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A state-of-the-art review of measurement and modelling of skid resistance: The perspective of developing nation

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ABSTRACT

A critical review of lab and field measurement methodologies, harmonization in measuring techniques, and modelling of skid resistance of asphalt concrete pavement have been provided. Although several past studies have provided literature review on general topics of skid resistance, to the best of the author's knowledge, none of them have compressively covered the topic considering the status & requirements of developing nations. There has been significant development in speed with the improvement in computational facilities. In modern times, with the improvement in infrastructure quality in developing nations, permitted speeds have also drastically increased. To avoid skid-related accidents, it is important to develop good practices in maintaining sufficient skid resistance. The requirements and the availability of technology might be significantly different in developing nations. The suitability and limitations of various methods used for capturing the skid characteristics of the surface have been outlined. The harmonization in skid resistance measurement using laboratory and field-testing methods has been summarized. Correlation analysis of various in-situ and laboratory test data has been made to maintain a better harmony of measurement either in the field or in the laboratory. In the subsequent sections, progress in the modelling approach (analytical to numerical) has been discussed in brief. Computational capabilities of an analytical and numerical modelling approach for predicting pavement skid resistance characteristics have been reviewed. These models have been developed to consider complex attributes of tire pavement interactions like hydroplaning, temperature rise in the tire, mix morphology, tire inflation, and vehicle acceleration and deceleration for predicting skid resistance. These attributes of skid resistance have been discussed in detail and presented a basic overview of the model development process which is missing in past review studies. Few recent studies on skid resistance measurement and modelling to highlight the use of new technology and improvements over conventional techniques have been presented in the manuscript which has not been reviewed earlier. Critical factors affecting the skid resistance model like hydroplaning, tire-related parameters, temperature, and surface texture have been highlighted in this manuscript. Few key research directions have been suggested as the scope of future study to predict a more reliable skid resistance model.

1. Introduction

When the rotation of a rolling tire is prevented, it starts sliding over the pavement surface. The resistive force developed to prevent

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this sliding of the vehicle is directly related to the available skid resistance between the tire and the pavement. Hence, the skid resistance is often measured by computing the frictional force at the tire pavement contact with the help of a friction-measurement device [1]. Pavement skid resistance value is often used as an indicator of safety for safe driving operations, and it plays a significant role in reducing accidents especially when the surface is wet [2]. Road accidents due to loss of skid resistance below its threshold are among major concerns for road authorities. These accident-prone pavement sections often have skid resistance values below a minimum safe level. Past studies show that approximately 14 % of the total fatal road accidents occur in wet weather conditions [3,4]. Apart from wet weather conditions, Najafi et al. [5] highlighted that frictional properties of pavement surface impact the rate of crashes in dry conditions as well.

Lack of sufficient skid resistance on a highway network is more noticeable in developing countries like India, Iran, and others where the fatalities rate in road crashes is higher than in developed countries. As per the World Bank report (2019), India tops the world with 11 % of global death in road accidents with just 1 % of the world's vehicles. Past studies [6] show that an increase in speed increases accident rates particularly when the infrastructure is not maintained properly. To avoid skid-related accidents, it is essential to develop good maintenance practices for producing sufficient skid resistance on the pavement surface. There is still a significant gap between the requirement and available technology to developing nations.

Maintenance of a pavement surface for ensuring sufficient skid resistance covers two broad aspects of tire-pavement interaction (a) maintaining tire tread and (b) maintaining pavement surface. Depending upon the rotational speed of the tire and the texture properties of the pavement, a tire may start skidding even on dry surfaces [5]. Skidding on the pavement surface may result in a loss of braking efficiency and steering capacity of the vehicle. It may lead to pavement damage and in extreme cases may result in human causalities [7]. Safety concern increases in presence of higher speed (>52 mile/hr), and rainfall intensity of 300 mm/hr [8] at which it may even hydroplane. A vehicle hydroplanes when a layer of water builds upward pressure between the tire rubber and pavement interface, leading to the loss of traction and thus preventing the vehicle to control inputs such as steering, braking, or accelerating [9].

Theoretically, the friction between a rubber tire and road surface comprises two components, namely adhesion, and hysteresis [10]. The hysteresis deformation is the result of damping losses and energy dissipation of tire rubber [10]. Frictional properties of asphalt concrete pavement are a time-dependent phenomenon and are affected by pavement surface characteristics, tire tread characteristics, loading conditions, and environmental parameters. The mechanism of skid resistance is complex and the reliability of its measurement is significantly affected by the variation in environmental parameters like temperature, wind speed, and humidity [11]. A few of the parameters like surface texture (micro-texture and macro-texture), tire slip speed, and asphalt mix properties can be easily controlled at the design stage of pavement and by regularizing the speed limits at different highway sections [7]. However, a few other parameters like evaluation of actual contact area at tire-pavement interface is important for prediction of a reliable skid resistance model. Effect of actual contact surface as a function of tire pressure and vertical load has been analyzed for this purpose [12]. The contact surface area is examined for anti-skid properties of the pavement. Some field studies used an innovative method of assessing anti-skid properties with simultaneous measurement of friction coefficient and continuous coefficient of average profile depth [13]. Micro texture and macrotexture of pavement is an important aspect of relating skid resistance characteristics with pavement conditions and results of field test were presented to analyze skid resistance properties [14]. For a better understanding of the frictional characteristics of pavement surface, it is necessary to evaluate it accurately as it will further help in preventing accidents and ensuring safe highway operations. Reliability and accuracy in the measurement of pavement surface frictional properties are important and can be ensured by the measurement devices used in-situ or in laboratory tests [15].

Previous research studies [6,16–20] have presented different aspects of skid resistance through various review articles. However, to the best of the author's knowledge, none of the existing articles provided a critical overview comprehensively covering both numerical and testing aspects, particularly from a stand point of a developing nation. The authors believe that a robust skid resistance management system requires both the modelling and measurements should support each other.

This paper tries to present a brief overview of various skid resistance measurement practices and computational advancement in modelling from the perspective of the developing nation. A brief discussion on recent advancements in numerical solutions of skid resistance at tire pavement interface using finite element analysis and capabilities of the developed model has been summarised and presented. A brief overview of current skid resistance measurement practices, challenges ahead and future scope from the perspective of a developing nation has been presented in Section 6. The authors hope that the presented article will help road authorities and road researchers in setting up a workable framework to minimize skid-related accidents in developing nations. Such frameworks are current need to help in building an accident-free high-speed pavement network.

1.1. *Skid resistance and accident data*

Though skid resistance is measured to assess the pavement roughness for safe vehicular operation and to reduce skid related accidents, however, to the best of author's knowledge there is no specific study available which provides accident-related data due to skid resistance only. It is very difficult to quantify accident data based on poor skid resistance of the pavement. There may be several reasons for a specific traffic accident. However, many researchers have tried to correlate surface roughness with skid resistance. The risk of wet pavement crashes increases when skid number is less than 40 while the risk is low when it is above 60 [21]. Wet pavement skidding contributes to 13.5 % of fatal and 25 % of all accidents [22]. Inadequate surface friction led to poor visibility due to splash and spray ultimately led to uncontrolled skidding are the main cause of wet weather crash. The splash and spray contribute to 10 % of wet weather accidents and can be minimised with deep surface texture [23]. As per the study of Larson et al. [24], 70 % of wet weather crashes can be prevented by improving the surface texture or friction level on pavement.

1.2. Paper organization

For easy understanding, the manuscript has been arranged into different sections and subsections. The first section introduces and provides information on various works of literature available on skid resistance measurement, and modelling. The second section provides a basic concept of skid resistance at the tire-pavement interface. The third section presents various ways of skid resistance measurement in the field and lab including the perspective of developing nation mainly focused on India. The fourth section discusses developments in analytical and numerical skid resistance modelling techniques and the challenges ahead. The fifth section presents a few recent advancements in skid resistance measurement and modelling. The sixth section discusses current practices, challenges in skid resistance measurement, opportunities ahead, and the way forward from the developing nation point of view. The seventh and eighth sections present critical conclusions and scope for future study respectively. The scope has been identified based on a few research gaps highlighted in the review study.

2. Skid resistance at the tire-pavement interface

Skid resistance at the tire pavement interface is defined as "the force that resists the relative skidding of tire rubber over pavement surface after applying brakes" [25]. The frictional force is usually characterized as the coefficient of friction (μ). Mathematically, the coefficient of friction, μ is given by Coulomb's definition:

$$\mu = \frac{F}{F_w} \tag{1}$$

where F is frictional force and F_W is the weight of the wheel load. The interaction at the tire-pavement interface is a complex phenomenon due to multi-body physics with different material behaviour and different surface morphologies. The visco-elastic nature of tire-rubber materials makes it harder for researchers to simply understand and analyse the interaction mechanisms. This is because empirical studies prove that the classical laws of friction are not valid for viscoelastic materials [26]. As per the United States based highway research agency, National Cooperative Highway Research Program (NCHRP) report 2009, the mechanism of friction at the tire pavement interface is not clearly understood but it is agreed that adhesion and hysteresis are the main components of friction as shown in Fig. 1 [7]. The shear force generated by the tire rubber is another component of friction but its magnitude is much lower and can be neglected as compared to adhesion and hysteresis [26]. Macrotexture is the prime factor that affects hysteresis mostly while tire speed has an almost negligible effect. In the case of adhesion, both micro-texture and sliding speed affect "$F_{adhesion}$" significantly. So, the force of friction can be expressed in a simplified form:

$$F = F_{adhesion} + F_{hysteresis} \tag{2}$$

In the case of asphalt concrete pavement, it is a common consensus that micro-level surface roughness mostly governs the magnitude of adhesion while hysteresis is governed by the presence of macro-level surface roughness [7]. Surface texture is the key parameter influencing all three mechanisms. Generally, micro-texture more closely relates to the adhesion component and macro-texture relates to the hysteresis component of friction.

Adhesion force is proportional to the real area of adhesion between tire and surface asperities. Adhesion component F_A is a function of the interface shear strength τ_s and contact area A_i. The hysteresis component, F_H of frictional force results from the energy loss (E_C –

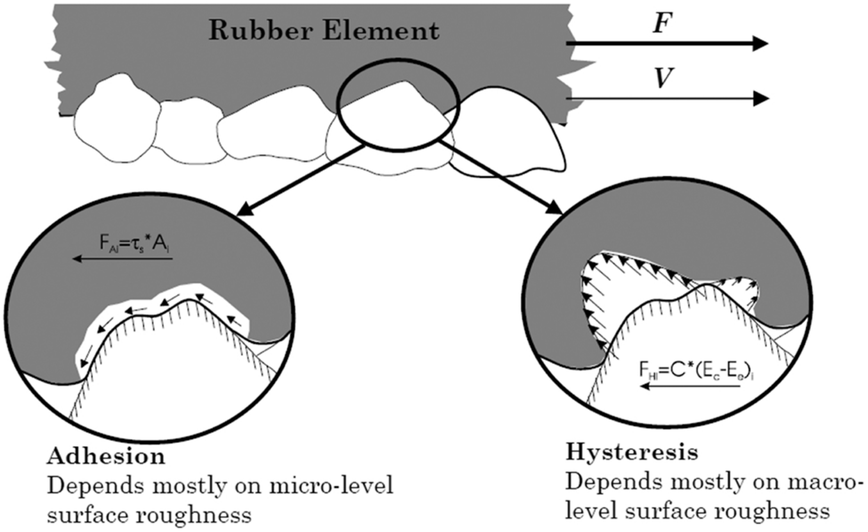

Fig. 1. Schematics of adhesion and hysteresis [7].

E_O) due to bulk deformation of tire. Where E_C and E_O is energy stored in the tire before and after compression respectively and C is coefficient of compression. Hysteresis is a function of speed and is generated within the deflecting and visco-elastic tire tread material.

2.1. Mechanism of pavement friction

As discussed earlier and expressed by Eq. (2), pavement friction is mainly composed of hysteresis and adhesion components. If one divides both sides of Eq. (2) by the normal load, the following result is obtained:

$$f = f_a + f_h \tag{3}$$

where f is the total friction coefficient and f_a and f_h are adhesion and hysteresis coefficients. Past studies [26] highlighted that f_a and f_h depend on the viscoelastic properties of tire rubber. This is also demonstrated in Eq. (4) & (5):

$$f_a = K_1 s \left(\frac{E'}{p^r}\right) \tan\delta \qquad (r < 1) \tag{4}$$

$$f_h = K_2 \left(\frac{p}{E'}\right)^n \tan\delta \qquad (n \geq 1) \tag{5}$$

where K_1 and K_2 are rubber constants, and tanδ is the tangent modulus of the elastomer. p is normal pressure, $E^{'}$ is storage modulus, s is the effective shear strength of the sliding interface, r is the exponent with a value of about 0.2, and n is an index greater than unity [26].

Adhesion at the interface appears due to the intermolecular force of attraction between the atoms exposed at the surface of pavement and tire rubber [27]. The hysteresis component is due to the continuous draping of tire rubber over the pavement surface. The study of mechanics of tire pavement interaction helps in understanding how various skid resistance measuring tools actually function, understanding their operating principles, and hence what they eventually measure. Other than these two components, rubber wear and surface hysteresis are other friction components with little significance [28].

3. Measurement of skid resistance

Many research studies as discussed in this section carried out on the measurement methods to evaluate the skid resistance on the asphalt pavement surface. With the advancement in laser technology and the computational power of high-speed computers, several systems are now available to measure skid resistance at traffic speed. This section presents a brief overview of various methods available for skid resistance measurements and their technology. Skid resistance is generally quantified using friction factor or skid number etc [29]. Roughness characteristics of asphalt concrete pavement are evaluated in the field as well as in the lab. Pavement macrotexture and micro texture are a good indicator of surface roughness. Wesolowski and Blacha [14] used sand patch method and profilometric method for surface texture evaluation. The sand patch method is used for 0.25–5 mm mean texture depth however, profilometric method is used for 0–5 mm mean profile depth. Zieja et al. [13] conducted surface roughness test using a measurement system based on the airport surface friction tester on a T-10 trailer additionally equipped with a 2D/3D high frequency laser scanner.

Friction-measuring vehicles can be used to evaluate pavement skid resistance in the field or lab. It can be observed that quite a few measuring techniques are available for in-situ or laboratory evaluation of skid resistance. For example, multiple testing methods for field evaluation have been developed which can be categorized by their ways of operation. Although the operating principles of a number of measuring devices are similar, still, it is very difficult to establish a good correlation among various methods [30]. The most common correlation method well established and recognised to date is the skid resistance index (SRI) and international friction index (IFI) [31]. Although researchers have tried to develop various correlations among different test methods for maintaining harmony in skid resistance measurement, there are still a few basic concerns that need to be addressed [32].

For comparing and harmonising surface texture and skid resistance data, world road association, Permanent International Association for Road Congresses (PIARC) developed the IFI in an extensive experimental program. The index allows for the harmonisation of skid resistance data measured from different devices to a common calibrated index (ASTM E1960–07(2015)). IFI is suitable for comparing and harmonising skid resistance when a smooth tire is fixed in a locked-wheel skid tester. The trend of friction speed curve changes when a ribbed tire is used in a locked wheel skid tester which poses a question on the suitability of usage of a ribbed tire. This disagrees with the assumptions made in IFI that pavement macro texture is the main parameter that affects the variation of the friction speed curve. Thus, the applicability of IFI, when the ribbed tire is installed, needs to be re-evaluated [33]. On the other hand, the calculation of SRI and calibration procedure, based on the outcomes of the HERMES project was published in European Committee for Standardization (CEN), 2010a. The SRI includes the effect of pavement texture employing the parameter mean profile depth (MPD). However, till date, this standard has not been put into practice [34].

These correlation methods developed for the harmonization of skid resistance measurement data are the outcomes of extensive research studies carried out in developed countries however in developing countries like India, no such program has been initiated. Looking into the importance of road user safety, the Indian Road Congress (IRC) released IRC:82 (1982) last revised in 2015 recommending a set of skid number values as obtained from a portable skid resistance tester for the different conditions of the pavement surface. For highways skid number (SN) of 60 for good roads, 50 for fair, and 40 represent poor at the other end for urban roads SN is 65 for roads having good skid resistance, 55 for fair, and 45 for poor. Compliance with these recommendations is the main concern for developing countries like India as many high-speed road network construction and maintenance agencies are still giving more weightage to visible distresses and ignoring the skid and related IRC recommendations.

3.1. Field measurement

The measurement of pavement skid characteristics is a tedious task as skid resistance measurements are affected by many parameters which cannot be easily controlled either in the field or in the lab. However, during the measurement of the friction coefficient, most of the parameters except surface texture are kept constant. There are various tools for the measurement of friction. Skid resistance in the field is generally computed by the force generated when a test tire with a varying slip ratio slides over the pavement surface [27]. Field test consists of rolling test tire to pavement surface with a control slip varying between 0 % and 100 %. Longitudinal friction coefficient (LFC), sideway force coefficient (SFC), and stationary or slow-moving measurement are major operating principles for various friction-measuring devices [6]. All the measuring devices using the LFC principle control the sliding of the test tire by the application of braking force. It restricts the rotation of the test tire to keep its rotation slower than the translational movement of the vehicle. The LFC principle offers a large array of slip ratios to measuring devices, which can be used to compare tire and vehicle speeds [35,36,37]. The following expression can be used to compute the tire slip ratio [7]:

$$SR = \frac{V - V_p}{V} \times 100 = \frac{Slip\ speed(S)}{Forward\ speed(V)} \times 100 \tag{6}$$

where slip ratio is denoted as SR, the translational speed or forward speed of the vehicle is V, in mile/h (1 mile = 1.61 km), the average circumferential speed of the tire (mile/h) is $V_{P,}$ and slip speed (mile/h) is S.

Under Indian conditions (IRC: 134–2022) it is recommended to use a locked wheel skid tester and a dynamic friction tester for the evaluation of skid resistance on urban roads. This method is meant to test the frictional characteristics of a surface under emergency braking conditions for vehicles without an anti-lock braking system (ABS) while the fixed slip method is used for cars with ABS. These are used for continuous and dynamic measurements of longitudinal skid resistance with a single wheel and fitted with smooth tread tires instrumented to measure longitudinal drag force and vertical load [38]. Slip speed in a locked wheel testing device is equal to tire speed, i.e., the test tire is completely locked and is prevented from rotation. Smooth (ASTM E 524) or ribbed tire (ASTM E 501) can be used to conduct the test at a test speed ranging from 64 to 96 km/h. The ribbed tires are insensitive to water film thickness and pavement macrotexture while the use of smooth tires makes them sensitive to pavement macro texture [39]. The pavement surface is made wet first for all kinds of tests in which, a locked wheel skid tester and variable slip of tires are used for the friction measurement [40]. Generally, the locked wheel testing method follows the specification of ASTM, and test results are presented as skid numbers, which can be calculated as:

$$SN = \left(\frac{F}{N}\right) \times 100 \tag{7}$$

where F is the frictional force and N is the vertical load on the test tire. The locked wheel testing device offers the advantage of understanding and controlling the test variable easily [20]. However, the primary disadvantage is that measurement of the friction coefficient is not continuous over the entire test section [20].

The side force test system maintains the test wheel in a plane at an angle (yaw angle) to the direction of motion, otherwise, the wheel is allowed to rotate freely. The test wheel moves at an angle to the direction of vehicle motion because a critical situation of skid resistance occurs during cornering motion [41]. The slip angle induces friction between the tire and the road as it makes the tire slip over the road surface. The side force is measured perpendicular to the plane of rotation [7,16]. Since the yaw angle is typically small, between 7.5° and 20°, the slip speed is also quite low; this means that the side force tester is sensitive to the pavement micro texture but is insensitive to changes in pavement macro texture. The two most common side force measuring devices are Mu-Meter and the Sideway-force coefficient routine investigation machine (SCRIM) [29]. Generally, tests are conducted by keeping the measuring device stationary based on the slow-moving operating principle. Various equipment mostly used in the lab for measuring skid resistance falls under this category.

In developing countries like India, the measurement for monitoring the in-service skid resistance of expressways/national highways/state highways should be made with SCRIM as per the recommendations of IRC 134:2022. For major district roads and rural roads in India, the British pendulum test is recommended. For urban roads, a dynamic friction tester and a locked-wheel skid tester are recommended. It also suggests a few empirical equations to estimate the SCRIM coefficient (SC) as follows:

$$SR(50) = SR(s)\frac{-0.0152s^2 + 4.77s + 799}{1000} \tag{8}$$

Where SR (50) is the value of SR(s) corrected to 50 km/h, SR(s) is the sideways-force coefficient measured at test speed s, multiplied by 100. SCRIM coefficient (SC) value should be calculated for each 10 m subsection for which a valid SR(s) value is available using the following equation:

$$SC = \frac{SR(50)}{100} \times index\ of\ SFC \tag{9}$$

The index of SFC (sideway-force coefficient) is currently taken as 0.78 for SC computations. In Indian conditions, IRC 134: 2022 recommends a minimum SCRIM coefficient based on investigatory and threshold skid resistance levels. It is adopted for national highways/expressways and state highways as presented in Table 1. Table 2 and Table 3 provide investigatory and threshold levels of locked wheel and dynamic friction tester (DFT) values corrected for Indian conditions. The investigatory and threshold level friction by locked wheel skid tester is presented by friction number (FN40R) normalised to 40 miles/hr and measured with ribbed tire. The skid resistance management system in India has been schematically presented in Fig. 2.

Other than the longitudinal and side friction measurement principle, many of the test equipment have been developed based on the fixed slip operating principle in which equipment utilizes vehicles with an anti-brake system for measuring friction. These devices operate at a constant slip between 10 % and 20 % [16]. The test wheel is driven at a lower angular velocity than its free-rolling velocity. These devices also measure low-speed friction.

A fixed slip device can be used to capture the effect of surface micro-texture on skid resistance as it is sensitive to micro-texture at low slip speed. Variable slip devices can switch between a wide range of pre-set slip ratios. ASTM standard E 1859 (16) has been developed for devices that performs controlled sweep through a pre-set slip condition. An advanced tool, the Norsemeter ROAR MkII variable slip skid tester can be operated on fixed slip mode also [16]. Testing results from Norsemeter ROAR MkII are useful for both road network condition monitoring and safety investigation of black spot areas through stopping distance analysis. It uses a hydraulic brake system for measuring friction coefficient. This equipment is designed to operate at normal traffic speed and uses shaft encoder technology and optionally GPS equipment to capture measurement location [42]. Variable slip testing machine uses automated braking of test tire from free rolling to fully locked for measuring skid resistance of the pavement surface.

Skid numbers obtained from locked wheel trailers were found to be consistently lower than corresponding numbers from fixed slip systems by a factor of 0.67 [43]. Skid number values as obtained in the locked wheel skid tester were found consistently lower than the sideway force coefficient as obtained in SCRIM [44–46]. It can be justified by the fact that when skid resistance is measured with a partially locked tire (slip ratio $<$ 100 %), is always higher than one tested with a fully locked tire (slip ratio $=$ 100 %). Unlike other methods, the test tire in the fixed slip method is completely locked and prevented from free rotation resulting in equal slip speed and vehicle speed [16].

Besides having certain advantages of a locked wheel tester, it suffers from various disadvantages as well (1) Continuous measurement of skid resistance is not possible (2) Besides having a lesser cost of locked wheel trailer (approximately 90 % of other measuring devices), total cost including the initial and operating cost of its accessories are still higher (3) Determination of skid resistance-speed trend is not possible unless repeated measurements are done on same sections of the pavement but with different speeds because the individual test is conducted at only one speed. A summary of pavement friction measurement methods has been presented in Table 4 for reference.

3.2. Laboratory measurement

The two most common devices used in the laboratory are the British portable tester (BPT) and the dynamic friction tester (DFT). Operational procedures of BPT have been specified in ASTM standard test method E 303 (18). BPT is used worldwide and is the basic skid resistance measuring tool in developing countries like India, Iran, Indonesia, and Brazil. These countries have set up various recommendations based on skid number values as obtained from BPT for different categories of road networks. For India, these values have been summarized in Table 5 as recommended by IRC:134–2022. Table 5 recommends skid resistance values as measured by a British pendulum tester for major districts roads and rural roads applies to newly constructed or strengthened roads. These values are categorized as good, satisfactory, and poor. Newly constructed or strengthened roads should meet the criteria under the “Good” category. Values for satisfactory and poor have been given for purpose of timely maintenance planning and interventions.

The BPT is low cost, easy to handle, and can be used for the field as well as a lab evaluation of frictional properties of surface at a low speed [41]. Previous studies show that BPT measurement is largely influenced by pavement micro structure due to its low operational speed however macro texture can also affect the measurement. Macro texture of asphalt mix surface and gap width of aggregates are the most effective factors controlling BPT measurement [42,47].

Some of the past studies have shown that BPT results are unreliable in coarse-textured measurements [39,48,49]. To overcome the shortcomings of BPT, DFT was developed as an alternative, which can measure pavement friction at higher speeds. The operation of

Table 1
Recommended SCRIM coefficient for network-level surveys for expressways/national highways/state highways.

Minimum scrim coefficient for investigatory and threshold levels		
Site description	Investigatory level	Threshold level
Expressways/national highways/state highways	0.35	0.30
Non-event carriageways with one-way traffic	0.40	0.30–0.35
Non-event carriageways with two-way traffic	0.45	0.35–0.40
Approaches to and across minor and major junctions, Approaches to roundabouts, and traffic signals	0.50	0.40–0.45
Approaches to pedestrian crossings and other high-risk situations	0.55	0.45–0.50
Roundabout	0.50	0.40–0.45
Gradient 5–10 %, longer than 50 M	0.50	0.40–0.45
GRADIENT > 10 %, longer than 50 M	0.55	0.45–0.50
Bend radius < 500 M – carriageway with one-way traffic	0.50	0.40–0.45
Bend radius < 500 M – carriageway with two-way traffic	0.55	0.45–0.50
Single carriageway non-event	0.40	0.35

Table 2
Recommended locked wheel and DFT values for investigatory level for urban roads.

Investigatory level		
Site description	FN40R-Locked wheel	DFT values
Signalized intersection		
Pedestrian/school crossing	45	0.55
Railway level crossings		
Roundabout approaches		
Curves with radius $\leq$ 250 m and $>$ 100 m	40	0.50
Gradients $\geq$ 5 % and $\geq$ 50 m long freeway/highways/off ramps		
Intersection	40	0.50
Manoeuvre–free areas of undivided roads	35	0.40
Manoeuvre–free areas of divided roads	30	0.35
Curves with a radius $\leq$ 100 m	45	0.55
Roundabouts	45	0.55

Table 3
Recommended locked wheel and DFT values for threshold level for urban roads.

Threshold level		
Site description	FN40R-Locked wheel	DFT values
Signalized intersection		
Pedestrian/school crossing	40	0.50
Railway level crossings		
Roundabout approaches		
Curves with radius $\leq$ 250 m and $>$ 100 m	35	0.40
Gradients $\geq$ 5 % and $\geq$ 50 m long freeway/highways/off ramps		
Intersection	35	0.40
Manoeuvre–free areas of undivided roads	30	0.30
Manoeuvre–free areas of divided roads	25	0.25
Curves with a radius $\leq$ 100 m	40	0.50
Roundabouts	40	0.50

DFT is specified in ASTM standard test method E 1890 (20). DFT is equipped with a rotating disc and three rubber pads mounted over it. It measures the friction force between the pavement surface and rubber pads. A motor is used to drive the rotating disc till the tangential speed of the slider is reached 90 km/h, water is then applied to the test surface, and the motor is then disengaged. The transducer is used to measure friction force as soon as the rubber sliders touch the pavement surface and the rotating disc spins down [41]. DFT has high repeatability in international friction index calculation and is recommended to be used for the calibration of friction measuring devices by ASTM E 1960 [16].

The measurement principle of DFT and BPT is based on computing the loss in kinetic energy of a rotating disc or pendulum when they are interacting with the pavement surface. DFT has an edge over BPT as it can measure friction at different speeds. However, DFT can be used for stationary measurements only and indicates friction coefficient. It cannot directly measure the pavement microtexture [50,51]. The incapability to consider all the field constraints affecting friction value reduces the accuracy of DFT measurement.

The latest development of skid resistance measurement devices includes an automatic Wehner/Schulze (W/S) device [52] and a Tire pavement dynamic friction analyser (TDFA) system [53,54]. Wehner machine has been designed to simulate the effect of traffic on pavement surface and friction measurement. Test measurement determines the friction dependency on speed. It can be used to forecast the skid resistance variation of pavement surface course with traffic loading which can be further used as a performance indicator of the pavement. Wehner/Schulze can be used to develop new materials and construction methodologies for a surface course [52]. The TDFA system has been designed to capture frictional characteristics at tire pavement contact in real-time [54]. Its measuring speed varies between 10 and 100 km/hr. TDFA is facilitated to simulate an anti-lock braking system which enables it to test the frictional properties of the road surface at different slip ratios, tire pressure, and speeds. It is a direct method to measure the friction coefficient of the pavement surface.

3.3. Harmonization of pavement friction measurement

With the above discussion, it is very clear that although there exists a number of skid resistance measuring tools but operating principles of many of them are similar and can be categorized under only a few. Various measuring devices can be grouped together based on the slip ratio and slip angles of the test tire. Nevertheless, establishing a relationship between these test methods is still very difficult [30]. The application of the international friction index (IFI) and skid resistance index (SRI) is of prime concern in the USA and European countries [31]. Looking at skid resistance measurement using smooth tires and ribbed tires installed on a locked wheel skid tester, it was observed that IFI is not appropriate for friction measurement as results obtained from the friction speed curve is different for ribbed tire and smooth tire [33,55]. SCRIM and locked wheel skid tester fitted with a ribbed tire were found highly correlated,

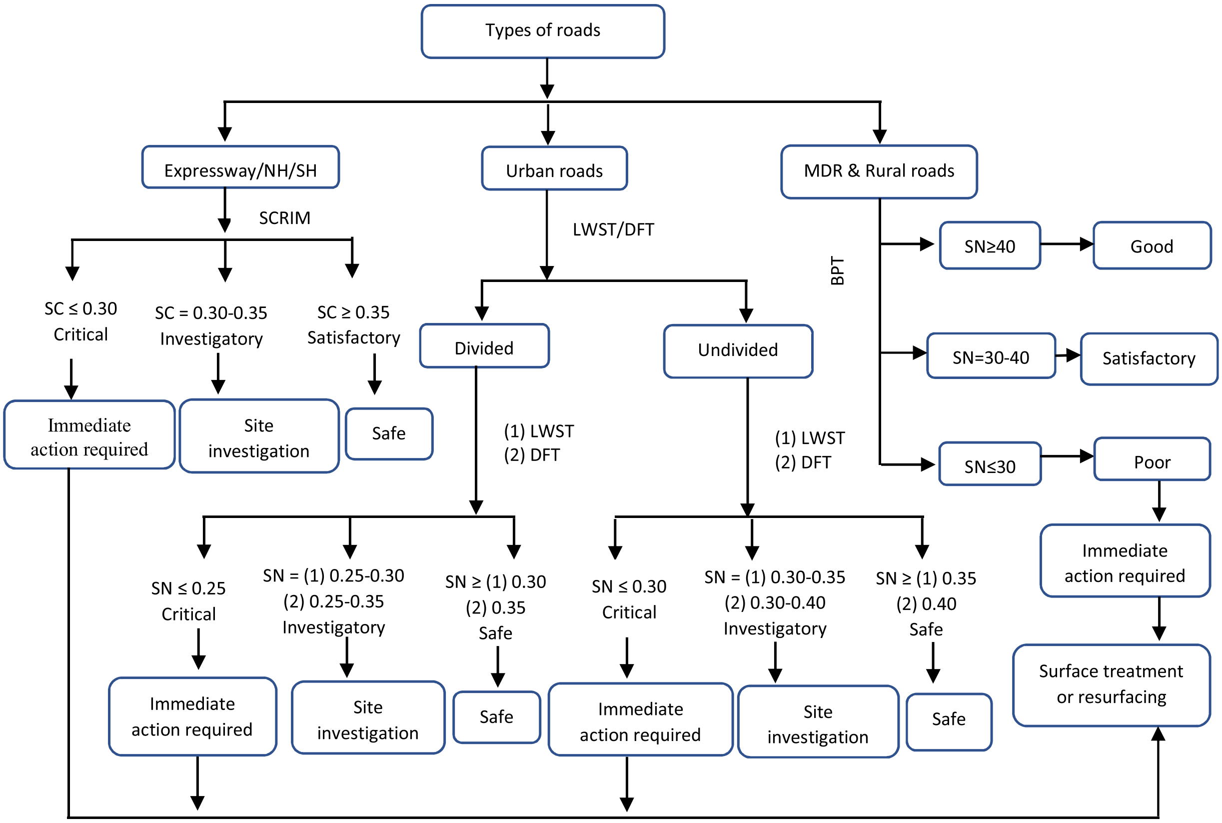

Fig. 2. Schematic diagram of skid resistance management system in India.

Table 4
Summary of field measurement methods of pavement friction [6,7,20,26].

Measuring method	Test principle	Test equipment	Advantage	Disadvantage
Sideway friction	It measures the average side force perpendicular to the rotational plane to quantify the sideway friction coefficient.	SKM, SCRIM, Mu-meter	The test can be done on complicated sections like curves and T sections. It performs continuous measurements.	The test wheel is prone to damage by pavement unevenness due to distinct slip angles.
Locked wheel method	Skid resistance is computed by measuring the resistive drag force and the wheel load applied to the pavement.	A locked wheel skid tester is used. It is connected behind a toe vehicle. Both ribbed or smooth tires can be used in a trailer.	It is simple, has less time-consuming performance, and user-friendly interface for operation.	It has test restrictions on curves, T sections, or roundabouts. It may miss slippery section data due to discontinued testing. It has a high operating cost.
Fixed slip method	The test is performed at a pre-set slip ratio, tangential rotation of the test tire is partially locked.	GripTester, SFT, RoadSTAR, ASFT, RFT, Skiddometer BV-11, ROAR	It performs continuous measurements. High-resolution friction data is obtained.	The difference between the slip speed of the device and critical slip is observed on snow surfaces. Long-distance measurement needs a high-volume water tank.
Variable slip method	Tire speed, vehicle speed, travel distance, friction force, and upper loading are recorded. The curve for the relation between slip ratio & skid number is obtained.	IMAG, RUNAR, SALTAR	It provides a continuous measurement facility. Friction can be measured at any slip ratio. The shape factor for the Rado model can be obtained.	Large test facility and complex in nature. High maintenance cost. Data processing and analysis are complex. A high-capacity water tank is needed.

Table 5
Recommended British pendulum number (BPN) for major district roads and rural roads.

Category	Site description	Good	Satisfactory	Poor	Intervention level (BPN)
A	Straight roads	≥ 40	40–30	≤ 30	30
B	Curves with tight radii & intersections	≥ 45	45–35	≤ 35	35

which do not vary with mean profile depth [44,56,57].

Harmonization is used to adjust the skid resistance measurement values reported by different equipments so that they all present the same value for similar pavement conditions [6]. Taking results of DFT and Grip tester as representative values, spot and continuous measurement of skid resistance data have also been studied [58]. It was observed that under different operating speeds, the friction number from the Grip tester is highly correlated with the friction number from DFT. Grip tester has a wide range of test scope and traffic control is not required during the measurement, therefore it can be used as a substitute for DFT. For establishing correlation analysis of the various friction testing devices mainly SCRIM and locked wheel skid tester, six highway sections were chosen for comparative study under different test velocities and repetitive runs [43]. It was found that the sideway friction coefficient from SCRIM is uniformly higher than friction number values from the locked wheel skid tester [44,46].

Testing of pavement friction using field measurement methods has certain advantages of high accuracy and efficiency however reliability of test results is affected by weather conditions and variations in measuring highway sections [32,59]. The test result may also be affected by the extent of surface texture polishing and wearing of asphalt mix due to traffic on any pavement section. Few studies have tried to establish a correlation between field test results and lab test results of skid resistance data, in an attempt to estimate the actual frictional characteristics of the pavement surface in the lab design stage [60].

4. Modelling of skid resistance

Although skidding characteristics of the pavement surface can be evaluated directly in the lab or field however experimental methods pose certain limitations and restrict its utility to a much wider framework. It is laborious, time-consuming, and very difficult to precisely capture the real-time contact pressure distribution on tire pavement interface for a better understanding of the pavement anti-skidding mechanism [61,62]. Test accuracy and repeatability of measurement using lab or field tests are also affected by operating conditions such as ambient temperature and wind speed. Therefore, modelling techniques have also been used as the essential computational method for pavement surface skid characteristics. Both analytical modelling and numerical modelling techniques are used for further investigation of surface skid characteristics and have been discussed in brief to gain insights into surface anti-skid performance.

4.1. Analytical modelling

Several factors affect the skid resistance of pavement and are needed to consider for an accurate estimation. It is very difficult to consider all factors in one model [63]. On the basis of various research perspectives, the skid resistance model can be grouped as pavement texture and speed-based, tire and rubber material-based, traffic volume, asphalt mix, and aggregate properties based.

4.1.1. Texture and speed-based modelling

Current practices for estimating skid resistance are mainly based on pavement textural characteristics and traffic volume. It has been observed that the accuracy of the prediction model can be considerably improved by taking into account both the texture parameters, macro texture and micro-texture in the analytical model. Early developments explain the Penn state model (PSU model) [64] as the widely used for modeling frictional properties at tire pavement contact. It relates pavement macro-texture to skid resistance. The PSU model constants are based on pavement surface texture and it estimates the variation of skid resistance with tire speed. The Penn State model is expressed by the following statistically developed exponential equation:

$$SN = SN_o e^{cv} \tag{10}$$

where SN (skid number or friction coefficient) represents skid resistance, SN_0 is skid number at 0 tire slip speed, v is slip speed, and c is the model constant that is independent of speed and is related to surface macro-texture [65]. However, the Penn State model and its derivate international friction index (IFI) have certain drawbacks which question its consistency [7]. For instance, the friction index measured at 80 km/hr slip speed from the same measuring device at two different speeds on the same pavement shows considerable variation. Further, the assumption of linearity between friction index and measured friction may not be valid in some cases, especially at higher slip speeds [6]. Karnopp [66] presented a different method that reduces the order of dynamic systems every time so that the relative speed is zero. Dynamic contact problems containing slip-stick friction can be effectively modelled using this method.

Furthermore, two similar models for simulating slip-stick friction, the reset integrator model and the bristle model, were proposed [67]. These models consider the dynamics of interacting bodies based on their principal mechanism. The former is a simple model with higher efficiency than the bristle model. As per the bristle model, friction is defined as the interaction between randomly located pliable bristles. In addition, Haessig et al. [67] showed that the bristle model is a suitable selection for simulating friction-based dynamic processes. On the other hand, the reset integrator model uses relative velocity (V_r) as the only parameter to estimate the friction coefficient.

LuGre model [68] has been recommended widely as it considers the dynamics of interacting bodies. Similar simulations of friction coefficient over relative velocity have been proposed and executed in the Dahl model [69]. LuGre model describes interface friction as an assembly of bristles indicating surface asperities. It simplifies the dynamics of 3-dimensional tire friction by assuming constant slip as well as the constant area of contact. The impact of lateral deformation of tire tread, which results in varying slip speeds with the tire pavement contact area, has been included in this model [70]. It demonstrates significant parameters that represent the prime attributes of tire pavement friction. The average deflection of the bristles and the friction force can be derived using the simplified form of the

LuGre model [71].

A very similar model to that of the PSU model (Rado model) investigated the variation of friction coefficient with the varying slip speed and developed an analytical model to explain the wide range of the friction coefficient as a function of slip speed. The model can further be used to understand the effect of surface micro-texture, macro texture, and tire properties on friction [72]. A study done by Serigos et al. [73] using field measurements of surface texture consisting of a number of texture-dependent parameters determined spatial and spectral domain concluded, that micro-texture can be visualised by the slope and y-intercept of a linear power spectral density function, facilitating the accuracy in skid resistance modelling. Speed and surface texture were considered in the above models for modelling skid resistance however wheel pressure and tire rubber temperature were missing. Wriggers model developed in this series served the purpose which estimates friction at different normal pressure and the corresponding speed [74].

4.1.2. *Tire-based modelling*

Skid resistance developed at the tire pavement interface is broadly a function of tire material properties, operating parameters of the tire (temperature, speed, slip ratio, etc.), pavement surface texture, and other environmental factors. LuGre model, Brush model, and Pacejka "Magic formula" are commonly in use for tire-based modelling for estimating the frictional behaviour of tires. The Pacejka "Magic formula" for tire modelling is suitable for a vast range of tire constructions and operating conditions. The general form of the Magic formula is given as:

$$F(\alpha) = D\sin(C\arctan(B(1-E)\alpha + E\arctan(B\alpha))) \tag{11}$$

where α is a slip parameter resulting in slip force F, and B, C, D, and E are model constants.

Different forms of analytical solutions can be developed for expressing slip force generated under different vertical loading conditions, tire slip angles, and camber angles [75]. With the knowledge of tire rubber material response, analytical modelling of frictional characteristics of tire rubber has gained significant accuracy. For example, Moore [76] proposed a binomial friction model, and Persson [77] proposed a fractional friction model for simulating the frictional properties of tire rubber. Grosch [78] and Savkoor [79] proposed a frictional model as a function of sliding speed and temperature, considering the effect of tire rubber material. However, analysing the skid resistance model restricted to tire properties only is of the limited application of the skid resistance model, since it alone cannot actually represent tire pavement contact behaviour.

4.1.3. *Traffic, asphalt mix, and aggregate properties-based modelling*

Current research practices for modelling skid resistance are not limited to tire rubber and surface texture but the effect of traffic

Table 6
Analytical skid resistance model.

Skid resistance model	Comments
The Penn State model [64]	It explains skid resistance variation with slip speed by means of two parameters: the skid resistance intercept at zero speed and a skid resistance gradient which describes the shape of the curve. Skid resistance at any speed can be estimated using surface texture parameters. It forms the basis of the International Friction Index (IFI).
The Rado model [72]	The model has been developed using PIARC experimental data. It presents the variation of slip speed data at a logarithmic scale with critical slip speed and shape factors. Critical slip speed & peak friction fixes the location of maximum friction. The prediction of skid resistance is based on the surface texture of the pavement and tire material properties. The model can explain the variation of skid resistance at different slip speeds.
Wriggers model [74]	In this model frictional characteristics based on the input parameters such as the maximum friction coefficient, vertical pressure, and speed is considered.
LuGre model [68]	It assumes a constant tire slip ratio to simplify the transient response of 3-dimensional tire friction. The model takes into consideration the effect of velocity and lateral deformation of tire tread to model and simulate frictional hysteresis. The model can also simulate slip-stick friction.
Bristle model [67]	The physical conceptualisation of the proposed model lies in a pair of surfaces facing each other with bristles extending from each face. This model can simulate friction-based dynamic processes but can be proved ineffective due to fine spacing between bristles.
Reset Integrator model [67]	The model considers relative velocity as an input parameter. Slip-stick friction is modelled as a function of relative velocity when the pavement surface and tire being simulated are slipping with respect to each other.
Dahl model [69]	The model is theoretically related to magnetic hysteresis. It incorporates dynamic effects in terms of speed in the proposed model. Variation in contact area forms the basis of a wide range of friction. Rolling friction is a result of tension and compression at tire-pavement contact and shearing force is the result of the tire sliding at the pavement surface.
Pacejka tire model [63]	The model is suitable for capturing longitudinal and lateral dynamic responses of the contact systems. Its application is restricted to relatively low time and path frequency. Commonly applied for analysing the frictional behaviour of tire rubber in vehicle engineering. Calibration of the model is tedious and time-consuming as it needs a huge amount of experimental data. Model parameters are random and have no physical significance.
Brush model [147]	This model assumes that the aligning torque, lateral force, braking force, and vertical load can be measured directly or predicted using some other models. The horizontal shear stress acting from the pavement to the tire is assumed to be linearly dependent on the relative displacement of the point of action of shear force with respect to its undeformed location. It presumes a number of bristles, which can be modelled as elastic components aligned with the tread base.
Binomial friction model [76]	Employed for the modelling of rubber friction and can be used to estimate kinematic friction for a tire rolling on a rough and hard surface. Input for the model is elastic modulus and information about the roughness of the road surface.
Fractional friction model [77]	It estimates frictional characteristics of various tire rubber as a function of sliding speed and temperature, considering pavement surfaces as rigid and hard. To neglect the frictional loss due to heating, the sliding speed is kept small. Friction was found to increase initially to a maximum as sliding speed increases and then it starts decreasing.

volume, aging of surface, mix type, and geological properties of aggregates are also being taken into consideration [80]. The magnitude of skid resistance, one obtained from regression analysis of cumulative traffic volume data and the other predicted from the skid resistance model can vary with each other although having the same material and equal traffic volume. This variation can be due to differences in loading distribution and maintenance history [81]. Masad et al. [82] studied the effect of aggregate texture, asphalt mix properties, and critical environmental conditions on skid resistance and proposed a model for its quantification. The type of aggregates was found to affect skid resistance significantly. Sandstone and quartzite can provide better skid resistance than gravel.

Considering the fact of non-linear variation of skid resistance with pavement age, few significant factors affecting skid resistance such as traffic load, pavement aging, area exposed by raveling and rutting cracks, % of aggregates passing 2.36 mm sieve, and % of aggregates retained on 4.75 mm sieve were considered [83]. Table 6. presents the important analytical skid resistance models.

4.2. *Numerical modelling*

Analytical solutions are good for a rough estimate of pavement friction, however, when dealing with contact modelling at tire pavement interface where large deformation of tire body and dynamic loading is involved, it is challenging to find analytical solutions [61]. Therefore, to deal with such complex models, numerical solution techniques gradually evolved. Usually, the finite element method is widely employed to deal with tire pavement friction. Estimation of pavement friction with higher accuracy and reliability needs a good representation of the tire and the pavement model in such analysis.

4.2.1. *Modelling of tire*

The complexities of a tire that contains the components such as tire tread, sidewalls, belts, plies, and inner-liners make the modelling of such a structure a very challenging task [84]. Large deformation, incompressibility, viscoelasticity, material nonlinearity, and its rolling/skidding contact with the pavement surface further make the numerical analysis more complicated. Zorowski [85] was among the first to utilise a FE modelling technique to analyse the dynamic behaviour of a tire. Following his work, Ridha [86] proposed a linear FE model to relate the deformed shape of the tire as the result of shrinkage to its original mould shape. In these studies, due to the symmetry of the tire geometry, 2D tire models were usually generated. However, applications of the 2D tire model are limited to central symmetrical loading/boundary conditions such as pressure inflation and rim mounting.

Oden et al. [87] modelled a 3-dimesional cylinder, a deformable body in steady-state rolling on rough surface assuming viscoelastic rubber material. Farroni et al. [88] presented a 3D mathematical-physical tire model to predict both the steady-state and transient behaviours of a tire. The coupling of multibody dynamics of the physical model can improve its capability further. Implementation of thermal, wear, and grip models can be taken into account for further analysis. Padovan et al. [89] developed a finite element scheme enabling the simulation of rolling tires. Padovan used the concept of moving Lagrangian to 3-dimensional isoparametric elements. The 3-D elements were used to model the tire tread, however, the inner components of the tire like a carcass, belt, and ply have been omitted from the study which can affect results considerably.

Recently, Srirangam [90] has investigated many complicated aspects of a tire simulation e.g., a tire in rolling/sliding/braking status. In this analysis procedure, finite element techniques such as generating a 3D tire model from the corresponding 2D schemes, the inclusion of the complex tire tread patterns in a virtual way, the use of the surface elements to simulate reinforcements, and the advanced contact modelling capabilities have enabled the analysts to study the mechanical behaviour of the tire during various operating and loading conditions in an easier and more accurate way.

4.2.2. *Modelling of asphalt pavement surface texture*

A simple approach for surface texture modelling is to transform the longitudinal road profile obtained from the laser profilometer into the frequency domain [70]. However, as the profile reduces to a spectrum, the pavement surface characteristics are idealized which may lead to the loss of data connecting pavement surface characteristics to skid resistance. Fractals have been used in pavement surface texture modelling as well. A fractal is a curve or pattern, each part of which has a similar statistical character as the whole. It has been shown that pavement surface texture can be described as a fractal surface. Profiles exhibiting self-similarity and the same appearance for any magnification can be characterised effectively by using fractal analysis [65]. Fractal texture models were developed based on the spectrum of a surface profile and can be used to describe the surface micro, macro, and mega texture in a multiscale [91,92].

The close-range photogrammetry (CRP) technique is another tool that can be used for surface texture modelling as well as 3D models of pavement surfaces. The procedure involves taking stereo image pairs using a high-resolution camera. The obtained sets of stereo images are then used to construct a 3D surface model. Close-up images can be taken to retain surface texture at the micro-scale. The resolution of the surface mesh is dependent on the size of the area under investigation.

CT-scanning technique combined with Finite element (FE) methods has also been used for simulating pavement surface morphology. Zhu et al. [93] used 3D optical scanning to capture the surface texture of the asphalt concrete samples. By post-processing, the coordinates of the surface points, and the 3D surface textures of different asphalt pavements can be displayed. However, this methodology neglects the inner microstructure of the asphalt mixture which is usually distinguished as air void, aggregate, and mastic. Most recently, Srirangam et al. [94] used the X-ray computed tomography method to acquire data pertaining surface morphology of the asphalt pavement. Advanced 3D image processing tools like Simpleware can be effectively used to capture surface texture images, and micromechanical finite element meshes can be produced for various types of asphalt concrete pavements.

Despite several studies conducted in past, the relationship between surface macrotexture and skid resistance is not completely understood. Few studies in last decade implemented 3D texture parameters or wavelet analysis for the prediction of pavement skid resistance. Wavelet is an irregular and asymmetric waveform within limited duration having average value of zero. It represents signal in frequency and time domain simultaneously. Yang et al. [95] implemented discrete wavelet transform to decompose pavement macrotexture profile into multi-scale characteristics and found suitability for surface friction prediction. Total energy and relative energy were calculated from decomposed macrotexture profile to represent characteristics of macrotexture at various wavelengths. Subsequently, multi-variate regression analysis was conducted to show relationship between pavement skid resistance and energy indicators derived from macrotexture data. Li et al. [96] implemented a 2D discrete wavelet transform to decompose surface texture at micro and macro level. In this study total energy and normalized energy was used as the wavelet-based indicators. Deng et al. [97] conducted a multiscale power spectrum analysis of a 3D surface to predict asphalt pavement friction. Pavement surface friction and texture data were collected in parallel using BPT and a portable ultra-high-resolution 3D laser scanner. Considering the effective contact area, friction was predicted as a function of surface power spectrum indicator under optimal micro texture and macrotexture wavelengths.

4.2.3. Tire-pavement interaction modelling

The FE method is the most widely used theoretical approach intended to provide approximate solutions to engineering problems. The popularity of FE method lies in its high computational efficiency in solving complex problems. It facilitates modelling the tire-pavement interaction for skid resistance-related problems with great accuracy. In the early years, internal codes such as NOSAP [98] and AGGIE [99] were used to model constitutive behaviours of rubber-like materials, but they were far from adequate in resolving the complexities associated with the tire-pavement interaction mechanism. The continuing increases in computing power and advances in numerical techniques have enabled the FE method to simulate complex phenomena such as frictional contact at the tire pavement interface. Many popular available FE tools such as ANSYS, ADINA, and ABAQUS have been widely used by both tire and pavement researchers [100].

Tires rolling/sliding over a dry smooth pavement have been modelled in past studies [101–104]. However, the primary focuses of these studies were on the mechanical behaviour of tires and little attention was paid to skid resistance. Cho et al. [105], developed a numerical–analytical solution for estimating the energy loss of a passenger car tire rolling on a flooded smooth pavement due to skid resistance. Based on NASA hydroplaning equation, Ong et al. [106] and Fwa et al. [107] performed the simulation of a locked wheel on the flooded pavement. The outcome of the analysis indicates, that with increasing sliding velocity; hydroplaning risk increases, and

Table 7

Overview of numerical solutions of skid resistance and model capabilities.

Parameters considered	Remarks
Surface texture, tire tread & operating conditions	Srirangam et al. [115] presented a 3-D finite element model to evaluate the surface macrotexture morphology effect on tire rubber friction due to loss of hysteresis. Analysis was carried out based on mix morphology, various pavement stiffness, tire load, and sliding speed conditions. A finite element model was developed by Peng et al. [116] to evaluate tire pavement interface friction by reconstructing the pavement surface model from high-resolution texture data. For deriving tire pavement interface friction, a binary search back-calculation method was used. For establishing the relation between surface texture and frictional characteristics, the PCA regression model was introduced.
Inflation pressure, load, speed	Thermo mechanical tire pavement interaction model [62] was developed. Analysis was carried out considering the effect of tire inflation pressure, tire load, speed variation, and temperature on skid resistance. Model capabilities for evaluating skid resistance was also presented using different tire size and configuration.
Hydroplaning	The fluid-structure interaction (FSI) model [117] was developed. The model presents the effect of tread depth, tire load, inflation pressure, slip speed, and water depth on skid resistance. The proposed model simulates the hydroplaning effect of rib trick tires on skid resistance. The model was validated using experimental data. Finite element-based software ABAQUS was used to develop an inflated tire model [118] to study the hydroplaning effect on different pavement surfaces of asphalt concrete, stone mastic asphalt, and open-graded friction course pavements. Surface adhesion coefficients were analysed under wet and dry conditions using coupled Eulerian-Lagrangian method and liquid surface tracking technique.
Stress distribution and slip ratio	Guo & Zhou [119] analyzed the tire pavement interface stress distribution and slip ratio were compared during braking and acceleration. Developed a tire pavement contact model using ABAQUS. The proposed model can effectively use the friction conditions and stability of vehicular manoeuvring.
Surface temperature, tire temperature, and environmental factors	Anupam et al. [11] utilized a thermo-mechanical contact algorithm. Model presented has the capability to evaluate the effects of surface temperature, environmental temperature, contained air temperature in the tire, texture characteristics, and tire slip ratio on pavement friction. Srirangam [120] developed coupled thermomechanical finite element model for estimating temperature rise in the tire. The effect of temperature rise on hysteretic friction was evaluated. Model capabilities include estimating the effects of the surface texture of pavement, tire temperature, and tire slip ratio on hysteretic friction.
Water film thickness	A simulation was performed using CFD (Fluent) [121] to obtain the critical effect of surface roughness on frictional characteristics and load-bearing capacity of the pavement surface. Variation in water film pressure with surface roughness was simulated and studied. Tang et al. [108] developed a tire-pavement model coupled with water interaction at the interface. It can estimate wet skid resistance at various rainfall intensities. The proposed model considers the effect of tire tread design, variations in pavement geometry, and different operating conditions of the tire. The proposed model captures the actual surface texture of the asphalt concrete mix.
Asphalt mix	Frictional characteristics of porous and nonporous pavement surfaces were evaluated by proposing a new framework of analysis [8]. Numerical simulation coupled with mechanistic modelling of skid resistance was conducted to develop the proposed framework considering the critical effect of wet weather conditions.

normal force between tire and pavement decreases thus skid resistance decreases. Tang et al. [108] proposed a finite element model for estimating wet skid resistance at different rainfall intensities. The effect of tire operating conditions, tire tread design, and pavement geometric design were mainly considered in the proposed model, and concluded that to ensure greater safety against skidding, a higher cross slope shall be provided. Additionally, an increase in rain intensity decreases skid resistance significantly.

Wang et al. [109] developed a model for simulating tire-pavement interaction to analyse the contact stresses and forces during vehicle manoeuvring. For investigating the structural response of the layered inelastic pavement, Wollny et al. [110] proposed a tire-pavement interaction model based on FE analysis.

However, most of the proposed models simulated the pavement sub-model considering it as a rigid flat plate that neglected the pavement surface characteristics. Anupam [111] proposed a finite element model for simulating asphalt concrete pavement as a grooved surface to analyse skid resistance. However, it simulates the tire sub-model as a non-rotating body which is a major limitation of this model. Additionally, few other studies investigated the pavement responses i.e., the stresses and strains developed within pavement materials under static or dynamic loads [112–114]. Although complex constitutive pavement material models were adopted in these models, the tire loading was idealized and represented by a uniform load patch. Such an unrealistic loading condition inevitably renders the capabilities of these models to accurately assess skid resistance. Table 7. summarises the recent various numerical solutions of skid resistance at tire pavement interface utilising finite element analysis.

4.3. Use of machine learning for skid resistance modelling

Machine learning can be used to solve highly complicated relationships between inputs (surface polishing, material properties, mix proportions) and output variables like surface texture and roughness. Machine learning (ML) algorithms in recent years has provided means for developing skid resistance models. The artificial neural network (ANN), one of the most powerful ML algorithms has been successively used for evaluating surface roughness of the pavement. The ANN model may be used for predicting time dependent friction and surface texture of the pavement using available experimental data from accelerated pavement polishing studies. Rajabipour and Yoon [122], developed an ANN based model for predicting long term friction and texture of concrete pavement. Using the ANN model, the long-term frictional properties of concrete pavement beyond accelerated testing range were estimated. Marcelino et al. [123] used a Python ML library, Scikit-learn to predict asphalt concrete pavement friction. Panahandeh et al. [124] proposed three classification models to frictional properties of road surface. The proposed models (logistic regression, support vector machine, and ANN) were evaluated under different setting including forecast time horizon, feature vector, and number of hidden layers. ANN method was found most stable under given conditions. Hu et al. [125] used light gradient boosting machine (LightGBM) algorithm with decision tree as the base learner for evaluating pavement skid resistance performance. Proposed ML algorithm was used for correlation analysis between 3D texture features and skid resistance.

4.4. Overview of skid resistance modelling in developing nations

Most of the advancement in numerical modeling of skid resistance as discussed earlier is an outcome of extensive research from highly advanced computational laboratories of developed countries. On the other end, the development of the skid model for facilitating advanced computations in developing countries is far beyond their requirement. India has the second largest road network in the world and tops with the highest death from road accidents and needs a comprehensive maintenance framework based on computational data.

Limited research studies are available for the computations and solutions of skid resistance in India. Studies are mainly focused on developing OGFC as wearing courses using oxygen furnace slag [126], copper slag, and reclaimed asphalt pavement [127]. Skid resistance models are derived from regression techniques based on BPT skid resistance measurement data [128]. Past studies are mainly focused to improve pavement surface roughness for ensuring a better skid-resistant surface. Similar studies are available in other developing countries like Iran, Brazil, and Indonesia. The effect of various waste materials like crushed glass [129], the effect of temperature for different asphalt mixes [130], and pervious macadam [131] on skid resistance were studied. Few studies in Brazil used the aggregate image measurement system (AIMS) [132] and the international friction index [133] for the computation of skid resistance. The development of techniques for improving skid resistance based on asphalt mix modification by other waste materials has been at the centre of the study of many researchers in Indonesia also. Some of the studies include the effect of short coconut fiber [134], the effect of gap-graded HMA with added crumb rubber [135], the effect of nano crumb rubber [136], and the influence of buton asphalt additive based on penetration index and temperature [137] on skid resistance.

5. Recent advancements in skid resistance measurement and modeling

Few efforts have been made in recent studies to develop some advanced, stable, and computationally efficient skid resistance models, which are discussed in this section briefly.

5.1. Texture-based modelling

Hu et al. [138], studied the effect of 3D macro-texture on skid resistance characteristics of open-graded friction course (OGFC-16 and OGFC-13) and asphalt concrete mix (AC-13 and AC-16). A British pendulum tester was used in this study to measure the friction coefficient of the specimen surface. An optimized Bayesian-Light Gradient Boosting Machine (LightGBM) model [138] was trained to predict the skid characteristics of the OGFC and AC mix. It was found that the OGFC-13 gradation type has the highest roughness and skid resistance followed by OGFC-16. The LightGBM algorithm has higher generalized stability and improved computational efficiency than other conventional regression models.

Liu et al. [139], used the concept of fractal dimension measurement for the evaluation of surface skid resistance. Fractal dimension is a good indicator of pavement surface texture. The differential box-counting method was used in this study to count the fractal dimension of 3-dimensional pavement surface texture based on the grid displacement mechanism.

Lu et al. [140], proposed a convolutional neural network to establish the relation between pavement surface texture and in-situ skid resistance. Effective surface texture in contact was extracted and analyzed to obtain a relation with skid resistance. Frictional characteristics of the pavement surface were measured using a British pendulum tester. Results of the analysis indicated that surface texture with a wavelength of more than 2.4 mm is the key to wet friction.

5.2. Wear and tire-pavement contact-based model

Kane et al. [141], proposed a skid resistance model to count for the polishing effect of the traffic. The polishing effect of pavement surface combines a wear model and tire pavement contact model. The wear law in the proposed model considers the aggregate type and resultant pressure distribution. The tire pavement contact model considers the tread material of the tire, operating conditions of the tire in contact, surface texture, and the presence of water. The “Wehner-Schulz” is used to measure the skid resistance of three different pavement surfaces.

5.3. Aggregate gradation, polishing, and traffic-based modelling

Zhan et al. [142] measured the skid characteristics of the pavement surface using LS-40 s and BPT to develop an integrated fast Fourier transform (FFT) and an extreme gradient boosting (XGBoost) framework [143] for the prediction of skid resistance. Four different models, the Multiple linear regression model [50,58,144], the Decision tree regression model [145], the Random Forest regression model [146], and the XGBoost regression model were developed for the modeling of skid resistance. Among these models, the Multiple linear models were found to be best followed by the XGBoost model.

6. Current challenges, opportunities, and way forward

There has been significant improvement in infrastructure quality to cater high traffic volumes in densely populated developing countries, like India. Modern infrastructure development ensuring a high-speed road network brings certain challenges with it. The design speed of the road and riding comfort are increasing as we are switching from conventional methods of construction toward expressways and high-speed roads. The stopping distance of the vehicle increases with an increase in speed which poses threat when a vehicle needs to stop suddenly. Looking into these concerns of pavement safety in terms of skid resistance, a pavement management system (PMS) needs to be developed for better management, maintenance, and planning for safe operation.

6.1. Current practices of skid resistance measurement and challenges ahead

Pavement friction is affected by several parameters. Surface macrotexture is one of the factors which contributes significantly to controlling friction. Macrotexture is found to be depleted over time with passing traffic and environmental degradation. Binder in asphalt mix is slightly viscous allowing aggregates to sink through it, especially in hot weather Indian conditions and passes of heavy loads. It brings the binder to the top which reduces surface friction over time (IRC:134–2022). Periodic treatment of road surface to restore surface macrotexture is necessary to maintain sufficient friction.

Developing countries have been late runners in developing and maintaining a PMS which can evaluate pavement conditions for its timely rescue. India follows the guidelines of IRC:82–2015 “Code of practice for maintenance of bituminous road surface” for defining serviceability indicators of different categories of roads. These indicators in terms of skid resistance have been specified for highways and urban roads adopted from ASTM-274. There was no specific standard code of practice till October 2022 which underlines investigatory level and threshold level skid resistance for different categories of road sites as listed in Table 1 to Table 3. Recently in October 2022, looking into the impact of skid resistance on road safety, IRC released a document IRC: 134–2022 which separately discusses types of measuring instruments to be used for various categories of roads. It further suggests investigatory level and threshold level skid resistance index values for various sites like intersections, crossings, curves, roundabouts, and divided and undivided roads. SCRIM skid coefficient for expressways/national highways/state highways, locked wheel skid tester, DFT coefficient for urban roads, and BPT values for rural roads are recommended.

6.1.1. Unavailability of advanced equipment and technology

It can be noted that for skid resistance measurement in developing countries, still, a conventional approach to the data collection system is adopted. Research studies related to available skid resistance at tire pavement interface, measuring technics as per the individual site requirements, and modelling efforts have been missing. Evaluation of skid resistance weather by field measurement or by developing a suitable model was not considered under PMS until the last decade. For a country like India where there is a great variation in road construction materials, type of roads, speed, and traffic, a specific methodology needs to be developed based on the location it is being to be used. The impact of skid resistance on road safety is well understood but the speed of technological advancement for its assessment is equally crucial to plan for suitable mitigation measures.

Although the use of advanced and computationally efficient tools like SCRIM has been well recognized and recommended by IRC:134–2022 for skid resistance measurement on expressways, national highways and state highways however need for specific tools based on various road categories, speed, and traffic is still there.

6.1.2. *Unavailability of sufficient scientific data*

For any developing nation to develop an efficient PMS for restoring sufficient skid resistance, the three basic challenges are reliable equipment for data capture, the sufficiency of data for every site as discussed in Tables 1–3, and a condition-specific index for presenting skid resistance data. A large-scale skid resistance data collection for each road class, speed class, curve, and roundabout is important for developing the skid resistance index.

6.1.3. *Development needs of skid resistance indices (SRI)*

The use of skid resistance indices has enabled the harmonization of the different sensitivities of the various skid measurement principles to micro-texture and macro-texture. Currently, developing countries are widely using skid numbers produced by ASTM 274 and SCRIM coefficient. The issue with data presented by skid number only considers the speed at which the test is conducted. Other factors like test temperature, tire type, inflation pressure, etc. are not considered which can affect data accuracy. Many developing countries have revised their skid measuring methodology and have moved from skid number to SCRIM coefficient. This facilitates data correction for temperature and speed. it is an advancement over the skid number test. Still, a great challenge lies in developing a comprehensive and more capable index that can be corrected for other influential factors like tire type, inflation pressure, contained air temperature, etc.

6.1.4. *Numerical simulation of skid resistance data*

Studies discussing various analytical and numerical modelling efforts have been reviewed based on surface texture, speed, tire tread, tire-pavement contact, and asphalt mix. However, research studies in these domains are still very limited in developing countries as discussed earlier. A computationally efficient system needs to be developed for the numerical modelling of various aspects of skid resistance at the tire-pavement interface. The development of an experimentally validated skid resistance model is a serious challenge, especially for warm surfaces. Numerical modelling of tires becomes tedious as rheological and geometric characteristics are complex to

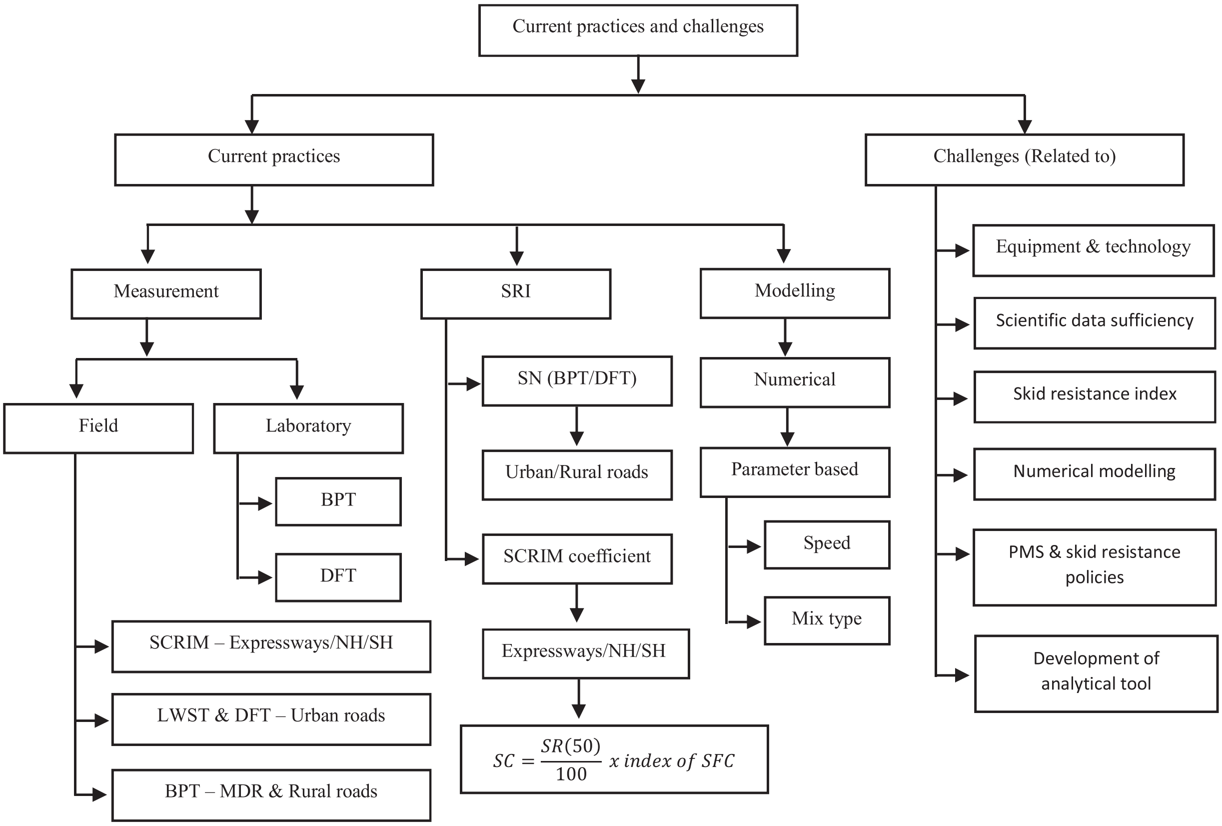

Fig. 3. Flowchart for current practices and challenges from the perspective of a developing nation. (LWST – Locked wheel skid tester, MDR – Major district road, SN – Skid number).

input. Numerical modelling of complex environmental parameters like temperature, rain intensity, wind speed, etc. needs to be simulated in a future study to quantify more robust and reliable skid data. Most of the test data available is a part of extensive work done against hot mix asphalt. With the growing knowledge, advancement, technical benefit, and field application of warm mix asphalt (WMA), a more extensive study for skid resistance modelling of WMA is required. A flowchart of current skid resistance measurement practices and challenges has been presented in Fig. 3 for quick review.

6.2. Opportunities and way forward

For developing countries having large road networks and a great variety of traffic, speed, construction material, and tire rubber-like India, Brazil, Indonesia, and Iran, there is a great scope of research in developing skid resistance measurement tools, skid resistance index, and numerical modelling of complex parameters.

6.2.1. Methodology for skid resistance measurement

IRC guidelines for measuring skid resistance are mainly based on BPT, DFT, and SCRIM. Continuous research in this field is required to develop a more comprehensive measuring equipment that considers Indian road conditions, source and properties of road construction materials, climate, and speed. India has the 2nd largest road network in the world which needs a large-scale PMS in which a continuous measuring system needs to be developed. It facilitates large-scale data collection and can cover higher road lengths in a short duration.

6.2.2. Correlation analysis and skid resistance index

Correlation analysis between lab and field data needs to be established for Indian conditions. The correlation of skid resistance data among various measurement principles needs to be developed. It is completely missing in most developing countries. Various indices used for skid resistance measurement have been discussed in the previous section. It can be noted that only skid number and SCRIM coefficient are available for data presentation with vast scope for future improvement. More such indices based on various road categories, type of site, speed, temperature, and texture need to be developed for better pavement management systems against skidding safety.

6.2.3. Need for PMS and skid resistance policies

Skid resistance policies have to be formulated and put in place to take an investigatory approach with targeted interventions on those sites where there is the greatest potential benefit. An effective asset management strategy needs to be developed to analyse the performance of aggregate over time as it is critical in deciding skid resistance at pavement surface.

6.2.4. Development of an analytical tool

There is no analytical tool available to derive theoretically the minimum skid resistance for safe driving in wet weather. The threshold minimum skid resistance has to be specified by highway agencies based on engineering judgment, and the deterioration trend of skid resistance. The more robust approach to establishing these thresholds requires the distribution of friction data versus crash rates. To achieve these threshold limits, more crash data will need to be collected. For defining the skid resistance deterioration model, a continuous measurement system needs to be developed and practiced.

7. Concluding remarks

The present article reviews a number of past and current studies on the frictional characteristics of asphalt concrete pavement primarily based on the various aspects of measuring and modelling methods of tire-pavement friction. It can be noted that although field measurements are rigorous, labour intensive, and costly yield a better assessment of frictional characteristics of pavement than available lab measurement techniques as it covers a wide range of slip ratio, slip angle, tire speed, actual tire pavement interaction, and vehicle load. There is still a considerable gap between field and lab testing on asphalt pavement frictional characteristics. For bridging the gap between field and lab measuring devices, additional facilities need to be developed in the lab for better simulating actual field conditions to realise frictional characteristics more accurately in the process of lab mix design. Additionally, field measuring devices, irrespective of their operating principles still need further improvement for increasing measurement scope.

Skid resistance measurement in a few developing countries especially in India has been reviewed and found that BPT is the most commonly used device in countries like India, Iran, Indonesia, Brazil, and others which is extensively used for low-volume roads. For high-volume roads like expressways, national highways, and state highways skid resistance measurement by SCRIM is recommended by IRC in India. Pavement surface has been categorized based on skid number values and recommendations have been standardized in the respective codes for pavement surface monitoring and maintenance. The development of other advanced and computationally efficient measuring tools is the need of modern times to overcome the limitations of conventional BPT and to produce much faster and more reliable measurements.

Harmonization of lab and field measurements of pavement friction has also been summarised to establish the relationship between multiple variables obtained from field measurements and lab test results. The international friction index widely used for the harmonization of friction measurements was found inconsistent and has been highlighted. As the test result obtained from the friction-speed curve, trends differently for a smooth tire than that of a ribbed tire, the applicability of the IFI model for ribbed tires needs to be reevaluated. Also, a locked wheel skid tester with a ribbed tire is found highly correlated with SCRIM which is insensitive to mean profile depth. Further, under different operating speeds, the friction number from the Grip tester is highly correlated with DFT, so the Grip tester can be used as an alternative for DFT considering its wider testing scope and needless traffic control during measurement. Nevertheless, it is still very difficult to establish a conclusive relationship between these testing methods.

Modelling efforts of tire-pavement interaction for predicting frictional characteristics were reviewed based on analytical and numerical modelling techniques. Analytical research practices for modelling skid resistance are considering the effect of traffic volume, aging of surface, mix type, and geological properties of aggregates. Numerical modelling includes the development of a 3D tire model and advanced contact modelling, which enables the analysts to study the mechanistic behaviour of the tire under different operating and loading constraints. CT-scanning technique combined with FE method has been adopted in recent years for simulating pavement surface morphology. Other CT-scanning tools, conventional way of laser scanning of pavement surface, and charged coupled cameras are in frequent use for the measurement of surface texture. Recent studies have considered the effective contact of the tire with pavement surface and started developing models to capture the effective contact of texture with the tire. With the advancement in computational technologies, pavement deformations are being taken into account in many recent studies.

But the phase of progress is much slower in developing countries like India, Brazil, Indonesia, and others where developed facilities are not enough for skid-related maintenance of roads. The computation of skid resistance mainly considers the effect of various constituents of asphalt mix or modified mix with certain waste additives as discussed and is far beyond the actual scenario in situ. For better simulation of the skid resistance model, the focus needs to be shifted to other complex attributes of tire pavement interaction as being developed in a few developed countries gradually.

For further investigation of tire-pavement frictional characteristics, critical factors such as ambient air temperature, surface temperature, and temperature rise in tire components shall be given due consideration for the analysis of the temperature effect. Investigation pertaining to hydroplaning risk, wearing of the tire, pavement rutting induced by traffic movement, variation in surface texture due to polishing effect, and the depth of water can be included in the modelling to increase the accuracy of tire-pavement skidding characteristics estimation. Finally, model validation shall be kept at the center of the study to verify the predicted outcomes.

8. Future research needs

In conclusion, a few areas have been identified for research as future scope. The gap in skid resistance results measured in the field and lab should be minimised by developing features in the lab measuring tools to cover different slip ratio, tire speeds, tire surface interaction, and loading conditions. Finite element computational capabilities need to be strengthened in future research studies to analyse the effect of complex parameters like tire-contained air temperature, pavement temperature, ambient air temperature, hydroplaning risk, tire wearing, surface polishing, and rutting on skid resistance in numerical analysis. While dealing with transient loading, modelling skid resistance is very challenging as it is a dynamic characteristic of tire pavement interaction and needs a better understanding of pavement lubrication, adhesion, and wearing. For developing a skid resistance model, actual contact area at tire pavement interface shall be considered and results obtained should be tested and verified based on data received from lab and field facilities. These facets shall be considered in future studies for a more reliable model.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

Data will be made available on request.

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