

Preface

The study of nonlinear science has evolved into a new era in last few decades, with fundamental theories established and some important results in the areas of chaotic dynamics such as chaos control, chaos synchronization and ultimate boundedness of chaotic attractors.

This thesis addresses the synchronization of chaotic systems in integer with delay as well as with fractional order derivative. Chaotic systems are characterized by their extreme sensitivity to initial conditions, deterministic randomness and long-term unpredictability. Synchronization of chaos refers to a technique in which two or more chaotic systems adjust a given property of their evolution to a common behaviour due to coupling. This phenomenon can be utilized in the chaotic communication system as a mechanism for information decoding. It has great potential applications in chemical reactions, biological systems, neon oscillators and so on.

The speciality of fractional order systems for which they have gained popularity in the exploration of dynamical systems is that they allow a higher degree of flexibility in the model. An integer order differential operator is a local operator whereas a fractional differential operator is a nonlocal operator in the sense that it takes into account the fact that the future states do not only depend upon the present state but also upon all of the histories of its previous states. For this practical property, the use of fractional order system becomes popular. It is to be noted that the present states of any real-life dynamical systems are dependent upon the history of its past states. The memory element is quite crucial.

This thesis consists of five chapters including introduction as chapter 1. This chapter starts with basics & brief history of dynamical systems, its stability analysis viz. equilibrium points, Lyapunov first and second method. Brief history and definition of chaos, strange attractors, Lyapunov exponent, chaos synchronization its various types and methods. Fundamentals of fractional calculus viz. history, applications, various definitions of fractional order derivatives and Leibniz rule. The essentials of the delay

differential equation are presented especially existence and uniqueness of solution by the method of steps with its numerical solution procedure. In the last part of this chapter, we present the methodologies to achieve synchronization like Active control method, Nonlinear control method. A scheme called Homotopy analysis method, which is used to analytically approximate the solution of a linear/nonlinear integer as well as fractional order differential equations. And numerical methods like Runge-Kutta method for delay differential equation and Adams-Bashforth-Moulton schemes. Each topic plays an essential role in the preparation of this thesis.

In chapter 2, the combined synchronization among time-delayed chaotic systems in the presence of uncertain parameters using nonlinear control method is accomplished. Control functions are designed to achieve combined synchronization using the Lyapunov-Krasovskii function for stability analysis. The synchronization between three and four time-delayed chaotic systems has been shown as examples of combined synchronization. Double delay Rossler system, the advanced Lorenz system, time delay Liu and Chen systems have been taken to show the combined synchronization. Numerical simulation and graphical results are carried out using Runge-Kutta method for delay-differential equations, which show that the designing of control functions are very effective and reliable and can be applied for combined synchronization among time-delayed chaotic systems.

Chapter 3 addresses the nonlinear control method for combination-combination phase synchronization between fractional order non-identical complex chaotic systems. The control functions are designed with the help of a new lemma and Lyapunov stability theory. The nonlinear control method is found to be very effective and convenient to achieve the said type of synchronization of the non-identical fractional order complex chaotic systems. The fractional order complex chaotic systems are taken to illustrate combination-combination phase synchronization process. The fractional derivative is described in Caputo derivative sense. Numerical simulations are carried out using Adams-Bashforth-Moulton method and the results are depicted through graphs for different particular cases.

In chapter 4, the triple compound synchronization among eight chaotic systems with external disturbances has been studied. Nonlinear approach is used to achieve triple

compound synchronization in the considered eight chaotic systems. The control functions are designed using Lyapunov stability theory. Numerical simulation and graphical results are carried out using Runge-Kutta method, which shows that the designing of control functions are very effective and reliable and can be applied for triple compound synchronization among chaotic systems. The salient feature of this chapter is the exhibition of complexity in the error function in triple compound synchronization for which the security of communication via signals will be more secured through this type of synchronization process.

In chapter 5, the homotopy analysis method is used to obtain approximate analytical solutions of the (1+1) dimensional nonlinear Boussinesq equation having fractional time derivative. By taking proper values of auxiliary and homotopy parameters, the numerical values of the state variables are computed and presented graphically for different particular cases. Nonlinear Boussinesq equation is used to discuss a two-dimensional flow of a volume of water over a flat bottom with air above the water. The motivation of considering the fractional time derivative form of Boussinesq equation is for their non-local behavior, which provides a lot of flexibilities in the model.