Chapter 5

INTELLIGENT & OPTIMAL CONTROLLER DESIGN FOR NONLINEAR SYSTEM

5.1 INTRODUCTION

One of the more popular new technologies is intelligent control, which is defined as a combination of control theory, operations research, and artificial intelligence (AI). Judging by the billions of dollars worth of sales and close to 2000 patents issued in Japan alone since the announcement of the first fuzzy chips in 1987, fuzzy logic still is perhaps the most popular area in AI. Thanks to tremendous technological and commercial advances in fuzzy logic in Japan and other nations, today fuzzy logic continues to enjoy an unprecedented popularity in the technological and engineering fields including manufacturing.

Fuzzy logic technology is being used in numerous consumer and electronic products and systems, even in the stock market and medical diagnostics. The most important issue facing many industrialized nations in the next several decades will be global competition to an extent that has never before been posed. The arms race is diminishing and the economic race is in full swing. Fuzzy logic is but one such front for global technological, economical, and manufacturing competition.

In order to understand fuzzy logic it is important to discuss fuzzy sets. In 1965, Zadeh [1] wrote a seminal paper in which he introduced fuzzy sets, i.e., sets with unsharp boundaries. These sets are generally in better agreement with the human mind that works with shades of gray, rather than with just black or white. Fuzzy sets are typically able to represent linguistic terms, e.g., warm, hot, high, low. Nearly ten years later Mamdani [2] succeeded in applying fuzzy logic for control in practice.

Today, in Japan, U.S.A, Europe, Asia and many other parts of the world fuzzy control is widely accepted and applied. In many consumer products like washing machines and cameras, fuzzy controllers are used in order to obtain intelligent machines (Intelligent Machine Quotient-$MIQ^{®}$) and user friendly products. A few interesting applications can be mentioned: control of subway systems, image stabilization of video cameras, image enhancement and autonomous control of helicopters [3]. Although the U.S and Europe hesitated in accepting fuzzy logic, they have become more enthusiastic about applying this technology. Fuzzy set theory is developed comparing the precepts and operations of fuzzy sets with those of classical set theory. Fuzzy sets will be seen to contain the vast majority of the definitions, precepts, and axioms that define classical sets.

In fact, very few differences exist between the two set theories [4]. Fuzzy set theory is actually a fundamentally broader theory than current classical set theory, in that it considers an infinite number of degrees of membership in a set other than the canonical values of 0 and 1 apparent in classical set theory. In this sense, one could argue that classical sets are a limited form of fuzzy sets. Hence, it will be shown that fuzzy set theory is a comprehensive set theory. Conceptually, a fuzzy set can be defined as a collection of elements in a universe of information where the boundary of the set contained in the universe is ambiguous, vague, and otherwise fuzzy. It is instructive to introduce fuzzy sets by first reviewing the elements of classical (crisp) set theory [5-7].

In this chapter the fuzzy logic control is used to control the nonlinear system. The nonlinear system considered in this chapter is the inverted-pendulum system mounted on a cart. The inverted pendulum system is one of the most important problems in control theory and has been studied extensively in control literature. The system is nonlinear, unstable, nonminimum phase and under-actuated. Because of their nonlinear nature inverted pendulum have maintained their usefulness and are used to illustrate many of the ideas emerging in the field of nonlinear control.

The inverted pendulum can be considered as the simplest robotic system, with only one rigid body and only one rotational joint. The inverted pendulum system has a stable equilibrium point when the pendulum is in a pending position and an unstable equilibrium when the pendulum is in an upright position. The model is strongly nonlinear when it is moved from the pending position to the upright position [8].

Since the 1950's, it is used for teaching linear feedback control theory to stabilize open-loop unstable systems [9]. The first solution to this problem was described in 1960 by Roberge [10] and then by Schaefer and Canon in 1966 [11]. Siebert [12] used this system as a typical model for root-locus analysis and Kwakernaak [13] used to solve the linear optimal control problem. The recent concise review in explains and discusses, for example, the design of type-2 fuzzy systems using optimization methods.

According to the control purposes of inverted pendulum, the control of inverted pendulum can be divided into three aspects. The first aspect is that it is widely researched for the swing-up control of inverted pendulum. The second aspect is the stabilization of the inverted pendulum. The third aspect is tracking control of the inverted pendulum.

5.2 MATHEMATICAL MODEL OF INVERTED PENDULUM SYSTEM

The inverted pendulum system is a perfect benchmark problem for the design of a wide range of control techniques. Inverted-pendulum system is mounted on a motor driven cart, said to be a highly nonlinear system because it may fall at any time and in any direction. To control such a system, a suitable controlled force is required, which is able to drive the cart in such a manner so that the pendulum gets stabilized within the stipulated time.

The free-body diagram of the system is shown in Figure 5.1, and assuming that the centre of gravity of the pendulum rod is at its geometric centre. The cart of the inverted pendulum system can move in both the direction connected through a belt. The movement of the belt is via pulley which is linked with a DC motor (24 V, 400 rpm).

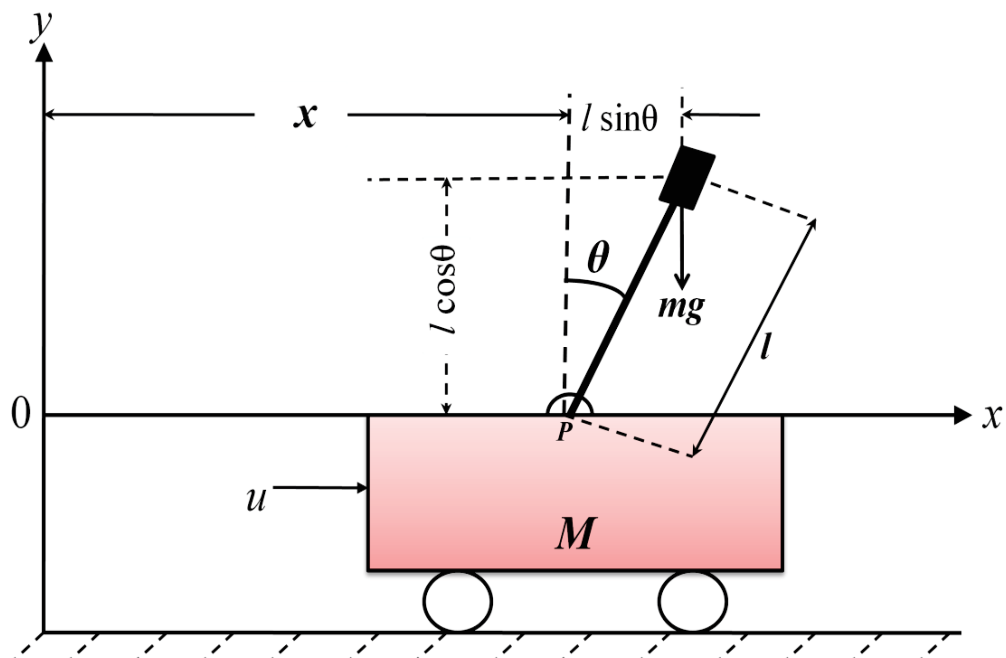

Figure 5.1 Free-body diagram of Inverted pendulum

The dynamics of the inverted pendulum system is obtained using the Lagrangian equation of motion.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=\tau \tag{5.1}$$

where L is the Lagrangian function, q is the position vector and $i = 1, 2, 3, \ldots, n$. The Lagrangian function is defined as the difference between the kinetic energy and potential energy of the system.

$$L = K_E - P_E \tag{5.2}$$

where K_E represents the kinetic energy and P_E is the potential energy of the system. Therefore, to determine the equation of motion of inverted pendulum system, we first defined the co-ordinates of the system. The x position of the pendulum is $x + l\,sin\theta$ and y position is $l\,cos\theta$, so the kinetic energy is-

$$K_E = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left(\frac{d}{dt}\left(x + l\sin\theta\right)\right)^2 + \frac{1}{2}m\left(\frac{d}{dt}\left(l\cos\theta\right)\right)^2 \tag{5.3}$$

First taking time-derivatives, then squaring, then noting that $cos2\theta + sin2\theta = 1$ gives-

$$K_E = \frac{1}{2}\left(M + m\right)\dot{x}^2 + ml\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}ml^2\dot{\theta}^2 \tag{5.4}$$

The potential energy of the inverted pendulum system is given as-

$$P_E = mgl\left(1 + \cos\theta\right) \tag{5.5}$$

Therefore, the Lagrangian function is given by-

$$L = \frac{1}{2}\left(M + m\right)\dot{x}^2 + ml\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}ml^2\dot{\theta}^2 - mgl\left(1 + \cos\theta\right) \tag{5.6}$$

The Lagrangian equation of motion for cart is defined as-

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right)-\frac{\partial L}{\partial x_i}=u \tag{5.7}$$

Since, $\partial L/\partial x=0$

and

$$\frac{\partial L}{\partial \dot{x}}=\left(M+m\right)\dot{x}+ml\dot{\theta}\cos\theta \tag{5.8}$$

Therefore,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=\left(M+m\right)\ddot{x}+ml\left(\ddot{\theta}\cos\theta-\dot{\theta}^2\sin\theta\right)=u \tag{5.9}$$

The Lagrange equation corresponding to the pendulum is given by-

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right)-\frac{\partial L}{\partial \theta_i}=0 \tag{5.10}$$

Whereas,

$$\frac{\partial L}{\partial \theta}=-ml\dot{x}\dot{\theta}\sin\theta+mgl\sin\theta \tag{5.11}$$

and

$$\frac{\partial L}{\partial \dot{\theta}}=ml\dot{x}\cos\theta+ml^2\dot{\theta} \tag{5.12}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)=\left(ml\ddot{x}\cos\theta-ml\dot{x}\dot{\theta}\sin\theta\right)+ml^2\ddot{\theta} \tag{5.13}$$

Therefore,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)-\frac{\partial L}{\partial \theta}=ml\left(\ddot{x}\cos\theta+l\ddot{\theta}-g\sin\theta\right)=0 \tag{5.14}$$

Therefore, the equations of motion for inverted pendulum system are-

$$\ddot{x}=\frac{1}{\left(M+m\right)}\left(u+ml\dot{\theta}^{2}\sin\theta-ml\ddot{\theta}\cos\theta\right) \tag{5.15}$$

$$\ddot{\theta}=\frac{1}{l}\left(g\sin\theta-\ddot{x}\cos\theta\right) \tag{5.16}$$

Both these equations have dependency on each other. Therefore, to cope up from this dependency, deriving the state-space equation of the inverted-pendulum system. Let us also assume that,

$$x_1=x \tag{5.17}$$

$$x_2=\dot{x}=\dot{x}_1 \tag{5.18}$$

$$x_3=\theta \tag{5.19}$$

$$x_4=\dot{\theta}=\dot{x}_3 \tag{5.20}$$

Therefore, the equation of motion for inverted pendulum system is

$$\dot{x}_2=\frac{1}{\left(M+m\right)}\left(u+mlx_4^2\sin x_3-ml\dot{x}_4\cos x_3\right) \tag{5.21}$$

$$\dot{x}_4=\frac{1}{l}\left(g\sin x_3-\dot{x}_2\cos x_3\right) \tag{5.22}$$

After simplifying the dependency of these two equations gives the equation of cart and pendulum-

$$\dot{x}_2 = \frac{\left(u + mlx_4^2 \sin x_3 - mg \cos x_3 \sin x_3\right)}{M + m - m\cos^2 x_3} \tag{5.23}$$

$$\dot{x}_4 = \frac{\frac{1}{l}\left(M + m\right) g \sin x_3 - mx_4^2 \cos x_3 \sin x_3 - u \cos x_3}{M + m - m\cos^2 x_3} \tag{5.24}$$

The linear model for the system around the upright stationary point is derived by assuming x_0=0, u_0=0. i.e.,

$$\delta \dot{x} = J\left(x_0, u_0\right)\Big|_{x_0=0, u_0=0} \delta x + J\left(x_0, u_0\right)\Big|_{x_0=0, u_0=0} \delta u \tag{5.25}$$

After simplifying and by doing partial derivative of each term the linearized state-space equation becomes:

$$\delta \dot{x} = \begin{bmatrix} \delta \dot{x}_1 \\ \delta \dot{x}_2 \\ \delta \dot{x}_3 \\ \delta \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{\left(M+m\right)g}{Ml} & 0 \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \\ \delta x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{Ml} \end{bmatrix} \delta u \tag{5.26}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \delta x + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \delta u \tag{5.27}$$

The above system is a single input multi output system. The control input u is the driving force.

Table 5.1: Parameters of Inverted Pendulum System

Mass of cart (M) in kg	2.4
Mass of pendulum (m) in kg	0.23
Length of the pendulum (L) in mts.	0.4
Moment of inertia of pendulum (I) in kgm^2	0.01
Friction coefficient of cart (b) in N/m/sec	0.05
Gravitational constant (g) in m/sec^2	9.8

The state-space equation of the inverted pendulum system is obtained by using the values as given in Table 5.1.

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -0.0195 & 0.2381 & 0\\ 0 & 0 & 0 & 1\\ 0 & -0.0132 & 6.8073 & 0\end{bmatrix} \qquad B=\begin{bmatrix}0\\ 0.3895\\ 0\\ 0.2638\end{bmatrix} \tag{5.28}$$

$$C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix} \qquad D=\begin{bmatrix}0\\ 0\end{bmatrix}$$

The open-loop response of the inverted pendulum system is shown in Figure 5.2 and Figure 5.3.

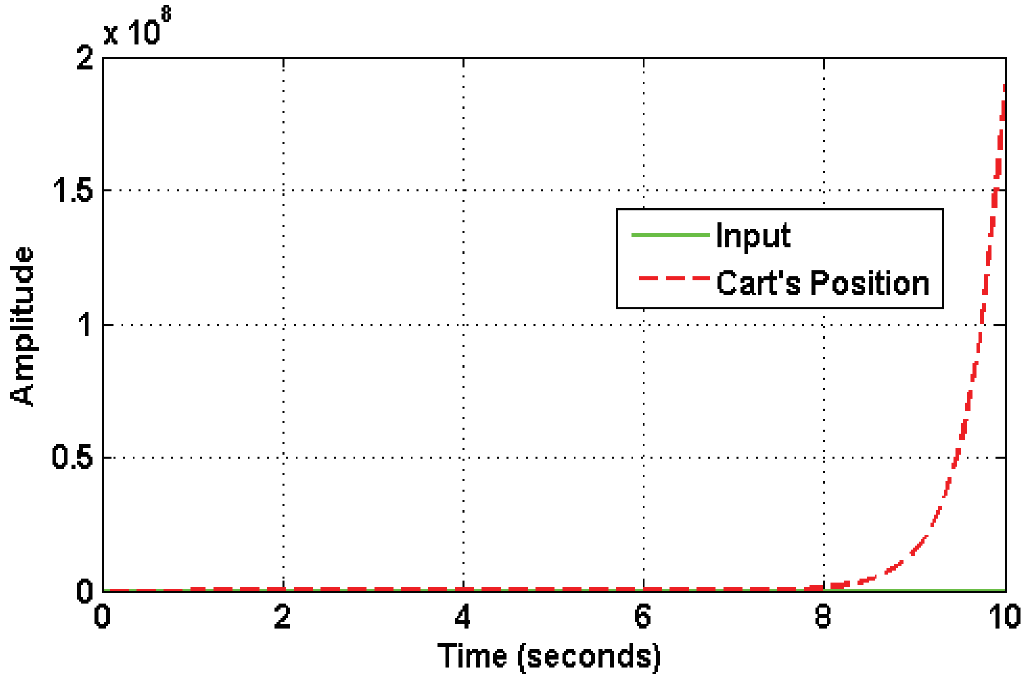

Figure 5.2 Open-loop response of cart's position

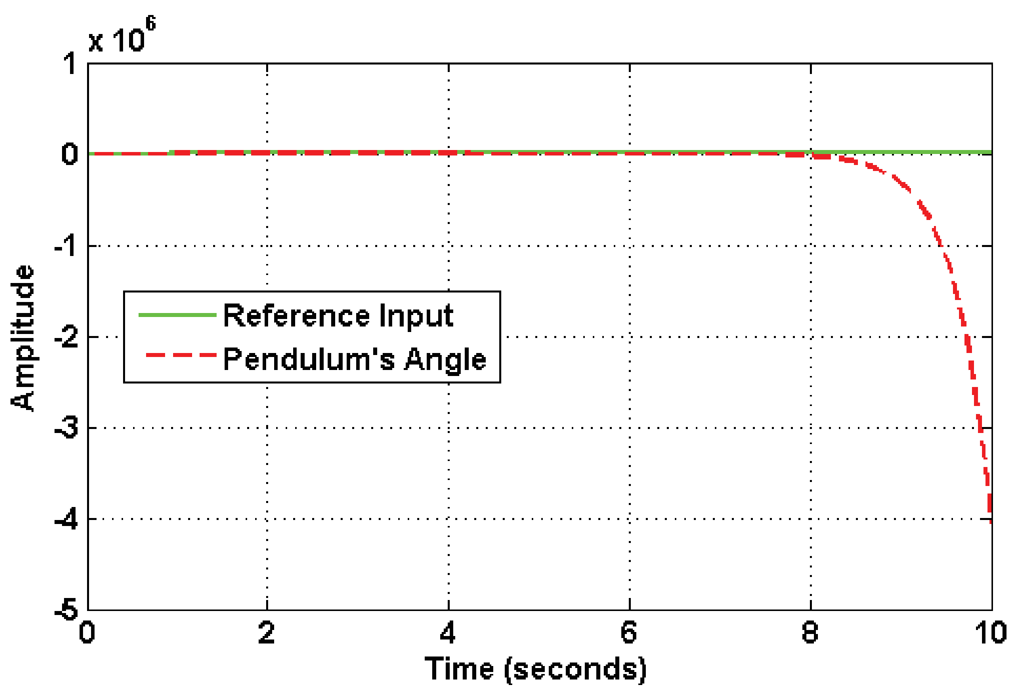

Figure 5.3 Open-loop response of pendulum's angle

The response of inverted-pendulum system is unstable as the output reaches to infinite with finite input signal. Therefore, in order to stabilize the pendulum in an upright position, the movement of the cart is to be regulated by using a proper control functioning. The control methodology the control design requirements are mentioned below:

5.2.1 Controller Design Requirements

To examine the performance of the controller following design specification are required:

- The percent overshoot of cart position *(x)* is ≤ 15%
- The rise time of cart position *(x)* ≤ 2 sec.
- The settling time of cart position *(x)* and pendulum angle ≤ 15 sec.
- Steady-state error is within 2%.

5.3 CONTROL METHODOLOGY

The following control methods are presented here to control the nonlinear inverted-pendulum system.

5.3.1 PID Control

To stabilize the inverted pendulum in the upright position and to control the cart at the desired position using the PID control approach, two PID controllers are required: Angle PID controller and cart PID controller. The equations of the PID control are given as:

$$u_P(t) = K_p e_\theta(t) + K_i \int e_\theta(t)\,dt + K_d \frac{de_\theta(t)}{dt} \tag{5.29}$$

$$u_c(t) = K_p e_x(t) + K_i \int e_x(t)\,dt + K_d \frac{de_x(t)}{dt} \tag{5.30}$$

where $e_\theta(t)$ and $e_x(t)$ are angle error and cart position error, respectively and whereas K_p, K_i and K_d are the parameters of PID controller. Since, the pendulum angle dynamics and cart position dynamics are coupled to each other, the change in any controller parameters affects both the pendulum angle and cart position, which makes the tuning tedious. The tuning of controller parameters is done by using trial and error methods and observing the responses of Simulink model to be optimal. Two PID controllers are required to control the inverted-pendulum system, as shown in Figure 5.4. The parameters of both the PID controller are shown in Table 5.2.

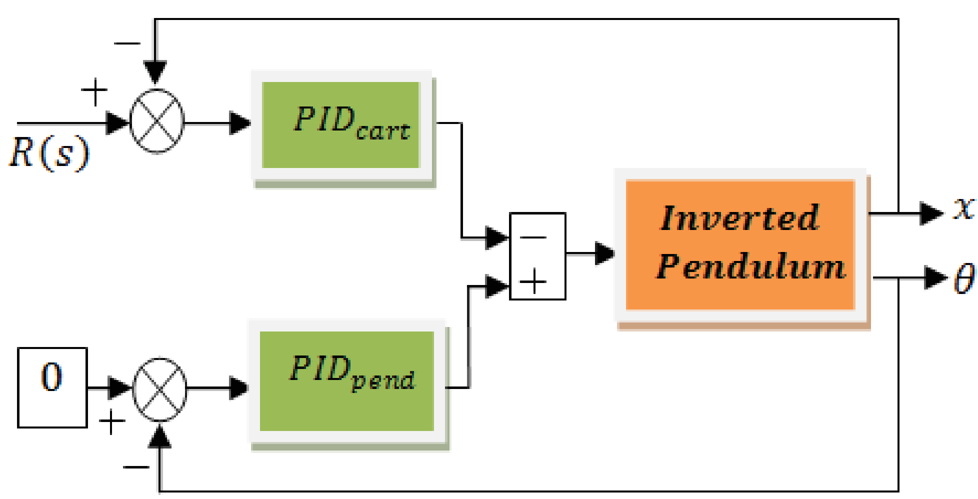

Figure 5.4 Block diagram of closed-loop control system

Table 5.2: PID controller parameters for controlling the inverted-pendulum system

For CART			For PENDULUM		
K_p	K_i	K_d	K_p	K_i	K_d
2.00	0.02	0.10	45	25	11

These parameters are used to control the movement of the cart which in turn stabilizes the pendulum's position. The pendulum gets stabilized in an upright position as per the design requirements but the PID controller fails in achieving all the specifications. Nevertheless, the position of the cart and angle of the pendulum reaches the target input but in achieving all these it has compensate with the oscillation of the system. Both the controllers posses additional swinging in the pendulum's movement and cart's position. And taking more time as per the controller design requirements.

The response of cart's position and pendulum's angle is shown in Figure 5.5 and Figure 5.6. The time-domain specification is given in Table 5.3. The settling time of the cart's position is around 36 seconds and pendulum's angle is around 32 seconds which is far more than the design requirements. The percentage overshoot of the inverted-pendulum system is also very large which is undesirable and therefore, the parameters are either required to be optimized or some better control technqiues is to be utilized for achieving all the design requirements.

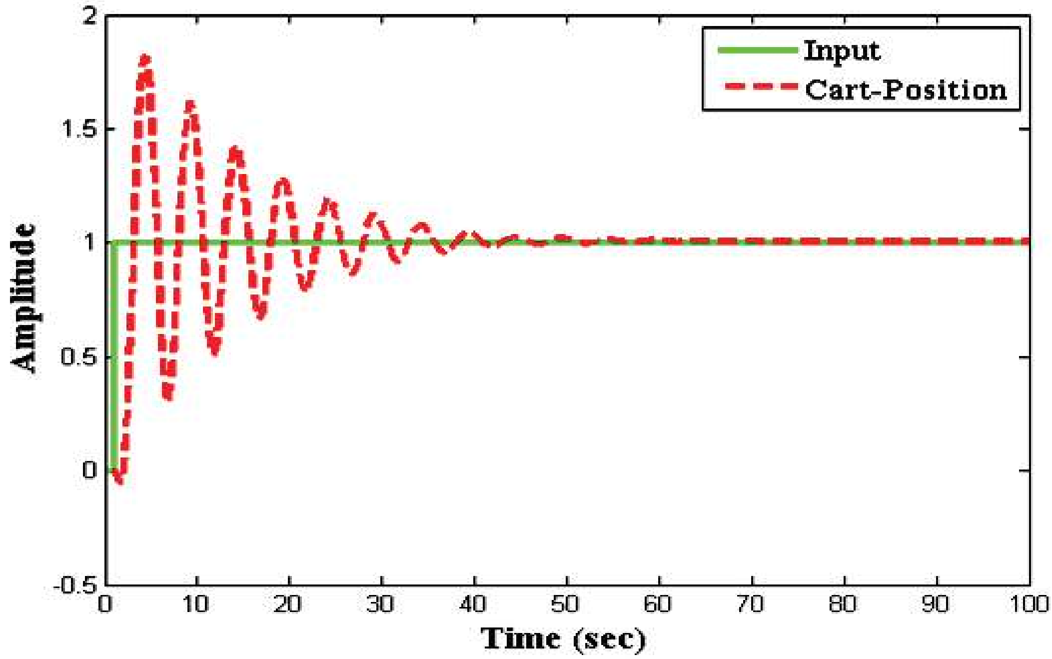

Figure 5.5 Closed-loop response of cart's position with PID controller

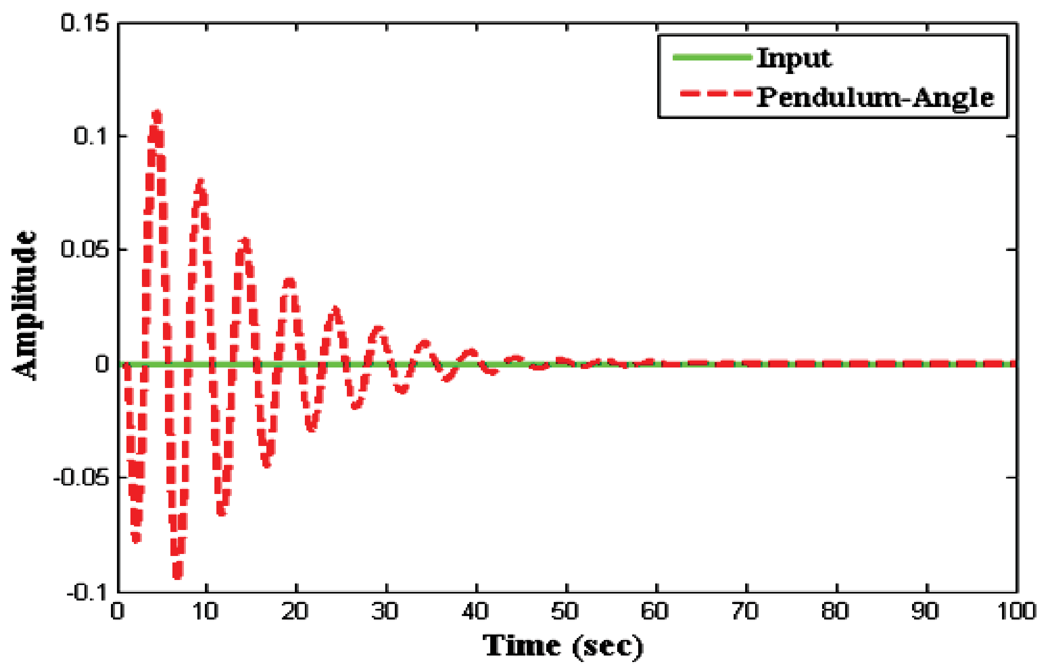

Figure 5.6 Closed-loop response of pendulum's angle with PID controller

Table 5.3: Time-domain specification of inverted-pendulum system with PID controller

	Rise Time (sec.)	Settling Time (sec.)	Peak Time (sec.)	Percentage Overshoot
Cart	3.5	36.23	7.65	65.50
Pendulum	5.4	32.45	8.40	12.50

5.3.1.1 Outcomes of PID Controller

Two PID controllers are designed to stabilize the pendulum's angle and the cart's position. The parameters of the PID controllers are obtained using trial & error method. The settling time and percentage overshoot of the system is too high. This is main drawback of the PID controller that it stabilizes the inverted-pendulum system but it fails in achieving the controller design requirements. The inverted pendulum system required an efficient controller so that it may able to reduce the oscillations and it could be able to control the system within the specified time.

5.3.2 OPTIMAL CONTROL

Optimal control refers to a class of methods that can be used to synthesize a control policy which results in the best possible behavior with respect to the prescribed criterion (i.e., control policy which leads to maximization of performance).

The main objective of optimal control is to determine control signals that will cause a process (plant) to satisfy some physical constraints and at the same time extremize (maximize or minimize) a chosen performance criterion i.e. the performance index (PI) or cost function. The optimal control problem is to find a control which causes the dynamical system to reach a target or follow a state variable (or trajectory) and at the same time extremize the Performance Index [14-20].

5.3.2.1 Linear Quadratic Regulator

Linear quadratic regulator (LQR) is one of the optimal control techniques, which takes into account the states of the dynamical system and control input to make the optimal control decisions. The Linear Quadratic Regulator (LQR) is design on the basis of algebraic Riccati equation (ARE). After linearization of nonlinear system equations about the equilibrium position having initial conditions as $x_0 = [0, 0, 0, 0]^T$, the linear state-space equation is obtained as-

$$\dot{x} = Ax + Bu \tag{5.31}$$

where $x = [x \quad \dot{x} \quad \theta \quad \dot{\theta}]^T$

The state feedback control $u = -Kx$ leads to

$$\dot{x} = (A - BK)x \tag{5.32}$$

where K is derived from minimization of the cost function.

$$J = \int (x^T Q x + u^T R u) dt \tag{5.33}$$

where Q and R are positive semi-definite and positive definite symmetric constant matrices, respectively. The LQR gain vector K is given by-

$$K = R^{-1} B^T P \tag{5.34}$$

where P is a positive definite symmetric constant matrix obtained from the solution of ARE given as-

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{5.35}$$

Under the specified conditions, it is desired to minimize J and regulate the states to zero such that $\lim_{t\to\infty} X(t) = 0$ with respect to control input 'u'. The solution obtained is the optimal control input and the trajectory obtained is the optimal trajectory.

- Selection for Weighting Matrices

  $Q \geq 0\ (psdf)$, $R > 0\ (pdf)$, are usually chosen as diagonal matrices, with

  $q_i = maximum\ expected/acceptable\ value\ of\ (1/x_i^2)$

  $r_i = maximum\ expected/acceptable\ value\ of\ (1/u_i^2)$

  $The\ pair\ \{A, B\}\ needs\ to\ be\ controllable\ and$

  $The\ pair\ \{A, \sqrt{Q}\}\ needs\ to\ be\ detectable.$

With the choice of weighting matrix as-

$$Q = \begin{bmatrix} 3000 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad R=1 \tag{5.36}$$

The LQR gain vector is obtained as

$$K = \begin{bmatrix} -54.77 & -59.34 & 340.65 & 131.90 \end{bmatrix} \tag{5.37}$$

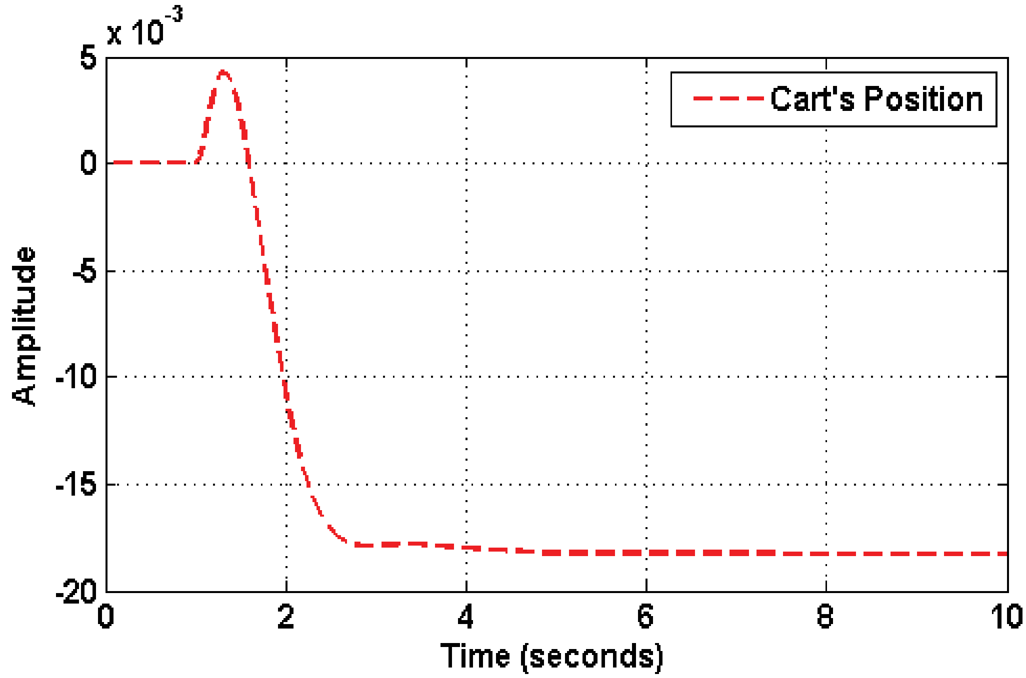

Figure 5.7 Closed-loop response of cart's position with LQR controller

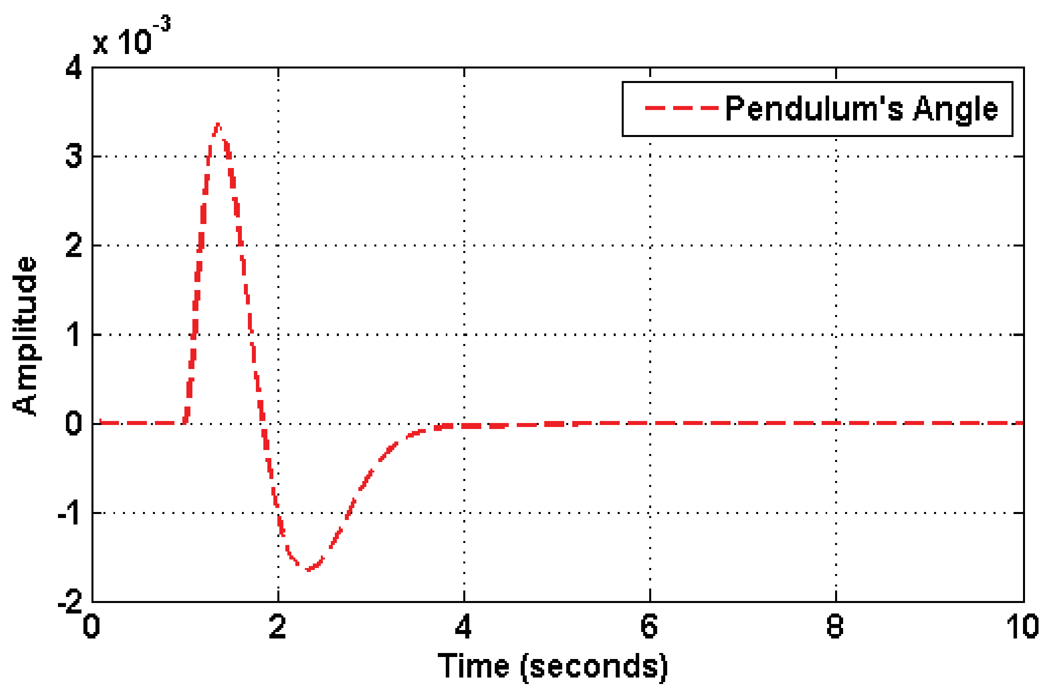

Figure 5.8 Closed-loop response of pendulum's angle with LQR controller

Table 5.4 Time-domain specification of inverted-pendulum system with LQR controller

	Rise Time (sec.)	Settling Time (sec.)	Peak Time (sec.)	Percentage Overshoot
Cart	1.0651	3.8877	4.61	34.81
Pendulum	1.000012	3.78	1.42	27.10

5.3.2.2 Outcomes of Linear Quadratic Regulator

The linear quadratic regulator successfully stabilized the pendulum in the upright position as per the desired requirements. The pendulum stabilized within 4 seconds with one overshoot and one undershoots. The cart's response is also satisfactory and it also stabilized within 4 seconds.

The only drawback associated with the LQR is that it is unable to bring the cart towards the reference point. Thus LQR fails in tracking the reference input.

The reference tracking can be obtained while designing an intelligent controller which can able to track the reference signal.

5.3.3 FUZZY LOGIC CONTROL

As we know, and pointed out explicitly by Driankov et al. (1995), conventional PID controllers are generally insufficient to control processes with additional complexities such as large time delays, significant oscillatory behavior, parameter variations, nonlinearities, and MIMO plants.

To improve conventional PID controllers, fuzzy logic is adapted. The fuzzy logic control was proposed by Lofti A. Zadeh in 1965. The fuzzy algorithm can make human knowledge into the rule base to control a plant with linguistic descriptions. It relies on expert experience instead of mathematical models. The advantages of fuzzy control include good popularization, high faults tolerance, and suitable for nonlinear control systems.

The primary motivation and “banner” of fuzzy logic is the possibility of exploiting tolerance for some inexactness and imprecision. Precision is often very costly, so if a problem does not require precision, one should not have to pay for it. The traditional example of parking a car is a noteworthy illustration. If the driver is not required to park the car within an exact distance from the curb, why spend any more time than necessary on the task as long as it is a legal parking operation?

Fuzzy logic and classical logic differ in the sense that the former can handle both symbolic and numerical manipulation, while the latter can handle symbolic manipulation only. In a broad sense, fuzzy logic is a union of fuzzy (fuzzified) crisp logics [2].

To quote Zadeh, “Fuzzy logic’s primary aim is to provide a formal, computationally-oriented system of concepts and techniques for dealing with modes of reasoning which are approximate rather than exact.” Thus, in fuzzy logic, exact (crisp) reasoning is considered to be the limiting case of approximate reasoning. In fuzzy logic one can see that everything is a matter of degrees.

5.3.3.1 Why Fuzzy Control?

Fuzzy logic is a technique to embody human like thinking into a control system. A fuzzy logic controller can be designed to emulate human deductive thinking, that is, the process people use to infer conclusions from what they know.

Traditional control approach requires formal modeling of the physical reality. Fuzzy control incorporates ambiguous human logic into computer programs. It suits control problems that cannot be easily represented by mathematical models:

- Weak model
- Parameter variation problem
- Unavailable or incomplete data
- Very complex plants
- Good qualitative understanding of plant or process operation

Because of its conventional approach, design of such controllers leads to faster development/ implementation cycles. Two typical fuzzy control systems are popularly known as Mamdani type and Takagi-Sugeno (T-S) type. Mamdani System: Mamdani type fuzzy systems employ fuzzy sets in the consequent part of the rules. T-S System: Takagi-Sugeno Fuzzy systems employ function of the input fuzzy linguistic variables as the consequent of the rules.

5.3.3.2 How Does Fuzzy Logic Controller Work

Determine the input and output relationship and choose a minimum number of variables for input to the FLC engine (typically error and rate of change of error for fuzzy PID controller). Using the rule-based structure of FLC, break the control problem down into a series of *IF X AND Y THEN Z* rules that define the desired controller output response for a given system input conditions. Create FLC membership functions that define the meaning of Input/Output terms used in the rules. Test the system, evaluate the results, tune the rules, membership functions, and continuously simulate until results are obtained.

5.3.3.3 Complete Architecture of Fuzzy Logic Controller

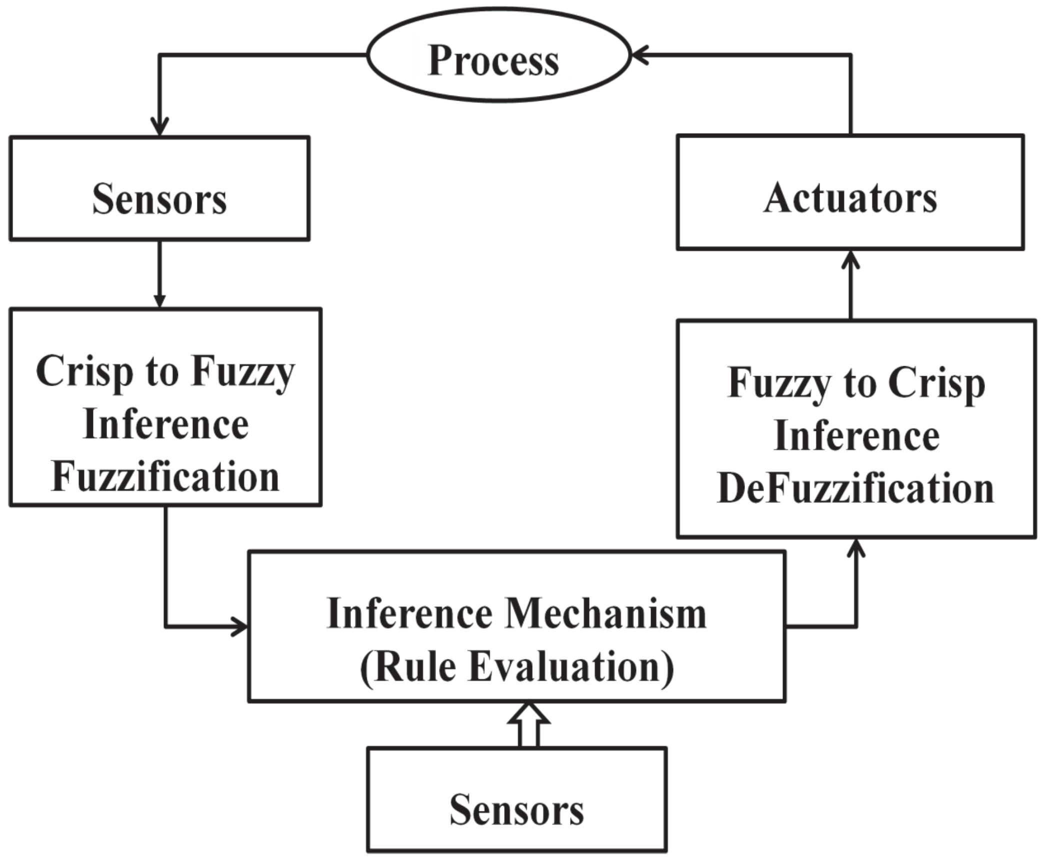

Figure 5.9 Architecture of Fuzzy Logic Control

The fuzzy controller is composed of the following four elements:

1. A *rule-base* (a set of If-Then rules), which contains a fuzzy logic quantification of the expert's linguistic description of how to achieve good control.

2. An *inference mechanism* (also called an "inference engine" or "fuzzy inference" module), which emulates the expert's decision making in interpreting and applying knowledge about how best to control the plant.

3. A *Fuzzification interface*, which converts controller inputs into information that the inference mechanism can easily use to activate and apply rules.

4. A *defuzzification interface*, which converts the conclusions of the inference mechanism into actual inputs for the process.

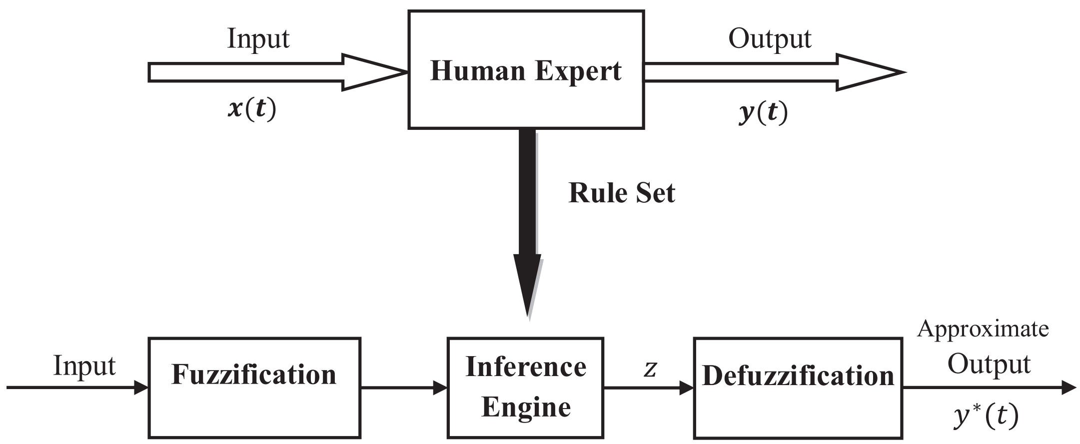

Figure 5.10 Conceptual Mechanism of Fuzzy logic control

5.3.3.4 Inverted Pendulum with Fuzzy Logic Control

Rules:

1. If error is negative large and change-in-error is negative large then force is positive large.

2. If error is zero and change-in-error is positive small then force is negative small.

3. If error is positive large and change-in-error is negative small then force is negative small.

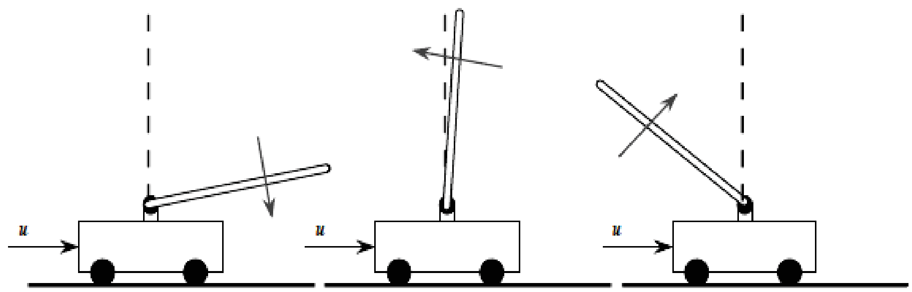

Figure 5.11 Movement of Inverted-pendulum system mounted on a cart

5.3.3.5 Simulink Model of Inverted Pendulum with FLC

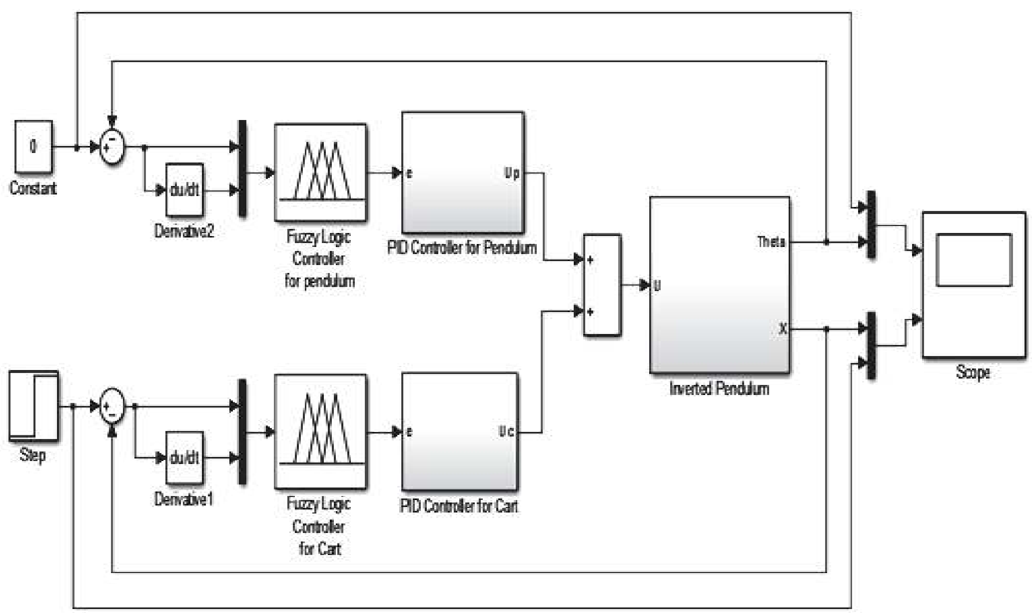

Figure 5.122 Simulink model of inverted-pendulum system with fuzzy logic control

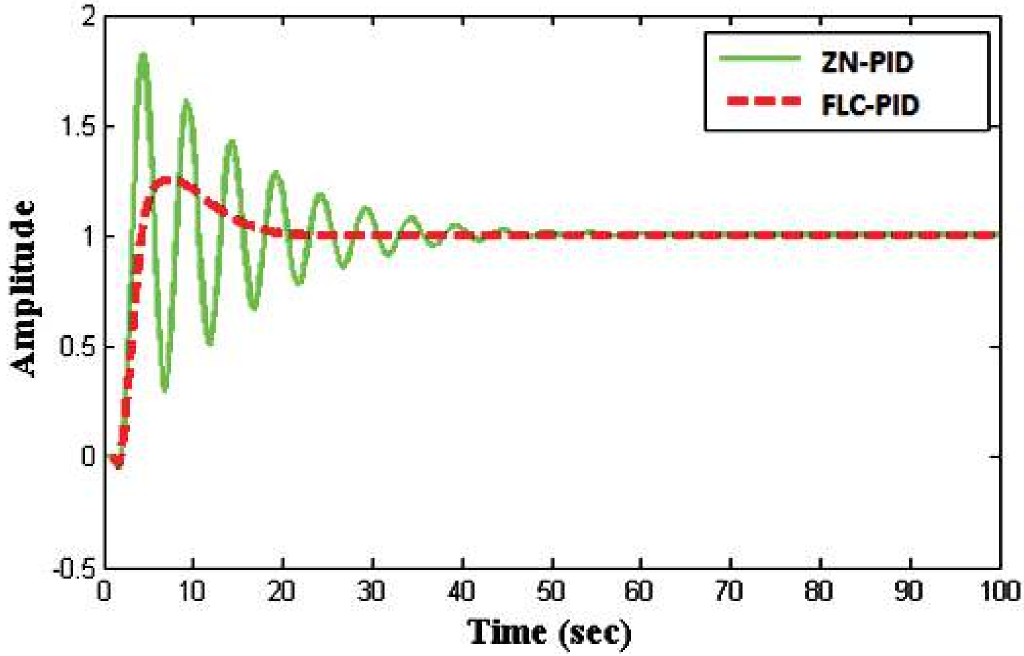

Figure 5.13 Closed-loop response of cart's position with Fuzzy Logic controller

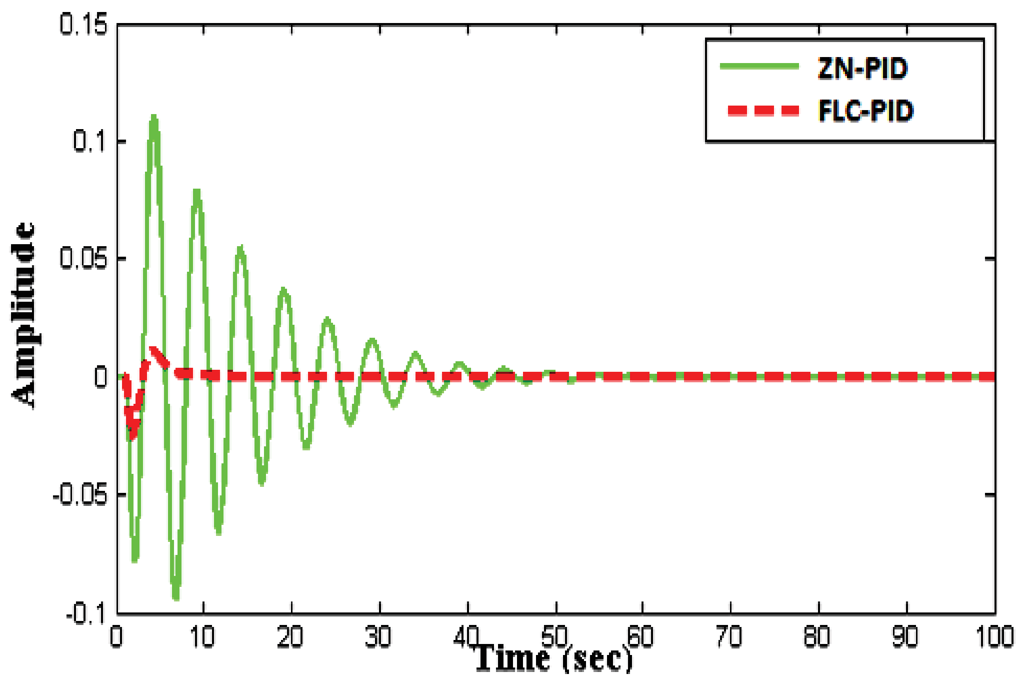

Figure 5.14 Closed-loop response of pendulum's angle with Fuzzy Logic controller

Table 5.5: Time-domain specification of inverted-pendulum system with Fuzzy Logic controller

	Rise Time (sec.)	Settling Time (sec.)	Peak Time (sec.)	Percentage Overshoot
FLC-PID-Cart	3.68	21.55	8.5	20.50
FLC-PID-Pendulum	5.3	8.45	7.85	1.45

5.3.3.6 Outcomes of Fuzzy Logic Control

The fuzzy logic controller successfully stabilizes the position of the cart and as well as the angle of the pendulum. Two FLC controllers are required to stabilize the inverted-pendulum system. Simulation result shows the effectiveness of the FLC in comparison with conventional PID control and LQR control. The angle of the pendulum stabilize within 10 seconds with proper overshoot and position of the cart shows better response then the above said controllers.

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