Preface

The thesis contains six chapters in which Chapter one is introductory and gives a general idea of when and how a discontinuity appears. Certain terminologies commonly used in the current work have been defined. The mathematical theory and their fundamental properties have also been briefly discussed. The physical properties of non-ideal gases, dusty gas and magnetogasdynamics are briefly reviewed.

In chapter two, an asymptotic approach is used to analyse the main features of weakly nonlinear waves propagating in a compressible, inviscid, non-ideal gas in the presence of magnetic field. An evolution equation, which characterizes the wave process in the high frequency domain and points out the possibility of wave breaking at a finite time, is derived. The growth equation governing the behaviour of an acceleration wave is recovered as a special case. Further, we consider a sufficiently weak shock at the outset and study the propagation of the disturbance given in the form of a sawtooth profile. It is observed that the non-idealness of the gas causes an early decay of the sawtooth wave as compared to ideal case however the presence of magnetic field causes to slow down the decay process as compared to non-ideal non-magnetic case. A remarkable difference in wave profiles for planar and cylindrically symmetric flows has been observed. The effect of non-idealness, in the presence of magnetic field, on the formation of shock is more dominant in case of cylindrical symmetry as compared to planar case.

In chapter three, a direct approach is used to solve the Riemann problem for a quasilinear hyperbolic system of equations governing the one dimensional unsteady planar flow of an isentropic, inviscid compressible fluid in the presence of dust particles. The elementary wave solutions of the Riemann problem, that is, shockwaves, rarefaction waves and contact discontinuities are derived and their properties are discussed for a dusty gas. The generalized Riemann invariants are used to find the solution between rarefaction wave and the contact discontinuity and also inside rarefaction fan. Unlike the ordinary gasdynamic case, the solution inside the rarefaction waves in dusty gas cannot be obtained directly and explicitly; indeed, it requires an extra iteration procedure. Although the case of dusty gas is more complex than the ordinary gasdynamics case, all the parallel results for compressive waves remain identical. We also compare/contrast the nature of the solution in an ordinary gas dynamics and the dusty gas flow case.

Chapter four is concerned with the existence of solution to the Cauchy problem for one dimensional, isentropic dusty gas. Using Riemann invariants, conditions for the existence of the solution are derived. It is shown that when the said conditions do not hold, the solution break up after a finite time. The dependence of break up phenomena on the parameters of dusty gas viz. mass fraction, volume fraction and specific heat of solid particles, is also observed.

In chapter five, the FORCE scheme has been used for the numerical solution of two dimensional shallow water equations with variable bottom geometry. Firstly, the one dimensional equations are solved and the source term is treated using time operator-splitting. The method is then extended to two dimensional problem using space operator-splitting. The method is applied to the various test problems in one and two dimensions. The results obtained are validated with the earlier works. It was observed that the FORCE method works faithfully to the test problems considered but source term must be treated according to the geometry of the problem.

In chapter six, a high resolution Godunov type method is used to solve the one dimensional Euler equations for non-hydrostatic gravity driven flow. The Riemann problem is solved by using the HLLC Riemann solver suggested by Toro, Spruce and Spears. The implementation of the HLLC Riemann solver with operator splitting for the numerical solution is described in detail. The scheme is then validated against the benchmark problems with gravitational potential. The test problems considered show that the scheme gives promising results.