theory introduced by Biot (1956).

The present thesis is divided into four different parts which are concerned with mathematical modeling on coupled thermo-mechanical problems in the contexts of some recent models of thermoelasticity. The thesis is aimed at analyzing various aspects of these recently proposed thermoelasticity theories by investigating some problems involving thermoelastic interactions inside a medium due to various types of thermo-mechanical loads. In the first part of the thesis, a problem of an infinitely extended thick plate is considered under homogeneous and isotropic thermoelastic medium. The boundary planes of the plate are subjected to thermo-mechanical loadings. We aim to study the behavior of all the physical field variables inside the plate in the contexts of various theories. The potential function concept along with the Laplace and Hankel transform techniques are used to solve the problem. We have also discussed the continuity and discontinuity of these field variables predicted by different models. In the second part, we have considered the plate problem subjected to axi-symmetric temperature distribution applied at the boundary surfaces and studied the effects of memory dependent derivatives in the generalized thermoelasticity theory and compared the results under the theory without memory effects. The third part of the thesis deals with mathematical modeling on coupled thermo-mechanical problems involving stochastic thermo-mechanical loading and discuss the effects of stochastic boundary conditions on the wave propagation in the elastic medium, where as in the fourth part, we discuss a problem of thermoelastic medium that contains a crack. The Mode-I crack in the material has been considered and the behavior of different physical fields near the crack region has been investigated in a detailed manner. Several points regarding the predictions of various heat conduction models for thermoelastic interactions have been highlighted.

The thesis consists of Six chapters. The **Chapter-1** is introductory in which the basics of coupled thermoelasticity theory along with its recent development is described. We have also given the brief discussion of stochastic process, Wiener process, memory dependent derivatives and basic features of crack of different modes in an elastic materials. A detailed literature review concerned with the present work has been included in this Chapter.

In **Chapter-2,** we analyze the thermoelastic interactions inside an infinitely extended thick plate due to an axi-symmetric temperature distribution applied at the lower and upper surfaces of the plate under recent heat conduction models, namely Green-Naghdi-I model, Green-Naghdi-II model, dual phase-lag model and Green-Lindsay model. In order to investigate the problem under all these four heat conduction models simultaneously, we consider the basic governing equations under all these models and formulate our problem in a unified way. The special findings and differences among the predictions by four models have been highlighted. The locations of discontinuities of physical fields under different theories have been identified.

In **Chapter-3,** the work is concerned with a recently proposed heat conduction model: an exact heat conduction model with a single delay term. The main purpose of this work is to examine the effects of the single phase-lag parameter/delay term on wave propagation in an infinitely extended thick plate due to axi-symmetric temperature distribution applied in the lower and upper surfaces of the plate. The problem is formulated by using the new heat conduction model in such a way that other existing models can be extracted as special cases. We have also discussed the discontinuity of different physical fields. Problem has also been solved numerically in order to compare the results predicted by different models.

In **Chapter-4,** we carry out the work under generalized thermoelasticity with memory dependent derivatives. The main purpose of this work is to analyze the thermoelastic interactions inside an infinitely extended thick plate due to axi-symmetric temperature distribution applied at the lower and upper surface of the plate under the generalized thermoelasticity with memory dependent derivatives. The formulation of the problem is done in the context of the theory of thermoelasticity under the memory dependent derivatives with inclusion of time delay parameter, τ and kernel functions that are defined in a slipping interval $[t-\tau, t)$. We have also compared the findings of the present work with the corresponding results under other models existing in the literature. An analysis and the comparison of all the physical fields inherent to the generalized thermoelasticity with one relaxation parameter is given in a detailed way. The special findings and differences of using different kernel functions are also highlighted.

The **Chapter-5** is further divided into two **Sections**: Section 5.1 and Section 5.2. In **Section 5.1,** we investigate the responses of stochastic type temperature distribution applied at the boundary of an elastic medium in the context of thermoelasticity without energy dissipation. An one dimensional problem of half space is assumed in such a way that the bounding surface of the half space is traction free and is subjected to two types of time dependent temperature distributions which are of stochastic types. In **Section 5.2,** we consider the stochastic mechanical load at the boundary of the elastic medium in the context of Green-Lindsay theory. Due to the following reasons, the stochastic boundary conditions are considered:

1. The system is not fully isolated, thus background fields give rise to additional noise.
2. Not all the variables that characterize the system are included in the model and these variables give rise to additional noise.
3. The accuracy of the measuring devices for the temperature, etc. are not 100% accurate.

In both the sections, a detailed comparison of the results of stochastic temperature, displacement and stress distributions inside the half space with the corresponding results of deterministic distributions is presented. We have shown both analytically and numerically that mean of the stochastic solution of all the physical fields in all cases coincides with the respective deterministic solution.

In **Chapter-6,** we consider a two dimensional dynamical problem of an infinite space containing a finite length linear Mode-I crack and employ the recently proposed heat conduction model with a single delay term. The thermoelastic medium is taken to be homogeneous and isotropic. However, the boundary of the crack is subjected to a prescribed temperature and stress distributions. Mathematical modeling of the present problem reduces the solution of the problem into the solution of a system of four dual integral equations. The solution of these equations is shown to be equivalent to the solution of the Fredholm's integral equation of the first kind which has been solved by using the regularization method. We solve the problem numerically and highlight the effects of the presence of crack in the behavior of thermoelastic interactions inside the medium in the present context and its results are compared with the results of the thermoelasticity of type-III.

The last chapter **(Chapter-7)** of the thesis incorporates the summary of the thesis work as well as scope for future research work in the relevant area. The list of references appeared in the thesis are appended at the end of the thesis.