

REDESIGN OF CONVENTIONAL CONTROLLER FOR NONMINIMUM PHASE SYSTEM

4.1 INTRODUCTION

Systems that are causal and stable whose inverses are causal and unstable are known as *nonminimum phase systems*. It is acknowledged that the nonminimum phase portion of a plant (explicitly the right-half zeros and time delays) can bound the intensity of attainable requirement of a feedback control system. These restrictions are measured by the Bode sensitivity integrals [81].

- *A nonminimum phase system (NMPs) is defined as a system having zeros in the right-half s-plane (RHP) or time delays or both (Morari & Zafiriou, 1989) [82].*
- *A nonminimum phase system is defined as a system having either a zero or a pole in the right-half s-plane (Kuo & Golnaraghi, 2010) [83].*

System with pole and zero in the RHP is an unstable NMPs and system with zero in the RHP is stable NMPs. Nonminimum phase system show similar behaviour as all-pass filters. All-pass filters carry mirror image zero in the right-half of the s-plane for every stable pole in the left-half plane.

$$G(j\omega) = \frac{1 - j\omega T}{1 + j\omega T} \quad (4.1)$$

The magnitude and phase of $G(j\omega)$ are

$$|G(j\omega)| = 1 \quad (4.2)$$

$$\angle G(j\omega) = -2 \tan^{-1}(\omega T) \quad (4.3)$$

The magnitude is always unity whereas the phase varies from 0^0 to -180^0 as ω is increased from zero to infinity. The complexity of the system increases when the system has long dead-time. Transport lag or dead-time has an excessive phase lag with no attenuation at high frequency. Pade's approximation is used to handle the delay term [84]. Transport lag normally exist in thermal, hydraulic and pneumatic systems.

The practical examples of nonminimum phase systems are

- Aircraft Trajectory control
- Continuous stirred tank reactor
- Bicycle Counter-steering
- Water heating system and many more

4.2 DESCRIPTION OF NON-MINIMUM PHASE SYSTEM

An important class of nonlinear systems that has been studied extensively within control theory is that of having one of the systems zero in the right-half plane. For the NMP systems, the inverse of the RHP zeros and time delay is physically unrealizable for its non-causal property. The internal stability is a basic requirement for a practical closed-loop system and it is verified by checking the controllability and observability Grammians of the system. The system considered in this context is well modelled by proper linear time-invariant (LTI) systems with possible non-minimum phase components and time delay. The transfer function of the LTI system $P(s)$ is defined to be

$$P(s) = \frac{KN(s)}{D(s)} = \frac{KN_{nmp}(s)N_{mp}(s)}{D_s(s)D_u(s)} e^{-sT} \quad (4.4)$$

where K is the gain, $N_{\text{nmp}}(s)$ is the non-minimum phase, $N_{\text{mp}}(s)$ is the minimum phase, $D_s(s)$, $D_u(s)$ is the stable, unstable polynomials and T is the delay time [85]. NMP systems are slow in responding because of their faulty behaviour at the start of the response.

4.2.1 Problem Formulation

Consider an input-output delayed SISO system as discussed by [3]:

$$P(s) = G(s)e^{-Ts} \quad (4.5)$$

where,

$$G(s) = \frac{0.1}{s(s+1)(0.5s+1)(0.1s+1)} \quad (4.6)$$

where $P(s)$ is the open-loop transfer function, $G(s)$ is the undelayed dynamics and T represents the delay-time. The above system is approximated by Pade's first order approximation and the transfer function is given by:

$$P_{\text{Pade}}(s) = \frac{-0.1s + 0.025}{0.05s^5 + 0.6625s^4 + 1.763s^3 + 1.4s^2 + 0.25s} \quad (4.7)$$

The controllability and observability grammians are determined to check the internal stability of the system. The controllability grammians is defined by:

$$W_c = \int_0^{\infty} e^{A\tau} B B^T e^{A^T \tau} d\tau \quad (4.8)$$

The controllability grammians is positive definite if and only if (A, B) is controllable.

The observability grammians is defined by:

$$W_o = \int_0^{\infty} e^{A^T \tau} C^T C e^{A\tau} d\tau \quad (4.9)$$

The observability grammians is positive definite if and only if (A, C) is observable.

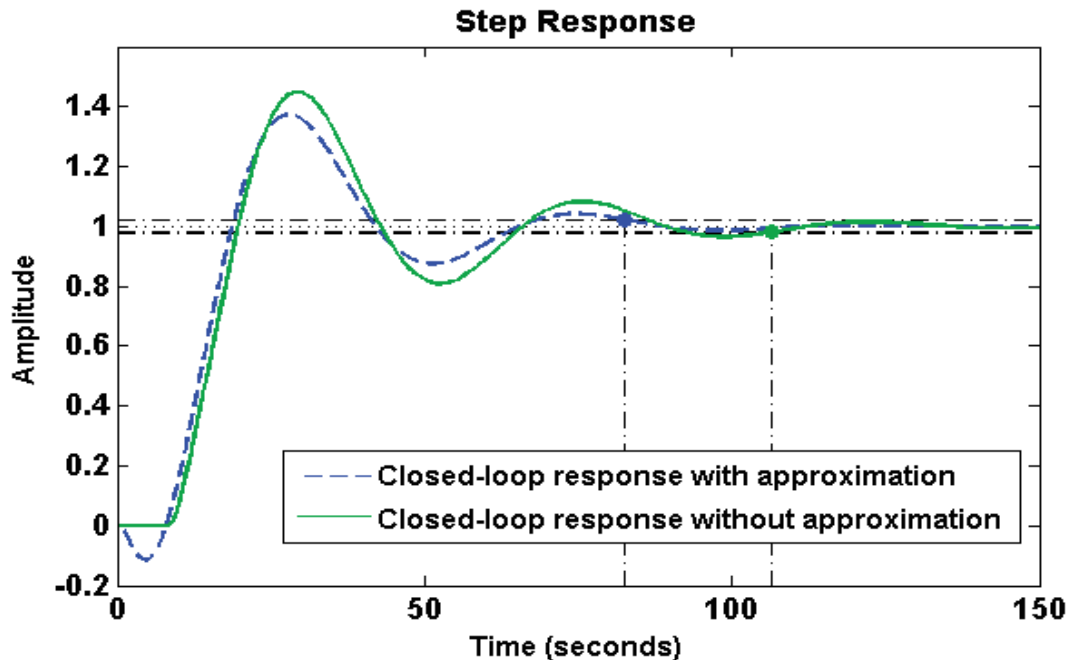


Figure 4.7 Step response of nonminimum phase system with and without Pade's first order approximation

The approximated system is both controllable and observable as the grammians are located in left-half of the s-plane. The step response of the nonminimum phase system approximated via Pade's first order approximation is stable as shown Figure 4.1. The step response shows a single zero crossing which implies that there is one of the system's zero is located in the right-half of the s-plane. As the settling time of the system is more as shown in Table 4.1 and also the oscillation in the system is large. Therefore, it requires an efficient controller to regulate the state in accordance to meet the design requirements which is shown in Table 4.2.

Table 4.1: Time-domain specification of nonminimum phase system

	Rise Time (sec.)	Settling Time (sec.)	Peak Time (sec.)	Percentage Overshoot	Steady-state error
$Y(s)$	8.2187	106.3853	29.4260	44.8311	0.0348
$Y_{Pade}(s)$	8.2241	82.5025	27.8863	37.2502	0.0133

Table 4.2: Controller design requirements

Time-domain specification	Rise Time (sec.)	$t_r = \frac{\pi - \cos^{-1} \zeta}{\omega_n \sqrt{1 - \zeta^2}} = \text{as small as possible}$
	Settling Time (sec.)	$t_s = \frac{4}{\zeta \omega_n} \leq 50$
	Peak Time (sec.)	$t_p = \frac{\pi}{\sqrt{1 - \zeta^2}} = \text{as small as possible}$
	Peak Overshoot (%)	$M_p = 100e^{-\pi \zeta / \sqrt{1 - \zeta^2}} \% \leq 15\%$
	Steady-state error	$e_{ss} _{\text{unit-step}} = \lim_{t \rightarrow \infty} [1 - y(t)] = 0$
Frequency-domain specification	Gain Margin (dB)	$GM = \frac{1}{ G(j\omega)H(j\omega) } \geq 6dB$
	Phase Margin (deg.)	$PM = 180^\circ + \angle G(j\omega)H(j\omega) \geq 45^\circ$

4.3 PID CONTROLLER

The complexity associated with PID controller is to find the starting solution. Initially, the parameters were tuned using trial & error method which is quite a tedious task to reach towards the finite solution. In 1942 Ziegler-Nichols (ZN method) solve this problem of fine-tuning the parameters of PID controller. ZN tuning formula is a heuristic method of determining the ultimate values of the controller.

At the ultimate value, the system is at the point of marginal instability and gives sustained oscillations in the output. The ultimate gain and ultimate frequency are used to get the PID controller settings. The PID settings proposed by ZN results in a large overshoot and an oscillatory response. The correlation between the ultimate period, the reset time and the derivative time was based on simulation of a large number of processes.

The key criterion is a quarter decay ratios. Many other researchers have modified the ZN method to obtain significant performance improvement. Tyreus-Luyben [86] proposed settings for PI and PID controllers, but the method results in a long settling time. Smith [87] and Yu [88] proposed modification in the tuning based formulae on the ultimate values. Furthermore, most of the proposed methods, based on ultimate values of controllers, are implemented mostly on stable processes. Controller design for the unstable process with time delay is difficult.

Many variants of the traditional Ziegler–Nichols PID tuning methods are available, to check the efficacy of the ZN method, Chien–Hrones–Reswick method is also used to control the nonminimum phase system. The Chien–Hrones–Reswick (CHR) method emphasizes the set-point regulation or disturbance rejection. In addition one

qualitative specification on the response speed and overshoot can be accommodated. Compared with the traditional ZN tuning formula, the CHR method uses the time constant T of the plant explicitly.

Table 4.3: Parameters of PID controller tuned via conventional tuning formulae

	K_P	K_I	K_D
ZN method	1.237	0.0651	6
CHR method	1.005	0.04331	4.054

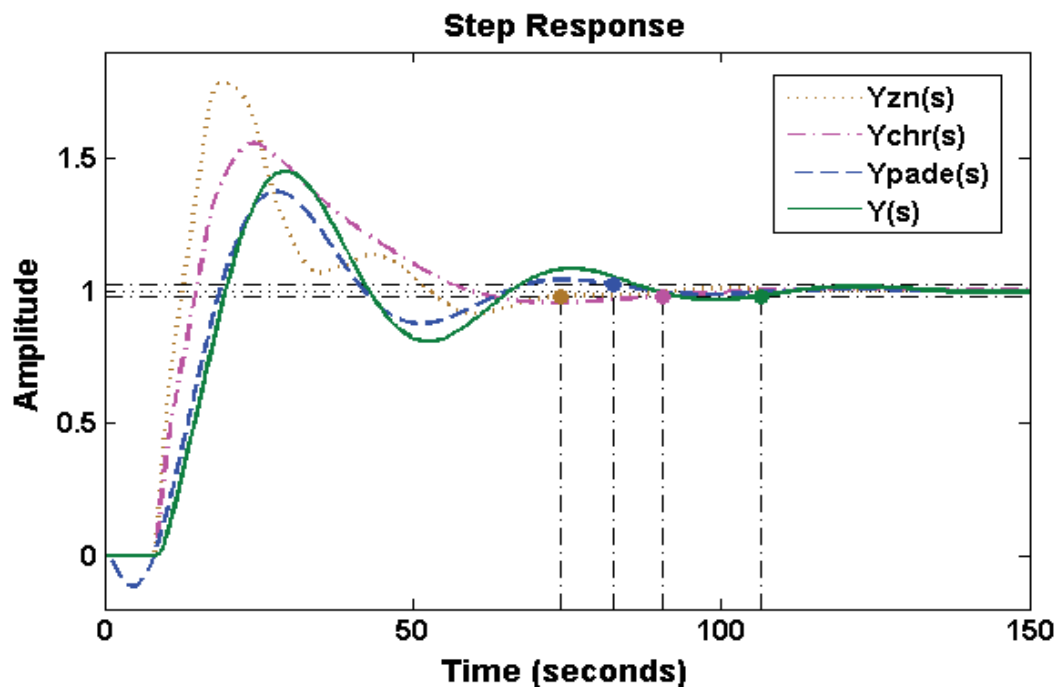


Figure 4.8 Step response of Nonminimum Phase System with PID Controller

Table 4.4: Time-domain specification of nonminimum phase system with PID controller

	Rise Time (sec.)	Settling Time (sec.)	Peak Time (sec.)	Percentage Overshoot	Steady-state error
$Y_{ZN}(s)$	3.4057	73.9244	18.8804	78.3240	0.0093
$Y_{CHR}(s)$	5.3288	90.6222	24.2146	55.5081	0.0056

where, $Y_{ZN}(s)$ and $Y_{CHR}(s)$ are the closed-loop transfer function of the nonminimum phase system when the parameters of the PID controller are tuned using Ziegler-Nichols and Chien-Hrones-Reswch method. $Y_{pade}(s)$ is the step response of approximate model.

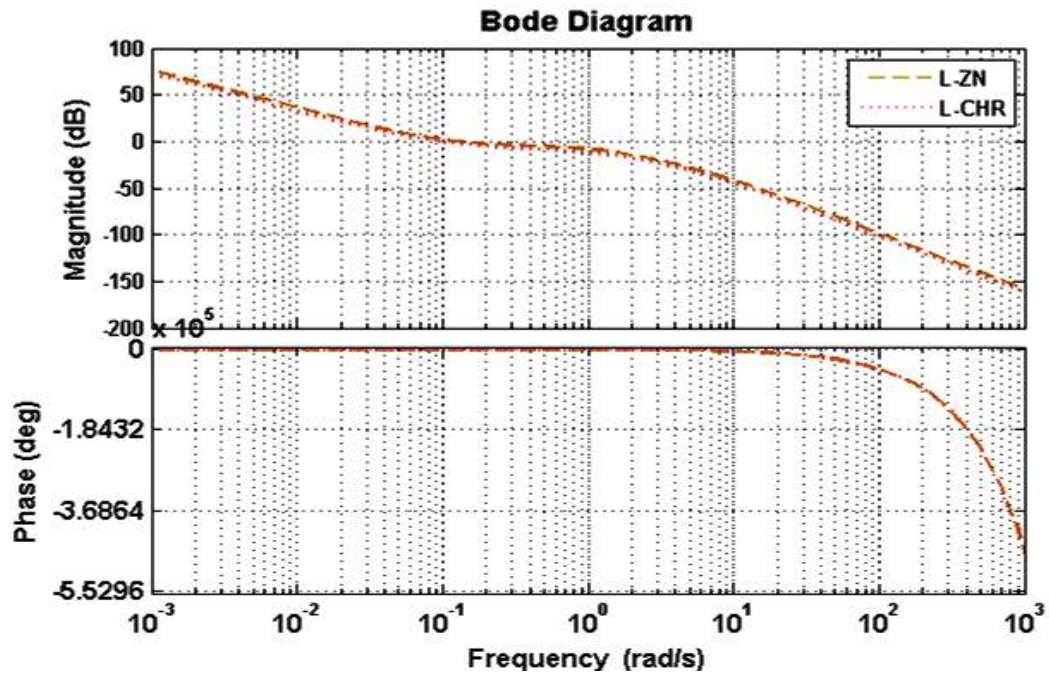


Figure 4.9 Bode Response of Nonminimum Phase System with PID Controller

Table 4.5: Frequency-domain specification of nonminimum phase system with PID controller

	Gain Margin	Phase Margin
$L_{ZN}(s)$	3.35 dB	31.8 ⁰
$L_{CHR}(s)$	5.59 dB	33.4 ⁰

where $L_{ZN}(s)$ and $L_{CHR}(s)$ are the loop gain of the nonminimum phase system controller via PID controller.

4.3.1 Outcomes of PID Controller

Both the techniques i.e. Ziegler-Nichols (ZN) and Chien–Hrones–Reswick (CHR) tuning formulae are fails to fulfill the controller design requirements. The ZN method provides large overshoot as compared to CHR method. Though, the settling time of ZN method is less compare to CHR method but it has poor steady-state error.

The CHR method is better in terms of frequency domain specifications as the gain margin and phase margin of the system is better than the ZN method. These tuning formulae perform even worse when the delay of the system increases. To cope up from these drawbacks and to achieve the desired design requirements, a PID controller is designed on the basis of the methodology proposed by Smith's in 1959.

4.4 SMITH PREDICTOR CONTROL

The control structure of Smith predictor [89-94] is shown below. The plant model is given by:

$$G_m(s) = \hat{G}(s)e^{-\hat{\tau}s} \quad (4.10)$$

If the model/ process are imperfect then the feedback signal $Y_f(s)$ is given by-

$$Y_f(s) = \hat{G}(s)U(s) + (G(s)e^{-Ts} - \hat{G}(s)e^{-\hat{T}s})U(s) \quad (4.11)$$

If a “perfect” model of the plant is considered then,

$$G(s) = \hat{G}(s) \quad (4.12)$$

$$T = \hat{T} \quad (4.13)$$

This means that the feedback is only dependent on the model of the plant-

$$Y_f(s) = \hat{G}(s)U(s) \quad (4.14)$$

The relationship between the control variable and the system output is

$$U(s) = \frac{1}{G(s)e^{-Ts}} Y(s) \quad (4.15)$$

Therefore,

$$Y_f(s) = e^{Ts} Y(s) \quad (4.16)$$

This shows that the internal loop containing the plant model feeds back a signal that is a prediction of the output, since e^{ts} represents a prediction $y(t + T)$ in the time-domain. The closed-loop transfer function of the system can be determined by using-

$$Y(s) = G(s)e^{-Ts}U(s) \quad (4.17)$$

Therefore,

$$U(s) = G_c(s)(R(s) - Y_f(s)) \quad (4.18)$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)e^{-Ts}G_c(s)}{1 + G(s)G_c(s)} \quad (4.19)$$

According to Dorf & Bishop-2011 [95], the sensitivity expression in this case can be defined as

$$S(s) = \frac{1}{1 + G(s)G_c(s)} \quad (4.20)$$

Hagglund in 1992 [96] and 1996 [97] combined the properties of the Smith predictor with a PI controller to control a first order plant with a time delay. In this thesis, a PID controller is designed using these properties. Firstly, a time-delay system is approximated using Pade's first order approximation.

$$\hat{G}_d(s) = \text{pade}(e^{-Ts}, 1) = \frac{-8s + 2}{8s + 2} \quad (4.21)$$

The transfer function is given as-

$$P_{\text{Pade}}(s) = \frac{-0.1s + 0.025}{0.05s^5 + 0.6625s^4 + 1.763s^3 + 1.4s^2 + 0.25s} \quad (4.22)$$

The PID controller for this approximated system is given as-

$$G_c(s) = \frac{1.088s^2 + 0.6408s + 0.0003766}{0.13s^2 + s} \quad (4.23)$$

A PID controller based on Smith predictive qualities is defined as

$$T_{\text{Smith}}(s) = T_{\text{PID}}(s) \quad (4.24)$$

$$C(s) = \frac{G_c(s)}{1 + \hat{G}(s)G_c(s) - \hat{G}(s)G_c(s)\hat{G}_d(s)} \quad (4.25)$$

$$C(s) = \frac{4.598 \times 10^{-5} s^{15} + 1.941 \times 10^{-3} s^{14} + 3.343 \times 10^{-2} s^{13} + 3.026 \times 10^{-1} s^{12} + 1.5580 s^{11} + 4.673 s^{10} + 8.276 s^9 + 8.584 s^8 + 5.003 s^7 + 1.469 s^6 + 0.1609 s^5 + 9.415 \times 10^{-5} s^4}{5.493 \times 10^{-6} s^{15} + 2.709 \times 10^{-4} s^{14} + 5.618 \times 10^{-3} s^{13} + 6.356 \times 10^{-2} s^{12} + 0.4269 s^{11} + 1.744 s^{10} + 4.315 s^9 + 6.388 s^8 + 5.423 s^7 + 2.354 s^6 + 0.3783 s^5 + 7.532 \times 10^{-5} s^4}$$

Therefore, such a higher order controller needs reduction. Applying, a generalized model reduction technique known as balanced realization to reduce to a lower order. The reduction techniques firstly balanced the system using 'balreal' command of MATLAB and then check the grammians of the system. The grammians with small Hankel singular values are neglected and the one with more dominance is selected.

The reduced order controller is given as:

$$C(s) = \frac{8.3717(s + 0.5225)(s + 0.0002302)}{(s + 7.592)(s + 9.457 \times 10^{-5})} \quad (4.26)$$

The controller has an additional filter which tries to compensate the system performance. Though, the structure of the controller is not the exact form of PID controller but it provides and fulfills all the design requirements. This controller is applied to the original nonminimum phase system. The step response of the closed-loop system and bode response of the loop transfer function is shown in comparison with the conventional control techniques.

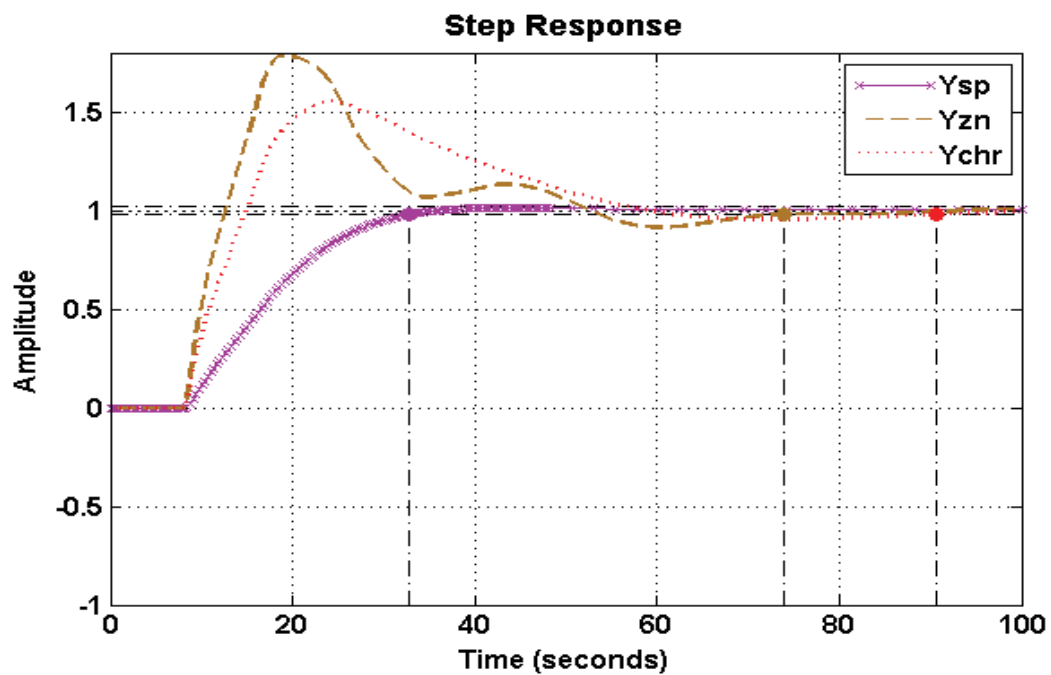


Figure 10.4 Step response of Nonminimum Phase System with Smith Predictor

Table 4.6: Time-domain specification of nonminimum phase system with Smith Predictor

	Rise Time (sec.)	Settling Time (sec.)	Peak Time (sec.)	Percentage Overshoot	Steady-state error
$Y_{sp}(s)$	17.1298	32.7164	43.7757	1.4133	0.0023

where $Y_{sp}(s)$ is the closed-loop transfer function of the nonminimum phase system with Smith Predictor.

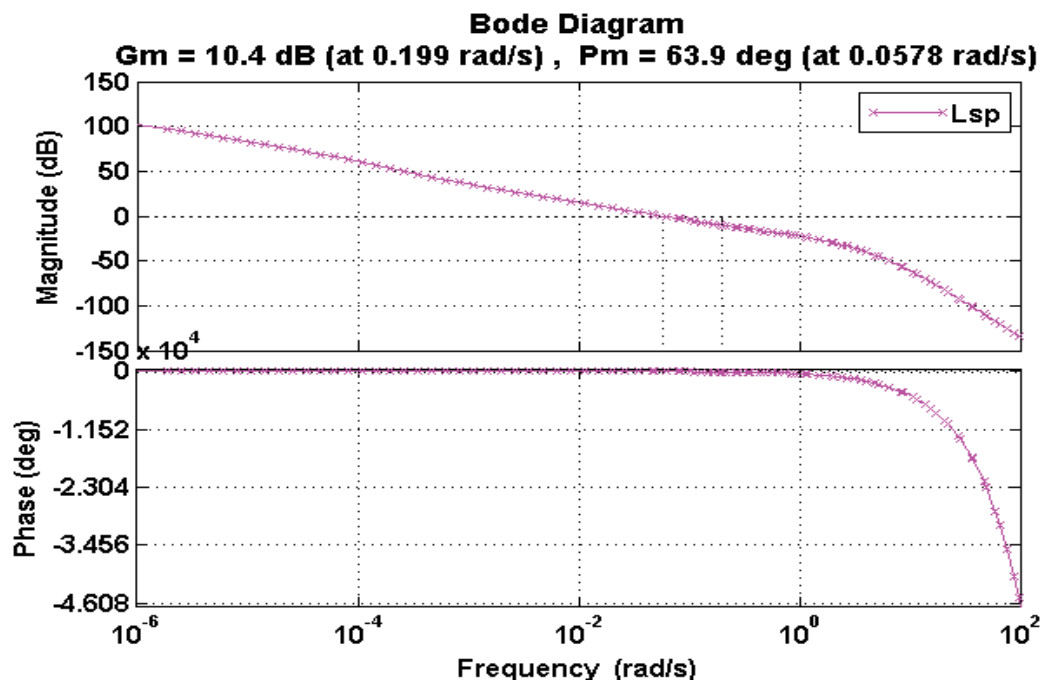


Figure 4.11 Bode response of nonminimum phase system with Smith Predictor

4.4.1 Outcomes of Smith Predictor

The PID controller designed using Smith predictor control structure consequent in an inexact PID form. Though, it achieves all the design requirements, but it also poses an additional feed-forward filter and required a tedious model reduction technique to reduce the order of the controller.

Therefore, in order to design an exact PID control structure and to fulfill all the design requirements, a meta-heuristic technique known as grey wolf optimizer (GWO) is used for the control of nonminimum phase system.

4.5 GREY WOLF OPTIMIZER

Grey wolf (*Canis Lupus*) belongs to a Canidae family. It is a general-purpose stochastic search method, offers several advantages like

- Robust and reliable performance
- Global search capability
- Little or no information required

The grey wolves are considered as apex predators, meaning that they are at the top of the food chain. They mostly prefer to live in a group. They are characterized by powerful teeth, bushy tail and they usually hunt in packs. Their natural habitats are found in mountains, forests, plain of North America, Asia and Europe. The technique is developed by Seyedali Mirjalili, Seyed Mohammad Mirjalili and Andrew Lewis in 2014.

4.5.1 Methodology of GWO

Grey wolves commonly have a group size of 5-12 wolves on an average. They have a very strict social dominant hierarchy as shown in Figure 4.6.

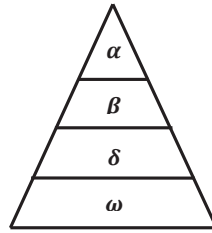


Figure 4.6 Hierarchy of GWO

The alphas either male or female is considered to be the leader among them and responsible for making decisions about the hunting of the prey. This makes alpha as the most dominating members of the hierarchy. The wolf that assists alpha in decision making is called beta, which is the second level of the hierarchy. The third level of hierarchy is known as delta, which acts as a subordinate and help alpha as well as beta during hunting process.

The level of respect is directly proportional to the dominance of the wolf. The delta wolf dominates the rest of the omegas. The omega wolves are the followers and have the lowest level in the hierarchy. They allowed to eat last and considered as an unimportant individual in the group. The hunting process of grey wolf incorporates the following steps:

- Tracking, chasing, and approaching the prey.
- Pursuing, encircling, and harassing the prey until it stops moving.
- Attack towards the prey.

4.5.2 Mathematical model of GWO

The mathematical model of GWO comprises of social hierarchy, encircling prey, hunting behavior, attacking prey, and search for the prey.

4.5.2.1 *Social Hierarchy*

The social hierarchy of GWO incorporated with four different levels of dominance. The most dominating wolf is considered as an alpha (α), then beta (β), delta (δ) which act as a subordinate and the rest of the wolves are the followers known as the omegas (ω). The fitness of the wolves is directly associated with the dominating behaviour of the wolf, i.e. alpha is considered to be the fittest wolf among them.

4.5.2.2 *Encircling Prey*

To model the encircling behavior of the wolves, following equations are required:

$$D = |C \cdot X_p(t) - X(t)| \quad (4.27)$$

$$X(t + 1) = X_p(t) - A \cdot D \quad (4.28)$$

$$A = 2a \cdot r_1 - a \quad (4.29)$$

$$C = 2 \cdot r_2 \quad (4.30)$$

where A and C are coefficient vectors, X_p is the position vector of the prey, X indicates the position vector of a grey wolf, t is the current iteration and components of a linearly decreased from 2 to 0 over the course of iterations and r_1, r_2 are random values in the range of $[0, 1]$.

4.5.2.3 *Hunting Behavior*

Grey wolves generally search for an easy prey and have the ability to recognize the location of prey and encircle them. The group of wolves is guided by alpha. Beta and delta plays a vital role during the hunting process and we assume that they have better knowledge about the potential location of the prey. Therefore, we save the first three

solutions and update the position of others on the basis of the best search agents and the formulas are:

$$\left. \begin{aligned} D_{\alpha_i} &= |C_1 \cdot X_{\alpha_i}(t) - X_i(t)| \\ D_{\beta_i} &= |C_2 \cdot X_{\beta_i}(t) - X_i(t)| \\ D_{\delta_i} &= |C_3 \cdot X_{\delta_i}(t) - X_i(t)| \end{aligned} \right\} \quad (4.31)$$

$$\left. \begin{aligned} X_{i1} &= X_{\alpha_i}(t) - A_1 \cdot D_{\alpha_i} \\ X_{i2} &= X_{\beta_i}(t) - A_2 \cdot D_{\beta_i} \\ X_{i3} &= X_{\delta_i}(t) - A_3 \cdot D_{\delta_i} \end{aligned} \right\} \quad (4.32)$$

$$X_i(t+1) = \frac{X_{i1} + X_{i2} + X_{i3}}{3} \quad (4.33)$$

where $i = (1, 2, \dots, \text{max. iteration})$ and $X_i(t+1)$ is the best search agent for iteration i . The three best solutions are updated as alpha, beta and delta. The other wolves update their positions on the basis of the current location of alpha, beta and delta.

4.5.2.4 *Attacking Prey*

The hunting process of grey wolf is completed when the prey stops moving. To shorten the gap between the position of the grey wolves and the prey is depend on the value of A . As A decreases the gap between the position of grey wolf and prey reduces. Therefore, to reduce A the value of a should be decreased. In this thesis, the value of a is calculated as:

$$a = 2 - \left(\frac{2}{J_{\max}} \right) \quad (4.44)$$

whereas a is decreased from 2 to 0 as the iteration increases and J_{\max} is the maximum value of the performance index. This encircling mechanism of GWO shows some properties of exploitation.

4.5.2.5 Search for Prey

Grey wolves search for the prey, according to the position of alpha, beta and delta. The searching for prey is diverged to different locations. This shows the exploration property of the GWO algorithm. The exploration is due to random variable C , which lies in the range $[0, 2]$.

4.6 VALIDATION TOOL FOR OPTIMIZATION

The objective of the optimization technique is to minimize the performance index as mentioned below:

- Integral Square Error

$$J_{ISE} = \int_0^{\infty} e^2(t) dt \quad (4.45)$$

- Integral Absolute Error

$$J_{IAE} = \int_0^{\infty} |e(t)| dt \quad (4.46)$$

- Integral Time weighted Square Error

$$J_{ITSE} = \int_0^{\infty} t.e^2(t) dt \quad (4.47)$$

-
- Integral Time weighted Absolute Error

$$J_{ITAE} = \int_0^{\infty} t \cdot |e(t)| dt \quad (4.48)$$

4.7 CONTROLLER DESIGN USING GREY WOLF OPTIMIZER

The GWO algorithm fine-tunes the parameters of PID controller as shown in the flowchart. The number of iteration required to search for the optimum solutions are 100 and number of wolves chosen are 12. The best parameter of the PID controller obtained using GWO algorithm is shown in Table and the procedure is shown in Figure 4.7.

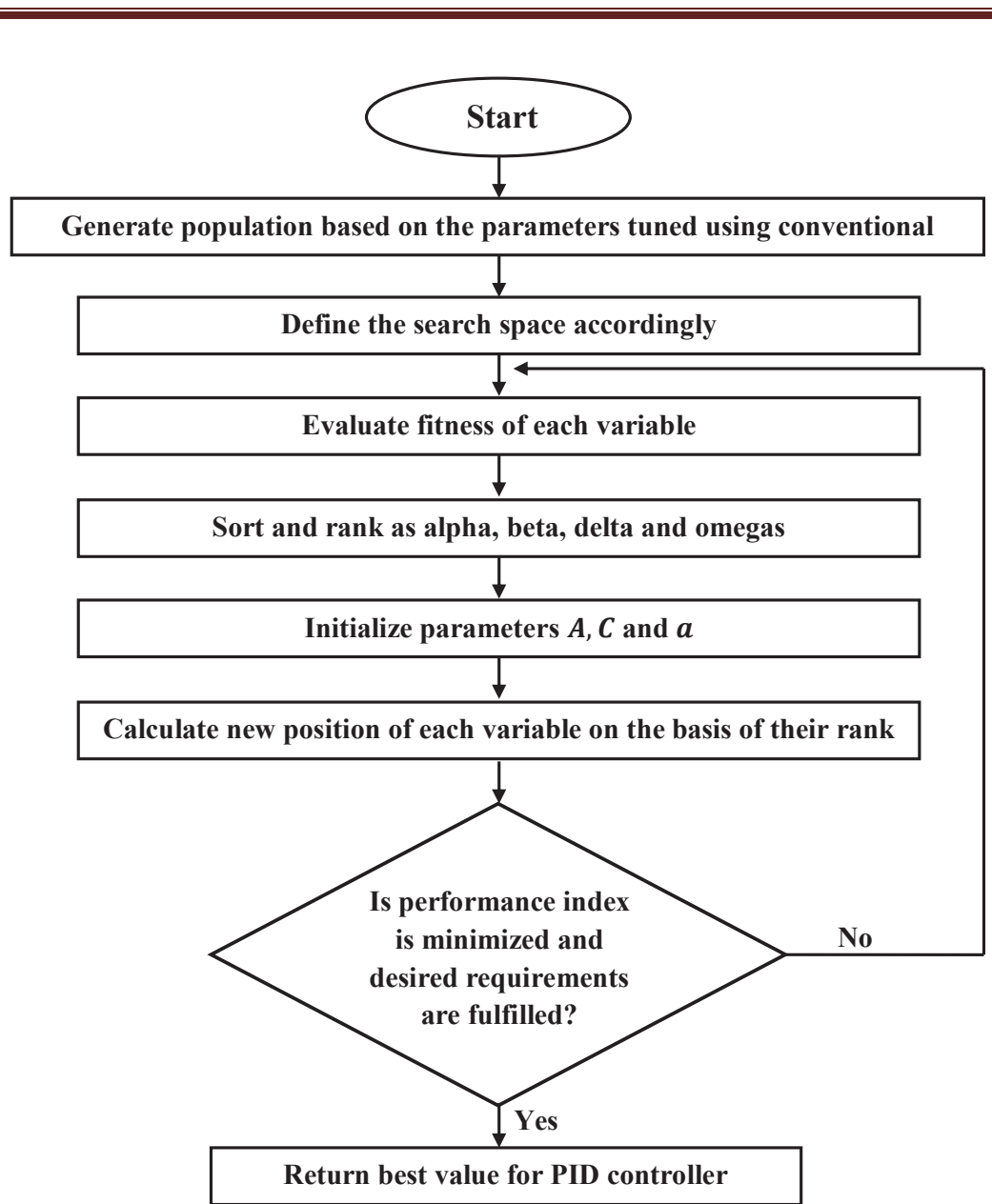


Figure 4.7 Flowchart for obtaining parameters of PID controller using GWO

Table 4.7: Parameters of PID controller obtained using GWO algorithm

	K_P	K_I	K_D
GWO	0.736799776	0.00001334	2.42268967

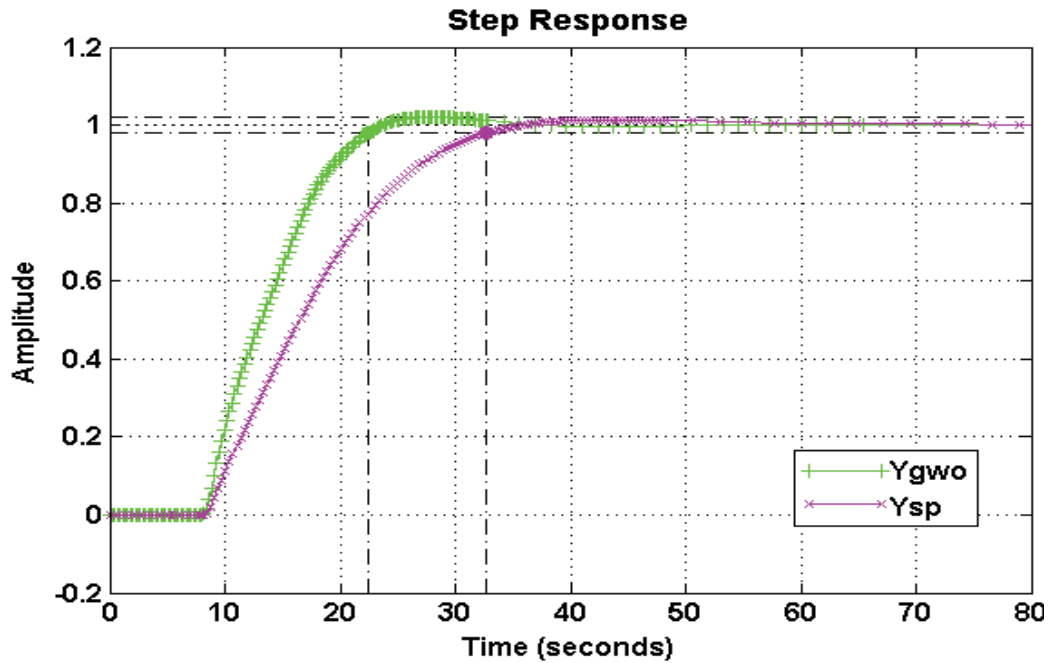


Figure 4.8 Step Response of Nonminimum Phase System with GWO algorithm

Table 4.8: Time-domain specification of nonminimum phase system with GWO algorithm

	Rise Time (sec.)	Settling Time (sec.)	Peak Time (sec.)	Percentage Overshoot	Steady- state error
$Y_{GWO}(s)$	10.4089	22.4561	28.003	1.983	0.000246

where $Y_{GWO}(s)$ is the closed-loop transfer function of the nonminimum phase system when the parameters are tuned using grey wolf optimizer.

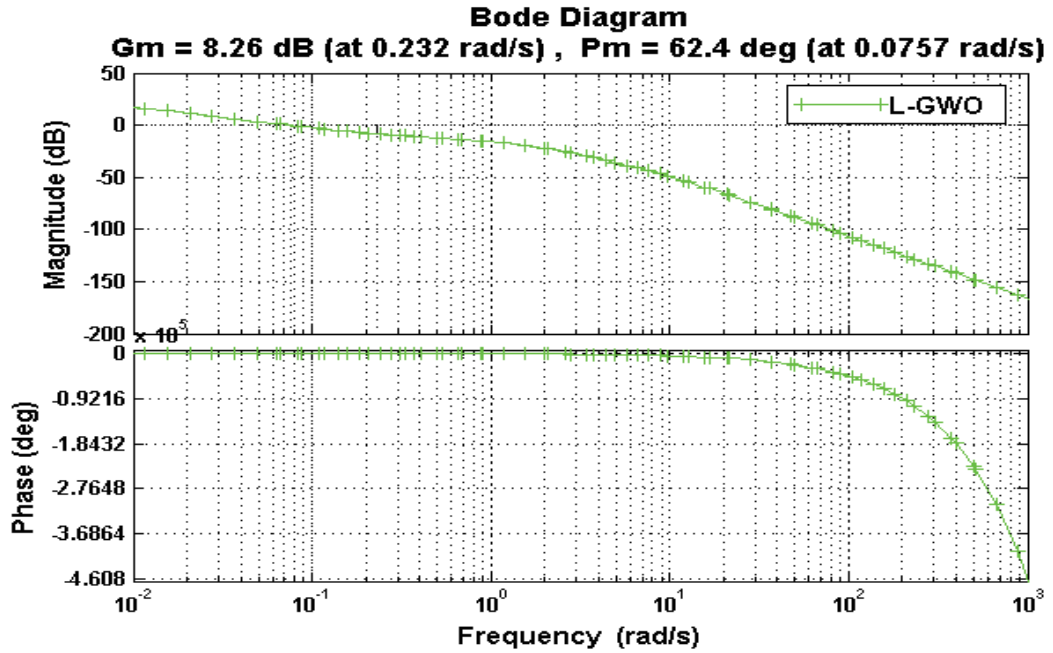


Figure 4.9 Bode Response of Nonminimum Phase System with GWO algorithm

Table 4.9: Performance indices of nonminimum phase system using GWO algorithm

	ISE	IAE	ITSE	ITAE
GWO algorithm	11.9738	14.3932	69.54	107.71

4.8 OUTCOMES OF VARIOUS CONTROLLERS

	Desired Specifications	Ziegler-Nichols method	Chien-Hrones-Reswich method	PID controller using Smith Predictor	PID controller using GWO
Settling Time	≤ 50	×	×	√	√
Peak Overshoot	$\leq 15\%$	×	×	√	√
Steady-state error	0	×	×	×	×
Gain Margin	$\geq 6\text{dB}$	×	×	√	√
Phase Margin	$\geq 45^\circ$	×	×	√	√

4.9 CONCLUSION

The controller is successfully designed based on classical and modern approaches to control the non-minimum phase system. Two popular classical tuning criterion known as Ziegler-Nichols and Chien-Hrones-Reswick methods were used for tuning the parameters of the PID controller.

The PID controller is also designed on the basis of Smith predictor approach. This method utilizes a plant model to predict the future output of the plant. This results in a control law that acts immediately on the reference input avoiding instability and

sluggish control. The PID controller is an inaccurate structure and therefore, an optimized technique is used to design a perfect PID controller for nonminimum phase system.

The Grey wolf optimizer successfully tunes the parameters of PID controller and stabilizes the system's performance both in time-domain as well as in frequency-domain. The eminent properties of optimization technique like exploration and exploitation are guaranteed by the GWO algorithm. The effectiveness of the proposed techniques is validated by comparing it with the conventional control techniques.

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