CONTROLLER DESIGN FOR SYSTEM WITH SATURATION

3.1 INTRODUCTION

Constraints are present in all control systems and can have damaging effects on the system performance unless accounted for in the controller design process. Most often constraints are identified as actuator magnitude and rate saturations. Actuator saturation is amongst the most common and significant nonlinearity in a control system. In the literature, numbers of examples are given where neglecting the saturation has led to crucial difficulties and endangered the overall stability of the system.

- *For a linear systems subject to actuator saturations, a globally asymptotically stabilizing feedback only exists if and only if the open-loop system itself is stable (Sussmann et al., 1994) [63].*

The physical system which pertains a problem of actuator saturation is known as Magnetic levitation system. The main issues that are coupled with the magnetic levitation system are inner coupling, misalignments between the sensors & actuators, and self-excited vibration.

Magnetic levitation is the process by which a ferromagnetic object is suspended in the air against gravity with the help of a magnetic field generated by the coil. Maglev system is becoming an attractive technology for application areas like high-speed trains, vibration isolation systems, magnetic bearings and suspension of wind tunnel.

3.2 MODELING OF MAGNETIC LEVITATION SYSTEM

The magnetic levitation system is highly unstable system consists of an electromagnet coil mounted on top of the box, infra-red sensor to sense the position of the metal ball [64]. The vertical and the horizontal movement of the ball are split by the infra-red sensor. The electromagnetic force is varying in such a way so that the metal ball can move in the air space in between 0.25 cm to 0.5 cm. The distance is measured from the electromagnetic coil, if the metal ball can cross this specified distance then there are chances of being fallen down or it attracts on outer surface of the coil [65-67]. The electromagnetic force is contradictory to the gravitational force g and sustains the metal ball in a levitated position. The electromagnetic force F depends on the electromagnetic current I, and the air gap X between the metal ball and the electro-magnet coil [68-70]. The movement of the metal ball in the air space is given by

$$F = Mg - K_m\left(\frac{i_m}{x_b}\right)^2 \tag{3.1}$$

where i_m is the current in the electromagnetic coil, x_b (m) is the distance between the ball and electromagnetic coil, g (m/sec2) is the acceleration due to gravity, K_m is the magnetic force constant, M (Kg) is the mass of the metal ball. Figure 3.1 shows the schematic diagram of the magnetic levitation system.

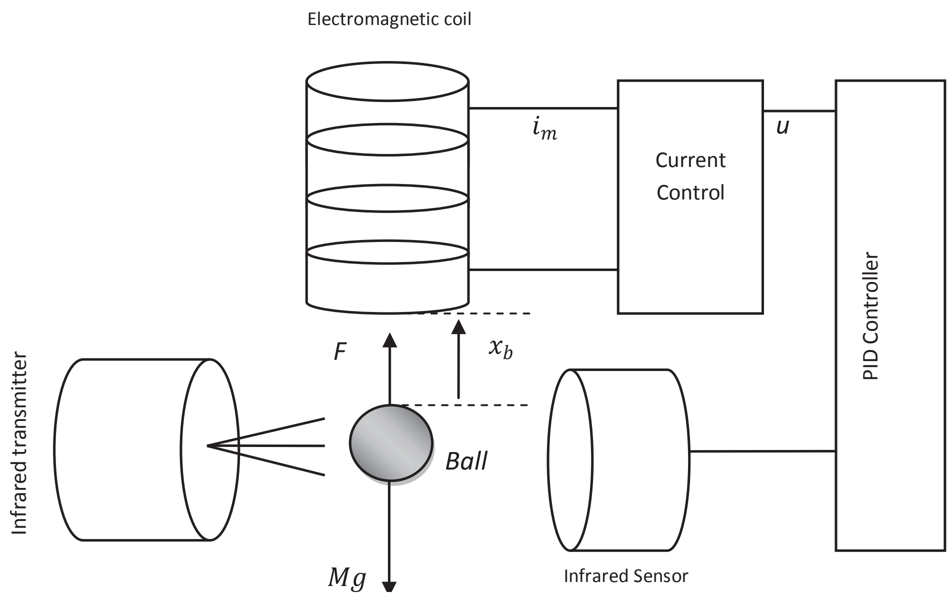

Figure 3.1 Closed-loop structure of magnetic levitation system

Newton's second law of motion is used to derive the differential equation of the magnetic levitation system given by

$$M\frac{d^2x_b}{dt^2} = Mg - K_m\left(\frac{i_m}{x_b}\right)^2 \tag{3.2}$$

The value of coil current and the position of the metal ball at the operating point can be derived by putting $\frac{d^2x_b}{dt^2} = 0$ in Eq. (3.2) gives-

$$x_{b_{ss}} = \sqrt{\frac{K_m}{Mg}}\, i_{m_{ss}} \tag{3.3}$$

where $x_{b_{ss}}$ and $i_{m_{ss}}$ are the value of position of metal and current of electromagnetic coil at the operating point. This coil current is sufficient in theoretical sense to levitate the position of the metal ball to the desired location but it fails practically due to variation at operating point because of external disturbances, parameter uncertainties and others. Therefore, there is a requirement of an efficient controller which is capable of handling such irregularities of the system. The magnetic levitation system is linearized by taking the approximates of x_b and i_m as

$$x_b(t) \triangleq \hat{x}_b + x_{b_{ss}} \tag{3.4}$$

$$i_m(t) \triangleq \hat{\imath}_m + i_{m_{ss}} \tag{3.5}$$

where $\hat{x}_b$ and $\hat{\imath}_m$ are the variations of metal ball position and coil current around the operating point. Thus the dynamic equation of the magnetic levitation system can be written as

$$M\frac{d^2\hat{x}_b}{dt^2} = Mg - K_m\left(\frac{\hat{\imath}_m + i_{m_{ss}}}{\hat{x}_b + x_{b_{ss}}}\right)^2 \tag{3.6}$$

Now, linearizing the above system using Taylor's series expansion method and assuming that $\hat{x}_b \gg x_{b_{ss}}$, $\hat{\imath}_m \gg i_{m_{ss}}$

$$\frac{d^2\hat{x}_b}{dt^2} = \frac{1}{M}\left\{\frac{\partial}{\partial\hat{x}_b}\left(Mg - K_m\left(\frac{\hat{\imath}_m + i_{m_{ss}}}{\hat{x}_b + x_{b_{ss}}}\right)^2\right)\Bigg|_{\hat{x}_b=0,\hat{\imath}_m=0}\hat{x}_b + \frac{\partial}{\partial\hat{\imath}_m}\left(Mg - K_m\left(\frac{\hat{\imath}_m + i_{m_{ss}}}{\hat{x}_b + x_{b_{ss}}}\right)^2\right)\Bigg|_{\hat{x}_b=0,\hat{\imath}_m=0}\hat{\imath}_m\right\} \tag{3.7}$$

Therefore,

$$\frac{d^2\hat{x}_b}{dt^2} = \frac{1}{M}\left(\frac{2K_m i_{m_{ss}}^2}{x_{b_{ss}}^3}\hat{x}_b - \frac{2K_m i_{m_{ss}}}{x_{b_{ss}}^2}\hat{\imath}_m\right) \tag{3.8}$$

By taking Laplace transform the transfer function of the magnetic levitation system is given as

$$G(s) = \frac{\hat{X}_b(s)}{\hat{I}_m(s)} = -\frac{K_2}{s^2 - K_1} \tag{3.9}$$

where $K_1 = \frac{2K_m i_{m_{ss}}^2}{Mx_{b_{ss}}^3}$ and $K_2 = \frac{2K_m i_{m_{ss}}}{Mx_{b_{ss}}^2}$ with $M = 0.002\ \mathrm{kg}$, $g = 9.81\ \mathrm{m/sec^2}$. The equilibrium point of the feedback make magnetic levitation system is $[x_{b_{ss}} =$

$-1.5\ \mathrm{V}, i_{m_{ss}} = 0.8\ \mathrm{A}]$. Therefore, the transfer function of the magnetic levitation system is given by

$$G(s) = \frac{-24.5250}{s^2+13.08} \quad (3.10)$$

The open-loop response of the magnetic levitation system is shown in Figure 3.2. The open-loop poles of the system are located on the imaginary axis i.e. at $s = \pm j3.6166$ and therefore the system lead to instability or having sustained oscillations. The complexity associated with this system is that the closed-loop system is unstable with one of the system pole lying in the right-half of the s-plane. The closed-loop poles are located at $s = \pm 3.3823$. Therefore, there is a requirement of an efficient controller which can effectively stabilize the position of the metal ball so that it can levitate in the air space.

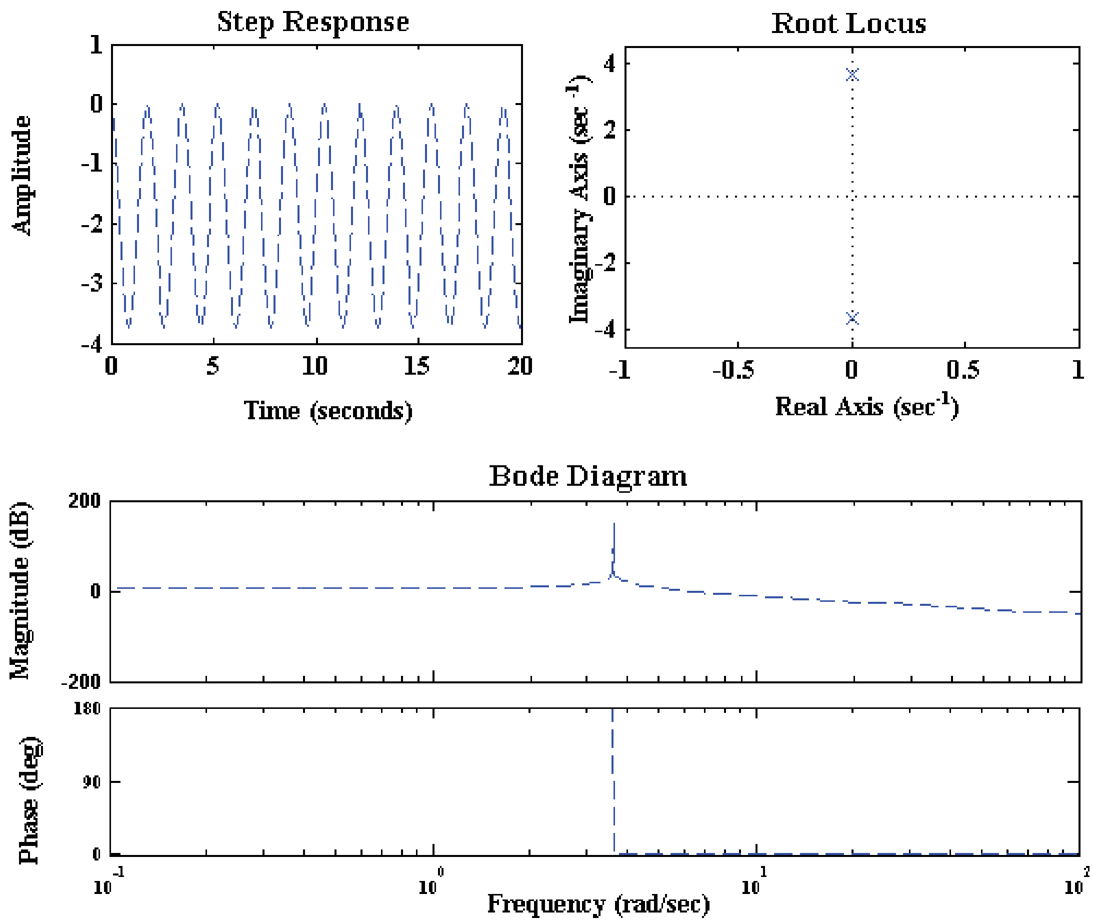

Figure 3.2 Open-loop response of magnetic levitation system

3.3 PID CONTROLLER

PID controller is known as the proportional, integral and derivative controller. It is very popular in the control system society because of its controlling capability. There are various combination of PID controller like PI, PD, PID-D, I-PD and many more depending on the system requirements [71]. Each term has its own role i.e. proportional controller is used to increase the loop gain of the system, thereby reducing its sensitivity to plant parameter variations. The integral controller increases the order of the system and reduces the steady-state error of the system by adding a pole at the origin of the s-plane. The derivative controller tends to stabilize the system by introducing derivative of the error.

The values of parameters of the PID controller can also be determined by trial & error method if the values of the open-loop transfer function are not exactly known. If the parameters of the plant are subject to large variations, the gain constants can be adjusted to improve the system performance [72]. The transfer function of the PID controller is given by

$$U(s) = \left(K_p + K_i \frac{1}{s} + K_d s\right) E(s) \tag{3.11}$$

where $K_p E$ is the proportional to the error, $K_i E/s$ is proportional to the integral of the error, $K_d sE$ is proportional to the derivative of the error, and U is controller output. To find out the parameters of the PID controller, the trial & error method is used because the conventional tuning techniques like Ziegler-Nichols, Cohen-Coon and approximated MIGO method are fails to give the approximate starting solution for the PID controller.

Though, these techniques would consider amongst the best conventional approaches but it fails in case of magnetic levitation system. The controller is designed for magnetic levitation system given in Eq. (3.10) in such a way so that it meets the following design requirements:

- Settling Time $(t_s) \leq 1$sec.
- Percentage Overshoot $(M_p) \leq 10$
- Gain Margin $(GM) \geq 6$ dB
- Phase Margin $(PM) \geq 60^0$

The parameters of the PID controller tuned using trial and error (T&E) method are given in Table 3.1, closed-loop responses like step response, bode plot are shown in Figure 3.3 and Figure 3.4. The time-domain characteristics like rise time, settling time, percentage overshoot, peak time and frequency-domain specifications like gain margin, phase margin are given in Table 3.2.

Table 3.1: Parameters of PID controller tuned using T & E method

K_p	K_i	K_d
-20.4793	-9.2557	-0.71604

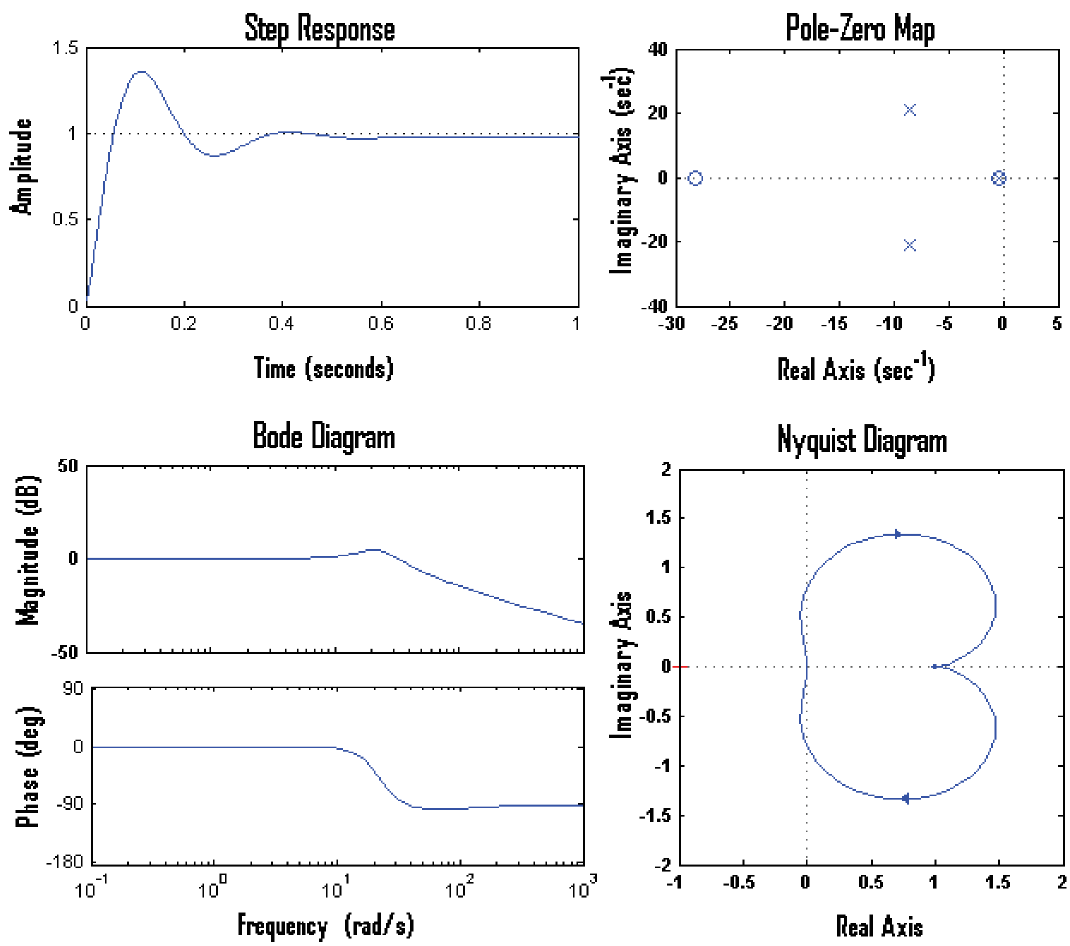

Figure 3.3 Step response of magnetic levitation system with PID controller tuned with T&E method

Table 3.2: Performance characteristics of magnetic levitation system tuned with PID controller

Rise Time (sec.)	Settling Time (sec.)	Percentage Overshoot	Peak Time (sec.)	Gain Margin (dB)	Phase Margin (deg.)
0.0443	0.656	35.5322	0.1130	Inf	42

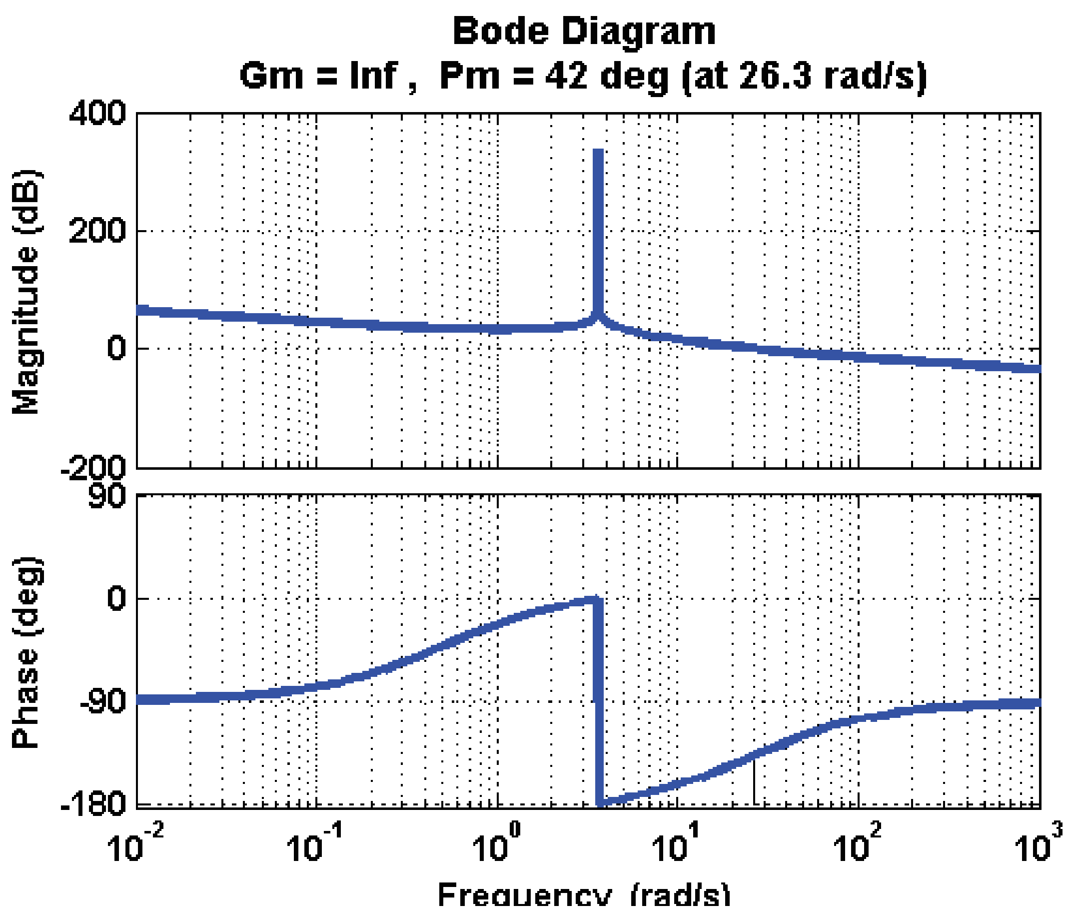

Figure 3.4 Bode plot of magnetic levitation system with PID controller tuned using T&E method

The performance characteristic shows that the controller successfully meets the settling time requirement but it fails to accomplish the percentage overshoot of the system. All Eigen-values of the closed-loop system are lying in the left-half of the s-plane which proves that the system is stable. The infinite gain margin shows that the system remains stable with any value of gain K, where K is the feed-forward gain of the closed-loop system varies from 0 to ∞.

The drawback of the PID controller tuned using T&E method is that it is unable to meet the phase margin requirement. In order to improve the performance of the system the parameters of the PID controller needs modification. The modification can be performed by updating the parameters using meta-heuristic approach. The meta-heuristic techniques searches for the optimum solution while minimizing the objective function i.e. the performance index of the system.

3.4 DESCRIPTION OF TLBO ALGORITHM

The teaching learning based optimization (TLBO) works on the effect of influence of a teacher on learners. Like other nature-inspired algorithms [73], TLBO is also a population-based method and uses a population of solutions to proceed to the global solution. The population is considered as a group of learners or a class of learners. The algorithm works into two successive phases; the “teacher phase” and the “learner phase” [74-79].

‘Teacher Phase’ means learning from the teacher and ‘Learner Phase’ means learning by the interaction between learners. Here, output is considered in terms of results or grades. The teacher is generally considered as a highly learned person who shares his or her valuable knowledge with the learners.

The quality of a teacher, affects the outcome of the learners. It is obvious that a good teacher trains learners such that they can have better results in terms of their marks or grades. If a teacher is more knowledgeable and has influence in the class then there are chances of improving the knowledge of the students/learners. It is not possible that all the learners grasp entire teaching material which the teacher taught them.

As shown in Figure 3.5 *curve-2* represents better results than *curve-1* and therefore it can be assumed that teacher T_2 is better than teacher T_1 in terms of teaching. The main difference between the results is their mean (M_2 for *curve-2* and M_1 for *curve-1*), i.e. a good teacher produces a better mean for the results of the learners. Learners also learn from the interaction between themselves, which helps to improve their results.

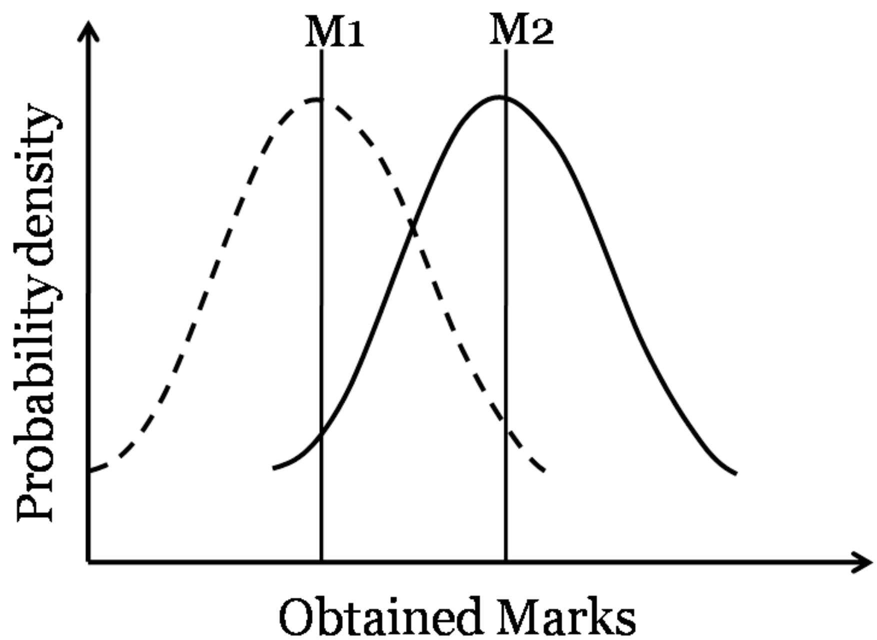

Figure 3.5 Distribution of marks obtained by learners taught by two different teachers

The Figure 3.6 demonstrate a model for the marks obtained for learners in a class with *curve-A* having mean M_A and *curve-B* with mean M_B. The teacher is considered as the most knowledgeable person in the society, so the best learner is mimicked as a teacher, which is shown by T_A. The teacher tries to disseminate knowledge among learners, which will in turn increase the knowledge level of the whole class and help learners to get good marks or grades.

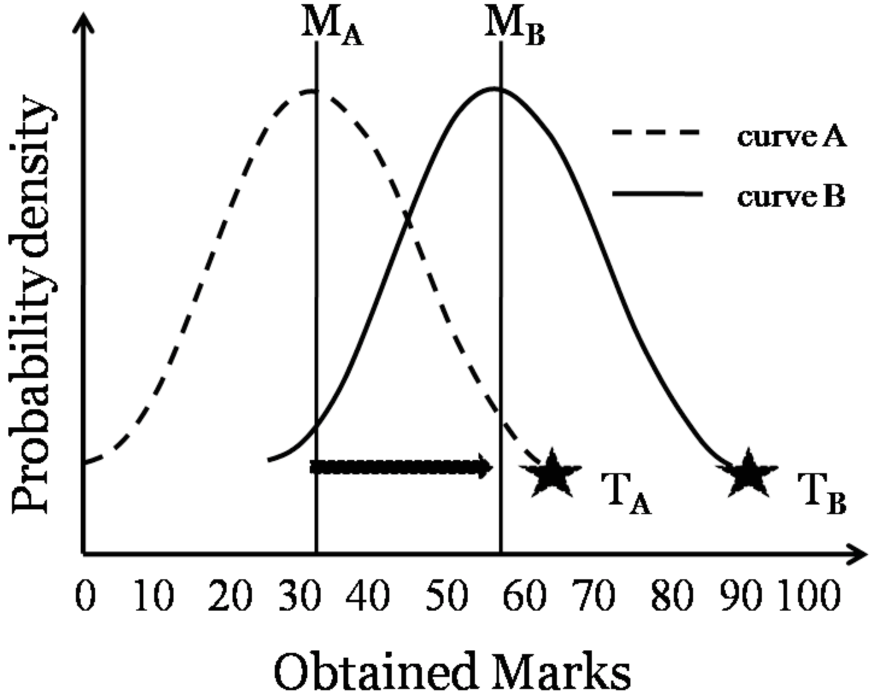

Figure 3.6 Model for the distribution of marks obtained for a group of learners

So a teacher increases the mean of the class according to his or her capability. In the above figure, teacher T_A will try to move mean M_A towards their own level according to his or her capability, thereby increasing the learner's to a new mean M_B. Teacher T_A will put maximum effort into teaching his or her students, but students will gain knowledge according to the quality of teaching delivered by a teacher and the quality of students present in the class.

The quality of the students is judged from the mean value of the population. Teacher TA puts efforts to increase the quality of the students from M_A to M_B, at this stage the students requires a new teacher of superior quality than themselves, i.e. in this case the new teacher is T_B. Hence, there will be a new *curve-B* with the new teacher T_B. In TLBO, different design variables will be analogous to different subjects offered to learners and the learner's result is analogous to the fitness, as in other population-based optimization techniques. The teacher is considered as the best solution obtained so far.

The quality of learners can be judged from the grades they obtained. If the mean of the learners are decreasing then it is required that the teacher changes the teaching pattern. Only few of them can understand the whole material and such students may consider as a best student and are allow to share his/her valuable thoughts with other learners. The second phase is the learner phase in which the best student (based on his grades) act as a teacher for other learners.

Therefore, the learners were allowed to interact with the acting teacher to clear his/her doubts for the improvement of their knowledge in the respective subject. The subjects are the parameters for the PID controller and the teacher/learner with maximum marks or minimum performance index are treated as the near optimum solution for the PID controller.

The overall block diagram of the magnetic levitation system combined with the TLBO algorithm is shown in Figure 3.7. The convergence of the TLBO algorithm on the way to global optimum solution is supervised by the performance index of the system. As the iteration increased, the parameters of the PID controller are modified in such a way so that they produce minimum performance index.

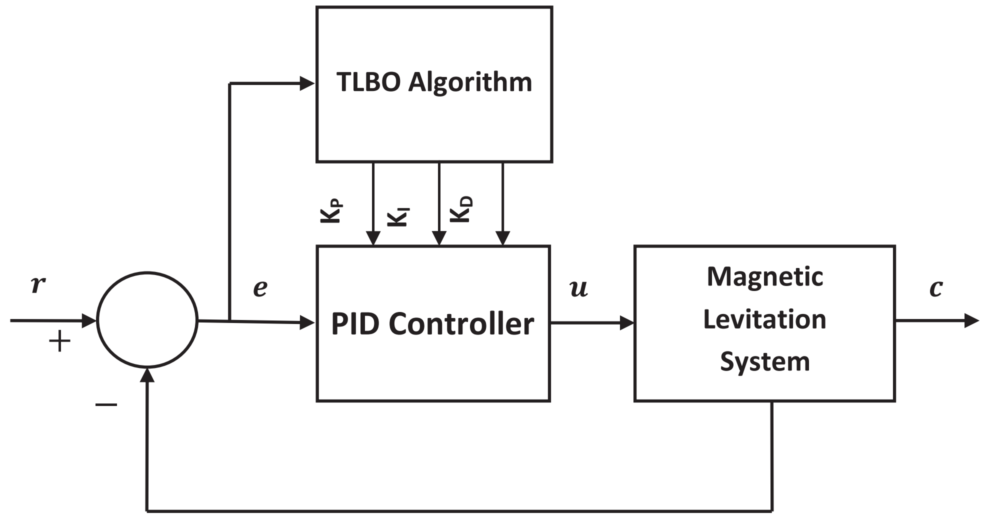

Figure 3.7 Block diagram for optimum search using TLBO

The TLBO algorithm incorporates the following steps:

Step 1: Initialize the number of students i.e. generates the population. Evaluate the objective function of each student. The initial population is generated on the basis of parameters tuned using conventional method.

$$PID_n = [K_p \quad K_i \quad K_d]_n \tag{3.12}$$

where n represents the total number of students in a class.

Step 2: Calculate the mean of each student. Here, performance index of the system is considered for calculating the mean of the generated population.

$$PID_{mean} = \frac{\sum_n ITSE}{n} \tag{3.13}$$

where ITSE is the integral time weighted square error.

Step 3: Identify the best student on the basis of minimum performance index achieved.

$$\mathrm{PID_{best}} = [\mathrm{K_p} \quad \mathrm{K_i} \quad \mathrm{K_d}]_{\mathrm{min\,.ITSE}} \tag{3.14}$$

and also calculate the teaching factor so that the best one of them would act as a teacher for the next iteration.

$$\mathrm{TF} = \frac{\sum_n \mathrm{ITSE} - n}{\sum_n \mathrm{ISE} - \min.(\mathrm{ITSE})} \tag{3.15}$$

Step 4: Now marks of all the students are modified and updated according to the marks obtained by the acting teacher as

$$\mathrm{PID_{new}} = \mathrm{PID_{old}} + r(\mathrm{PID_{best}} - \mathrm{TF} * \mathrm{PID_{mean}}) \tag{3.16}$$

The above equation restructured the marks of all students and updates their grade sheet on the basis of their present performance. For these updated set of variables the performance index is calculated and compared with the ITSE of old students. If new solution is better than the previous one then it is stored for next iteration otherwise it is rejected. Here, r is the random variable lies between 0 and 1.

Step 5: Select any two variables randomly from $\mathbf{PID_{new}}$ and compare the performance index of each student with the performance index of these two variables. If $\mathbf{PID_{new_i}}$ is better then $\mathbf{PID_{new_j}}$

$$\mathrm{PID_{new}} = \mathrm{PID_{old}} + r\left(\mathrm{PID_{new\ i}} - \mathrm{PID_{new\ j}}\right) \tag{3.17}$$

Else

$$\mathrm{PID_{new}} = \mathrm{PID_{old}} + r\left(\mathrm{PID_{new\ j}} - \mathrm{PID_{new\ i}}\right) \tag{3.18}$$

Step 6: Save the updated marks of the students and compare their performance index with the existing one. Opt the new marks if they are better in the sense of ITSE otherwise continue with the previous solution for next iteration. Stop the process if all the design requirements are fulfilled or the maximum number of iteration is reached.

3.5 SIMULATION RESULTS AND DISCUSSION

In this section, the parameters of the PID controller are tuned by using TLBO algorithm is discussed. Initially the parameters are randomly selected on the basis of the parameters tuned using trial & error method. For TLBO algorithm, eighteen students are selected and one of them who have minimum performance index would consider as teacher for the next iteration. The teacher updates the knowledge of other students and guided them to secure good marks in the form of minimum performance index, so that all the controller design requirements are fulfilled. The performance is calculated for every parameter and they were sorted accordingly. The performance index or objective function chosen here is the ITSE of the system given as:

$$J = \int_{t_1}^{t_2} t.\, e^2\,(t)dt \qquad (3.19)$$

where e is the error of the system, t is the time period $(t_1, t_2 \in 0, 100)$. The TLBO algorithm required around hundred number of iteration to fine-tune the parameters of the PID controller. The tuned parameters of the PID controller are shown in Table 3.3.

Table 3.3: Parameters of PID controller tuned using TLBO

	Randomly selected Parameters			*Parameters tuned with TLBO*		
S. No.	K_p	K_i	K_d	K_p	K_i	K_d
1	-20.40	-9.25	-0.71	-184.8	-249.96	-9.60
2	-9.40	-19.80	-0.05	-181.7	-249.30	-9.79
3	-10.25	-15.00	-0.80	-154.6	-249.66	-12.5
4	-22.00	-30.00	-0.12	-174.6	-249.78	-10.9
5	-15.06	-8.00	-0.32	-187.1	-249.97	-10.4
6	-11.00	-12.00	-0.12	-188.1	-249.50	-10.2
7	-21.00	-27.00	-0.10	-174.1	-249.90	-10.9
8	-31.00	-15.50	-0.31	-185.2	-249.67	-10.8
9	-23.00	-19.00	-0.52	-190.3	-249.08	-10.8
10	-40.00	-30.00	-5.10	-185.6	-249.75	-11.3
11	-30.00	-28.00	-4.10	-185.8	-248.87	-10.7
12	-25.00	-12.00	-0.82	-207.3	-249.91	-10.9
13	-10.50	-14.00	-0.20	-203.0	-249.71	-10.9
14	-10.20	-15.00	-0.80	-176.2	-249.87	-12.0
15	-20.00	-30.00	-0.01	-185.6	-248.42	-10.7
16	-15.10	-8.00	-0.32	-186.0	-249.22	-11.3
17	-10.20	-12.00	-0.12	-194.0	-249.28	-11.2
18	-20.00	-27.00	-0.10	-39.7	-258.31	-11.1

The issue associated with the optimization techniques of being struck at local optima is resolve by calculating the error, time-domain and frequency-domain characteristics at each & every instants. Also, two students are selected randomly from the class and performance of every student is compared with them to ensure that every student benefited with knowledge of the teacher.

This shows the exploration property of the TLBO algorithm. The step response of the best three performers is shown in Figure 3.8. The best one of them is considered as a teacher and others are as learners. The parameters tuned via TLBO algorithm fulfils all the design requirements i.e. both the settling time and percentage overshoot of the system gets improved and is shown in Table 3.4. The Bode diagram is shown in Figure 3.9. The gain margin of the best three performers is infinite which shows that the system is highly stable and can handle any value of gain.

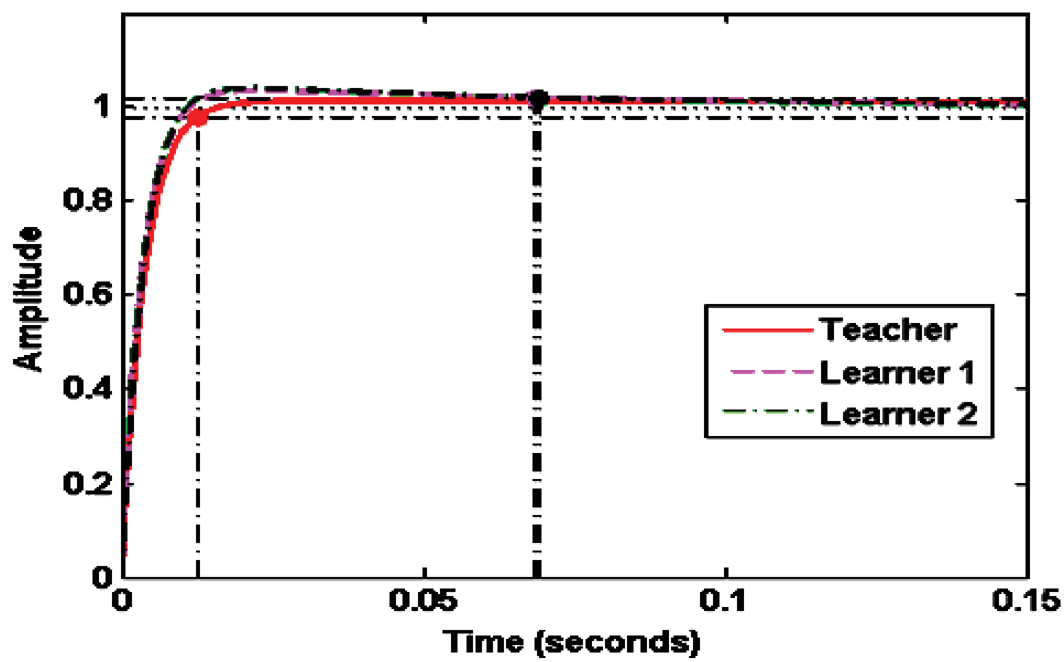

Figure 3.8 Step response of magnetic levitation system with parameters of PID controller tuned using TLBO algorithm

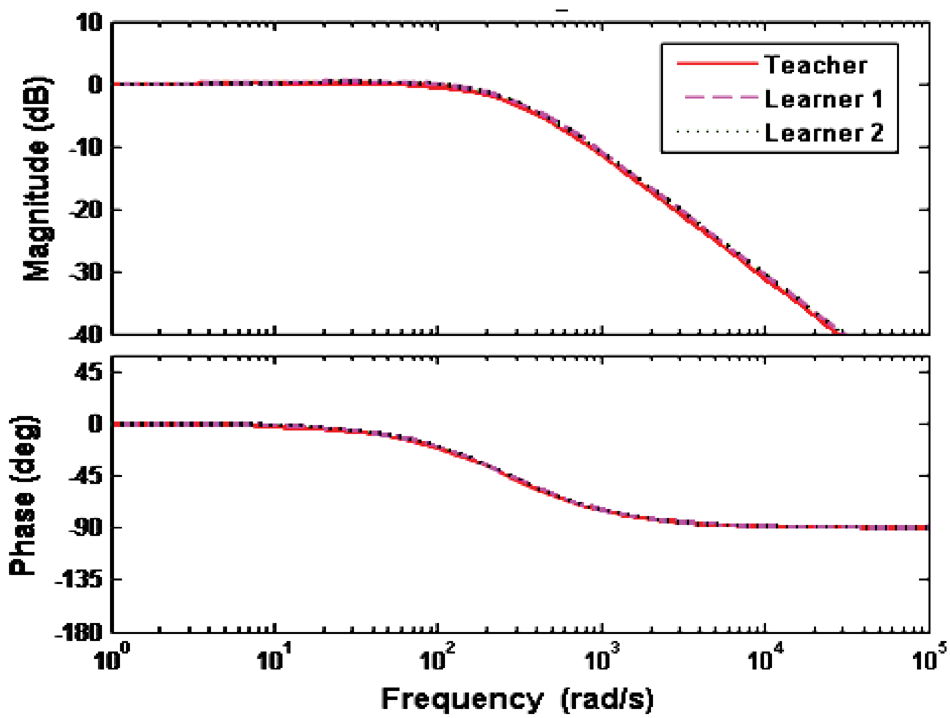

Figure 3.9 Bode diagram of magnetic levitation system with parameters of PID controller tuned using TLBO algorithm

The higher the phase margin the more stable is the system and for these tuned parameters, the phase margin is around 170^0. Some researcher given their theory on the phase margin that there are changes of getting sluggish response for larger phase margin but using TLBO algorithm the settling time and as well as peak overshoot of the system shows better response as compared to conventional techniques.

Table 3.4: Performance characteristics of TLBO algorithm

	Teacher	Learner 1	Learner 2
Rise Time (seconds)	0.0077	0.0066	0.0065
Settling time (seconds)	0.012	0.068	0.069
Max. Overshoot (%)	1.28	3.53	4.00
Peak Time (seconds)	1.012	1.035	1.040
Gain Margin (dB)	∞	∞	∞
Phase Margin (degree)	170	163	161
ITSE	9.36×10^{-7}	9.39×10^{-7}	9.67×10^{-7}

3.6 CONCLUSION

This chapter presents a new meta-heuristic technique known as teaching learning based optimization for controlling the position of the metal ball via magnetic levitation system.

The parameters of the PID controller are tuned effectively by TLBO algorithm. The controller is used to levitate the position of the metal ball in the air-space by controlling the electromagnetic coil current. The coil current is controlled by sensing the position of the metal ball through infra-red sensor.

The effectiveness of the proposed controller is validated by comparing it with the conventional tuning method. The simulation results performed in MATLAB shows that all the design requirements are successfully achieved.

The TLBO technique ensures the improvement of the time domain and as well as the frequency domain specifications by minimizing the performance index of the system.

The renowned properties like exploration and exploitation are handled appropriately by the TLBO algorithm. Though the TLBO algorithm fulfilled all the design requirements but still the tuning process is more iterative. Presently a new algorithm based on parameters-less Jaya algorithm proposed by Rao et al. is easier to implement as discussed in [80]. The algorithm moves towards the best solution while rejecting the worst solution. The author would like to apply such a simple and effective technique for the future work.