# Chapter 3: Design and analysis of rectenna

#### 3.1. Introduction

In the previous chapter, various key issues of the SSPS space section have been discussed and transmission of high power microwave beam has been explored. But in the SSPS ground section, there is a requirement of a large size rectenna array to collect microwave energy and transform it back into DC power. Thus an efficient RF-DC power conversion is desired, which after appropriate power management can be connected to conventional grids [59]. Several rectenna elements are combined in series and parallel to form an array which generates significant dc power for reliable operation [68-70]. Also, cost effective rectenna element with reduced size is required [70]. For the design of an efficient rectenna element, it's modeling and circuit analysis is desirable. A number of circuit analysis models was proposed to analyze the performance of a rectenna [70, 76]. Analysis of rectenna can be done either by using a linear circuit model or a nonlinear circuit model [76].

In this chapter, rectenna performance analysis is presented based on it's linearized circuit model. Furthermore, it is required to accurately formulate an equivalent circuit model of Schottky diode. Schottky diode is a nonlinear device, which may produce harmonic signals. The non-linear model of Schottky diode is more accurate, especially when individual rectenna elements have different power ratings and its characteristics that highly depend on receiving RF signal. Thus the analytical model of the Schottky

diode is presented and compared with ADS harmonics balance simulated model. Furthermore, a CP rectenna with reduced size is designed based on the analysis.

# 3.2. Rectenna operation

A rectifying antenna (rectenna) is depicted in Figure 3. 1. Rectenna is the key component for converting RF-DC power [71]. The microwave energy is collected by the antenna, and the integrated rectifying circuit through filters, and a matching circuit converts the received microwave energy into direct current (DC). Typically, a rectenna is composed of five components: (1) An antenna, (2) a harmonics rejection filter, (3) an impedance matching, (4) a rectification diode, and (5) an output DC filter [71-75]. A harmonic rejection filter is required to suppress harmonics which can re-resonate antenna and efficiency is highly reduced [72]. An impedance matching block is also required which can match the antenna output with the rectifier's input in order to achieve high RF to DC conversion efficiency [72]. A load resistor is connected at the output terminal to measure the DC output voltage.

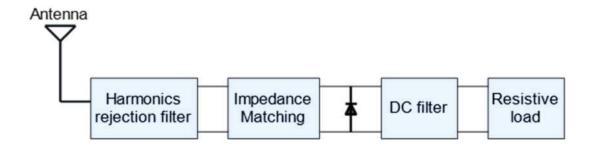


Figure 3. 1 Rectenna block diagram [72]

The rectifying diode is the key element of the rectifier circuit [25]. Its threshold or turnon voltage  $V_T$  is the most important parameter for diode efficiency. This parameter limits
efficiency at low powers as noted by  $V_T$  Effect' in Figure 3. 2. When low input RF power
is incident on the rectenna, there is not enough energy to overcome this barrier and charge
the output capacitor [25, 26]. Diode reverse breakdown voltage  $V_{br}$  also limits diode
efficiency as this will allow energy to short the diode demonstrated by the curve  $V_{br}$ Effect.' This situation will occur at larger RF input power levels. Note that while the
efficiency curve decreases above this point, DC output power will remain constant. Here
the maximum power a rectifying diode can provide is  $\frac{V_{br}}{4R_L}$ , because  $\frac{V_{br}}{2}$  is the maximum
DC output voltage that can be produced by rectifying diode [25, 26].

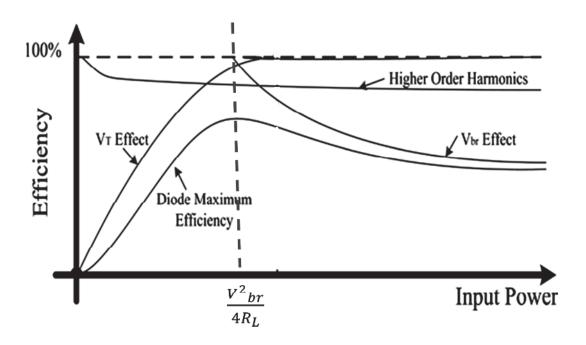


Figure 3. 2 RF-DC conversion Efficiency of rectifying diode with different input power

#### 3.2.1. Equivalent circuit model of a rectenna system

In this section, linearized circuit model of rectenna is presented, and its circuit analysis is performed to optimize the microwave to Dc power conversion. The equivalent circuit model of conventional rectenna is presented in Figure 3. 3. Here for the impedance matching either a lumped element matching network or a distributed network can be used. There is a requirement of optimal impedance matching network between the antenna and the rectifier for the maximum power transfer which will be found in this analysis, and the condition for high voltage gain will be derived.

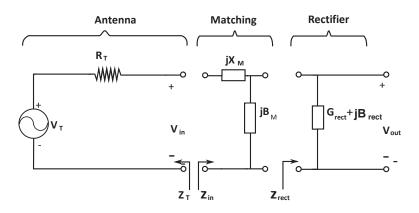


Figure 3. 3 Equivalent circuit model of a rectenna system

In the Figure 3. 3, the antenna is represented by its Thevenin equivalent as a voltage source  $V_T$  with a series impedance  $R_T$ . In this equivalent model of the antenna, the voltage  $V_T$  can be expressed as a function of  $P_{in}$  as given in equation (3.1). Here  $P_{in}$  is the maximum power available at the antenna's output.

$$V_T = \sqrt{8R_T P_{in}} \tag{3.1}$$

In the generalized rectenna system shown in Figure 3. 3, impedance matching is done by using an 'L-network' between the antenna and the rectifier. The input power  $P_{in}$  at the input of the matching network is expressed as in equation 3.2.

$$P_{in} = \frac{1}{2} \frac{|V_{in}|^2}{\Re\{Z_{in}\}}$$
 (3.2)

Here  $Z_{in}$  is the input impedance as shown in Figure 3. 3. The function  $\Re\{x\}$  denotes the real part of x.  $V_{in}$  is the voltage across the input impedance  $Z_{in}$  see Figure 3. 3. Now  $V_{in}$  is calculated as

$$V_{in} = \left(\frac{Z_{in}}{Z_{in} + R_T}\right) V_T \tag{3.3}$$

$$P_{in} = \frac{1}{2} \frac{\left| \left( \frac{Z_{in}}{Z_{in} + R_T} \right) V_T \right|^2}{\Re\{Z_{in}\}}$$
(3.4)

Putting equation (3.3) in (3.2) results in equation (3.4)

The output power  $P_{out}$  at the rectifier output terminals is given by

$$P_{out} = \frac{1}{2} \frac{|V_{out}|^2}{\mathcal{R}\{Z_{rect}\}}$$
(3.5)

Here  $Z_{rect}$  is the input impedance of the rectifier  $Y_{rect} = 1/Z_{rect} = G_{rect} + j B_{rect}$  and  $V_{out}$  is the output voltage across the rectifier terminals.  $V_{out}$  and  $Z_{rect}$  are indicated in Figure 3. 3.

#### 3.2.2. Antenna matched to the rectifier

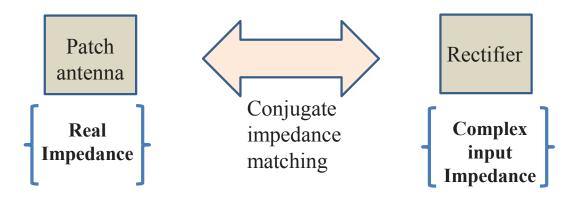


Figure 3. 4 Antenna matched to rectifier using conjugate impedance matching [76]

The patch antenna has a real impedance as shown in Figure 3. 4 [76, 77]. Therefore a conjugated matching circuit is required for connecting antenna with the rectifier. The 'L-network' shown in Figure 3. 3 is used to match the antenna impedance to conjugate of the rectifier. If a lossless network is assumed, the output power  $P_{out}$  is equal to input power  $P_{in}$  and, consequently, the voltage gain G is given by

$$G = \frac{|V_{out}|}{|V_T|} = \frac{1}{2} \sqrt{\frac{\mathcal{R}\{Z_{rect}\}}{\mathcal{R}\{Z_{in}\}}}$$
(3.6)

The input impedance  $Z_{in}$  is calculated using the network model shown in Figure 3. 3 as

$$Z_{in} = jX_{M} + \frac{1}{G_{rect} + j (B_{M} + B_{rect})}$$

$$= \frac{G_{rect}}{G_{rect}^{2} + (B_{M} + B_{rect})^{2}}$$

$$+ j \left(X_{M} - \frac{B_{M} + B_{rect}}{G_{rect}^{2} + (B_{M} + B_{rect})^{2}}\right)$$
(3.7)

For matching the conjugate of rectenna impedance to rectifier impedance, following conditions should be met,

$$\mathcal{R}\{Z_{in}\} = R_T = \frac{G_{rect}}{G_{rect}^2 + (B_M + B_{rect})^2}$$
(3.8)

$$X_{M} = \frac{B_{M} + B_{rect}}{G_{rect}^{2} + (B_{M} + B_{rect})^{2}}$$
(3.9)

From equation (3.7), the real part of the input impedance is calculated and is found to be equal to  $R_T$  as given in equation 3.8. Since  $Y_{rect} = G_{rect} + jB_{rect}$ , the real part of the rectifier's admittance is  $\{Y_{rect}\} = G_{rect, and} \{Z_{rect}\}$  is given in equation 3.11.

$$Z_{rect} = \frac{1}{G_{rect}} + \frac{j}{B_{rect}} \tag{3.10}$$

$$\mathcal{R}\{Z_{rect}\} = \frac{1}{G_{rect}} \tag{3.11}$$

$$G = \frac{|V_{out}|}{|V_T|} = \frac{1}{2} \sqrt{\frac{\Re\{Z_{rect}\}}{\Re\{Z_{in}\}}} = \frac{1}{2} \sqrt{1 + \left(\frac{B_M + B_{rect}}{G_{rect}}\right)^2} = \frac{1}{2} \sqrt{1 + (Q)^2}$$
(3.12)

By putting equation 3.8 and  $\{Z_{rect}\}$  in equation 3.6 result in equation 3.12. The voltage gain can be expressed as a function of Q. Where

 $Q = \frac{B_M + B_{rect}}{G_{rect}}$  is the quality factor at the output of the matching circuit.

From Eq. (3.12) it is noticed that, by a proper choice of the matching network susceptance of  $B_M$ , one can choose the output voltage  $V_{out}$  and then design the antenna having an impedance  $R_T$  and take  $X_M$  as dictated by Eq. (3.8) and Eq. (3.9), respectively. It is

implicitly assumed in this analysis that  $V_T$  and  $R_T$  can be chosen independently, while in fact, they are dependent on the available power from the antenna as specified in Eq. (3.1).

#### 3.2.3. Maximizing the output voltage

Assuming a lossless network, one can express the available power from the antenna by using equation (3.1)

$$P_{in} = \frac{V_T^2}{8 R_T} {(3.12)}$$

$$V_T = \sqrt{8 R_T P_{in}} = 2 \sqrt{2 P_{in} \frac{G_{rect}}{G_{rect}^2 + (B_M + B_{rect})^2}}$$
(3.13)

By the substitution of equation (3.8) in equation (3.12),  $V_T$  is expressed as in equation 3.13.

$$V_{out} = G V_T = \sqrt{\frac{2P_{in}}{G_{rect}}}$$
(3.14)

As described in Eq. (3.14), the output voltage is thus dictated by the available power from the antenna and the real part of the input admittance of the rectifier.

 $G_{rect}$  is the real part of the rectifier admittance as shown in Figure 3. 3. After substituting equation (3.15) in equation (3.14), the output voltage is described in terms of the input impedance of the rectifier as follows:

$$G_{rect} = \mathcal{R}\{Y_{rect}\} = \frac{R_{rect}}{R_{rect}^2 + X_{rect}^2}$$
(3.15)

$$V_{out} = \sqrt{2P_{in} \frac{R_{rect}^2 + X_{rect}^2}{R_{rect}}}$$
(3.16)

To maximize the output voltage  $V_{out}$  (i.e., the sensitivity) and the power transfer between the antenna and the rectifier, Eq. (3.16) should be satisfied.

# 3.3. Schottky diode

For higher frequency operation, i.e., 2.45 GHz application Schottky diode is used for rectifying AC to DC [78]. Schottky diodes are formed by plating a metal contact on n or p-type semiconductor [78-80]. This metal-semiconductor junction acts as a diode with lower forward voltage drop. Small junction capacitances of a Schottky diode make it faster switching between conducting and non-conducting mode [78, 79, 81].

### 3.3.1. Schottky Diode Equivalent Circuit Model

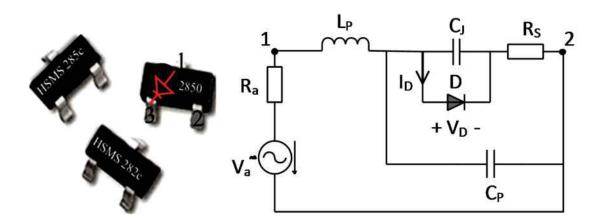


Figure 3. 5 Commercially available, packaged Schottky diodes (SOT- 23), and their equivalent circuit model [80]

In this section, Schottky diode equivalent circuit model is presented and compared with Harmonic Balance simulation model in ADS. Commercially available, packaged Schottky diodes (SOT- 23), and their equivalent circuit model is shown in Figure 3. 5 [80]. Kirchhoff's current and voltage laws are used to determine the voltage  $V_D$  and current  $I_D$  across the diode D. These found values are further used to determine the impedance of the rectifier. A voltage source with a single frequency  $f_0$  as input,  $V_a = |V_a| \cos(2\tau r f_0 t)$  is used as shown in Figure 3. 5. The electrical behaviour of this circuit can be described with the following expressions, found by applying Kirchhoff's relations:

$$V_a = I_a R_a + L_p \frac{\partial I_a}{\partial t} + V_{C_P} \tag{3.17}$$

$$V_{C_P} = V_D + V_{R_S} (3.18)$$

$$V_{R_S} = R_S(I_{C_i} + I_D) (3.19)$$

$$I_{C_i} = C_j \frac{\partial V_D}{\partial t} \tag{3.20}$$

$$I_D = I_s(e^{\alpha V_D} - 1) \tag{3.21}$$

so that

$$\frac{\partial V_D}{\partial t} = \frac{1}{R_s C_i} \{ \psi(\frac{\partial I_a}{\partial t}) - R_s I_s (e^{\alpha V_D} - 1) \}$$
 (3.22)

where 
$$\psi(\frac{\partial I_a}{\partial t}) = V_a - R_a I_a - V_D - L_p \frac{\partial I_a}{\partial t}$$
.  $\alpha = \frac{q}{nKT}$ .

where  $\frac{q}{KT}$  is the thermal voltage and n is the ideality factor. The above differential equation (3.22) have been solved by the 4<sup>th</sup> order Runge- Kutta method as in [80], and the voltage  $V_D$  across the diode D is calculated. Further equation (3.21) is used to

determine the current  $I_D$  flowing through the diode. After the evaluation of  $V_D$  and  $I_D$  the input impedance of the diode D is found using Ohm's law:

$$Z_D = \frac{V_D}{I_D} \tag{3.23}$$

Table 3. 1 Spice parameters for commercially available Schottky diodes, Avago HSMS-286X [82]

Parameter	Units	HSMS-286X
$B_{ m V}$	V	7.0
C <sub>J0</sub>	pF	0.18
E <sub>G</sub>	eV	0.69
IBV	A	10 E -5
$I_{S}$	A	5.0 x 10E -8
N		1.08
Rs	Ω	5.0
P <sub>B</sub> (VJ)	V	0.65
P <sub>T</sub> (XTI)		2
M		0.5

Table 3. 1 summarizes the Spice parameters for the commercially available Schottky diode, Avago HSMS- 286X [82]. Using the values of parameters as in Table, the input impedance of the Schottky diode  $Z_{in}$  including its series resistance and packaging parasitic is calculated. To verify the accuracy of the equivalent circuit model, the Schottky diodes HSMS- 286X ADS harmonic balance simulations model will be investigated. The input impedance versus frequency for different power levels is calculated with the aid of the equivalent circuit model and is compared with harmonic balance simulation results [44]

Figure 3. 6 and Figure 3. 7 show the impedance of the rectifier as calculated by using ADS harmonic balance simulations and the presented analytical expressions. Figure 3. 6 show the real and imaginary parts of the input impedance versus frequency at an input

power level of 0 dBm. Figure 3. 7 show the real and imaginary parts of the input impedance versus frequency at an input power level of 5 dBm.

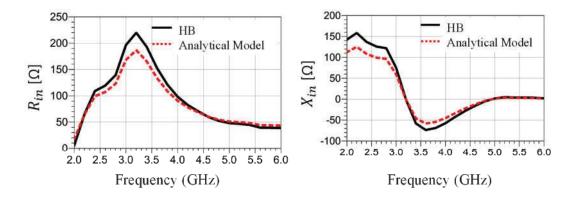


Figure 3. 6 Real and imaginary parts of the input impedance of the Schottky diode HSMS-286X versus frequency for a maximum available power level of 0 dBm calculated using the analytical model and harmonic balance (HB) simulations. Ra =  $50~\Omega$ 

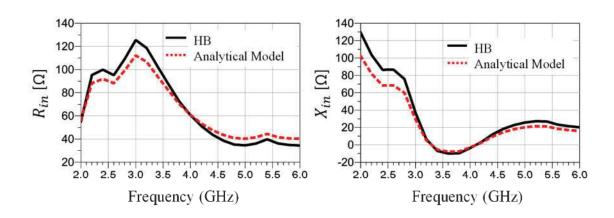


Figure 3. 7 Real and imaginary parts of the input impedance of the Schottky diode HSMS-286X versus frequency for a maximum available power level of 5 dBm calculated using the analytical model and harmonic balance (HB) simulations. Ra =  $50 \Omega$ 

It is shown in the Figure 3. 6 and Figure 3. 7 that the equivalent circuit model can predict the impedance of the rectifier with a relative difference of less than 10 % for the real and imaginary parts of the input impedance. The input impedance is again calculated using

ADS harmonic balance simulations and the equivalent circuit model. It is shown in Figures that the results of the analytical equations are matching the results obtained using harmonic balance simulations.

# 3.4. Rectenna element design

In this section, an improved rectenna with reduced size is proposed as shown in Figure 3. 8. It consists of a circular polarized (CP) truncated corner microstrip patch antenna, a compact microstrip resonant cell (CMRC) to suppress harmonics, a high Q microstrip impedance matching network, a detector diode for RF-DC conversion, and a DC pass filter. The output voltage and RF-DC conversion efficiency of rectenna depend on the resistive load. The CP truncated patch is fed by a microstrip line. The CMRC passes the generated frequency, i.e. 2.45 GHz and blocks higher order harmonics signal. For the maximum power transfer, a microstrip single stub and high Q impedance matching is also designed and used between antenna output and rectifier diode. After rectification, RF power is converted into DC power.

The proposed rectenna is simulated and designed in Agilent design system (ADS) 2011. The ADS has two solvers, by the methods of the moment and finite element method (FEM) solver and ADS FEM simulation have been used here. The designed rectenna is fabricated on FR4 substrate with dielectric constant ( $\mathcal{E}r$ ) 4.4, height (h) 1.6 mm, and a loss tangent of 0.02; the copper conductor thickness is 35 um.

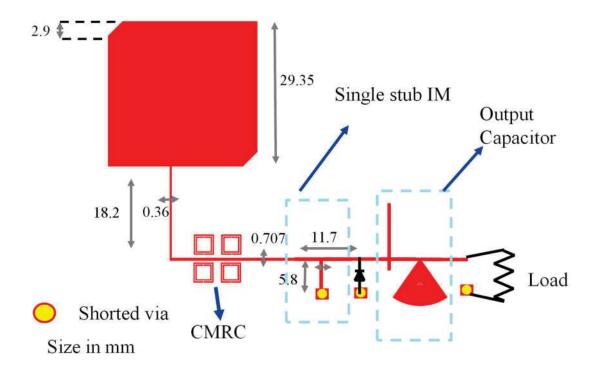


Figure 3. 8 Design layout a CP 2.45 GHz rectenna with CMRC

#### 3.4.1. Circularly polarized truncated patch antenna

CP is produced by exciting two orthogonal resonating mode with 90° phase difference [71, 74]. The truncated corner patch is a popular method to produce CP with single microstrip feed [71]. However it is desired that the dimension of patch antenna and its truncated corners are precisely designed for good antenna performance. The geometry of truncated corner patch antenna is shown in Figure 3. 8. The benefit of CP is that the performance of rectenna is not affected due to rotation [74]. The CP antenna has layout coupling effect from CMRC; subsequently, it will influence the antenna radiation pattern. Therefore, the CP antenna and CMRC should be combined to measure antenna performance. The truncated patch CP antenna with CMRC is shown in Figure 3. 9. The

measured return loss of the truncated patch antenna and CMRC combined circuit is shown in Figure 3. 10. The measured return loss is 14.23 dB at 2.45 GHz. The truncated patch can provide the circular axial ratio (AR) below 3 dB. Measured gain of the CP antenna with the CMRC and its axial ratio are shown in Figure 3. 11. The antenna has a maximum gain of 3.07 dBi at the fundamental resonant frequency, i.e., 2.45 GHz. The circular polarization 3 dB axial ratio bandwidth is from 2.419 GHz to 2.461 GHz frequency that is found to be 42 MHz, with the minimum axial ratio of 1.23 dB. The high performance CMRC is designed in ADS to pass 2.45 GHz of the generated signal from the antenna to the rectifier. The return loss and insertion loss of CMRC are shown in Figure 3. 12.

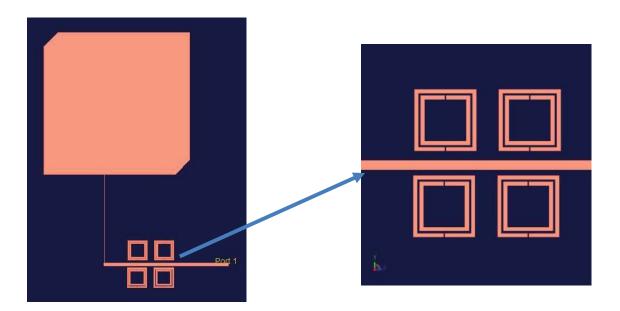


Figure 3. 9 CP 2.45 GHz rectenna with CMRC and Zoomed view of CMRC

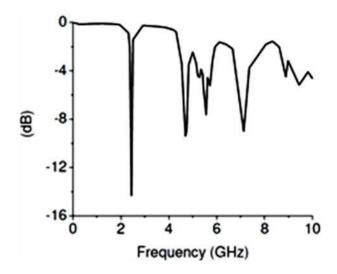


Figure 3. 10 Antenna with CMRC measured return loss

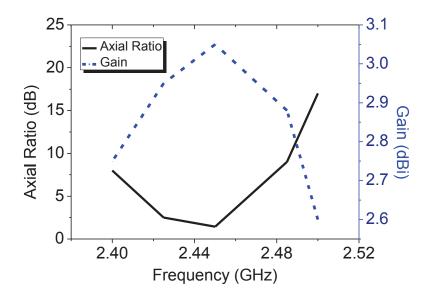


Figure 3. 11 CP antenna measured axial ratio and gain

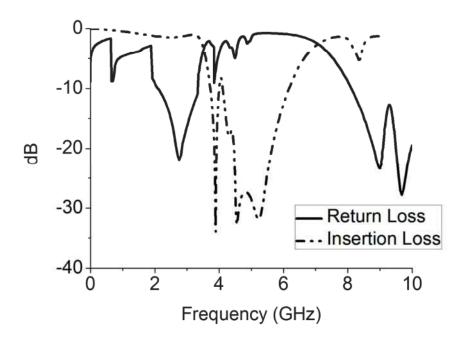


Figure 3. 12 Measured return loss and insertion loss of CMRC

# 3.4.2. Rectifier circuit and impedance matching

Schottky diode HSMS-2860 (threshold voltage 350 mv & break down voltage 7 V) is used for rectification here. Due to its nonlinear property, it has complex input impedance, while on the other hand, the patch antenna with feed has real impedance. Therefore a conjugated matching circuit is required for connecting antenna with rectifier. The variation of rectifier's input impedance with RF input power is shown in Figure 3. 13. Here, ADS Smith chart toolbox is used to design impedance matching network at 9 dBm. ADS Smith chart utility is shown in Figure 3. 14.

pin	zim3	
-20.000 -19.000 -18.000 -17.000 -16.000 -15.000 -14.000 -13.000 -11.000 -10.000 -10.000 -7.000 -6.000 -5.000 -1.000	61.278 / -85.632 61.289 / -85.385 61.379 / -84.989 61.602 / -84.385 62.008 / -83.539 62.618 / -82.467 63.418 / -81.225 64.385 / -79.882 65.504 / -78.501 66.777 / -77.127 68.220 / -75.791 69.858 / -74.508 71.714 / -73.285 73.813 / -72.123 76.173 / -71.018 78.813 / -69.964 81.758 / -68.948 85.030 / -67.953 88.634 / -66.961 92.555 / -65.952 96.708 / -64.915 100.868 / -63.881 104.701 / -62.942 108.052 / -62.210 111.072 / -61.726 114.239 / -61.376 117.828 / -61.085 121.260 / -60.998 124.943 / -60.916 129.539 / -60.578	
(a)		

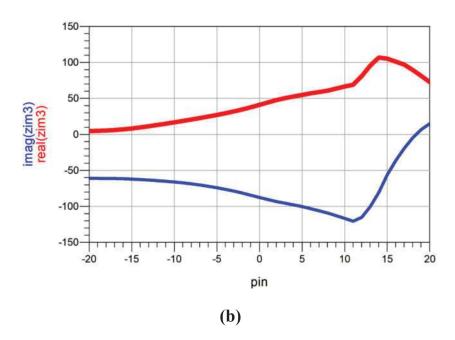


Figure 3. 13 Rectifier's input impedance (a) complex (b) real and imaginary value variation with input power (dBm), here zim3 is input impedance of the rectifier and pin is RF input power

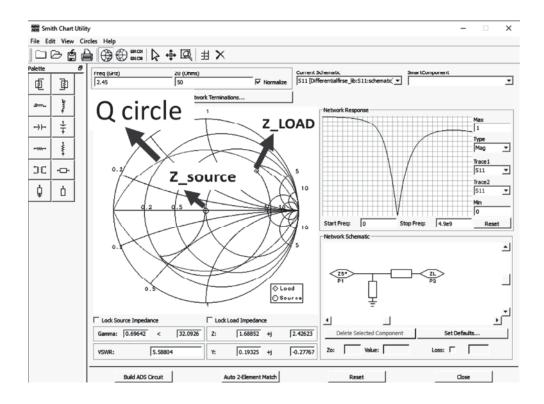


Figure 3. 14 Smith chart utility in ADS

#### 3.4.3. Rectenna Measurement

A rectenna using microstrip is designed and fabricated at 2.45 GHz. The circuit schematic and detailed parameters are shown in Figure 3. 8. The rectenna is printed on an FR4 substrate, 1.6 mm height, dielectric constant 4.4 and loss tangent 0.02. In the measurement, RF-DC conversion efficiency can be calculated as

$$\eta(\%) = \frac{Output\ DC\ power}{RF\ Input} \times 100$$
(3.24)

For the measurement, APLAB 2130 Series Signal Generator with 9 kHz~3002 MHz frequency coverage with 50 Ohm VNA output port is used as RF source; it can excite RF power of 15 dBm. A high-directivity horn antenna (about 7 dBi gain at 2.45 GHz) is used,

it has larger dimension D 20 cm. Then rectenna under test is placed at a distance of 70 cm from transmitting horn antenna. The distance in between satisfies far field condition  $(R > 2D^2/_{\lambda} \sim 65 \ cm)$  at 2.45 GHz, The simulated and measured RF-DC conversion efficiency of the rectenna versus input power level are shown in Figure 3. 15. The designed rectenna has a maximum efficiency of 36.75 % at 10 dBm input power. It is noticeable here that due to polarization mismatch loss, the received power is decreased about 3 dB as this is a CP rectenna. However, their efficiencies are stable irrespective of the rotation.

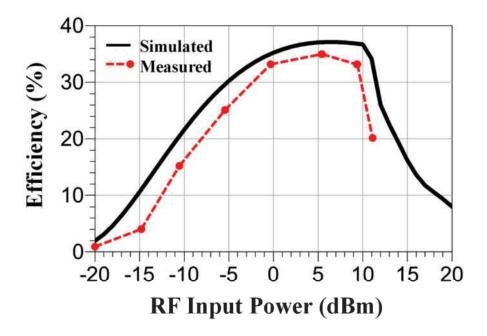


Figure 3. 15 The simulated and measured efficiency of CP rectenna with CMRC

# 3.5. Summary

In this chapter, rectenna is first introduced, and its components requirement for SSPS application are discussed. Then rectenna analysis is presented for the efficient rectenna

design using linearized circuit model. As there is a requirement of impedance matching component which is more focused here and findings are obtained to connect an antenna with a nonlinear rectifier to achieve high RF to DC conversion efficiency. A nonlinear model of Schottky diode is also presented. The nonlinear model provides comparable electric behavior and its performance is compared with HB simulation model in ADS. The comparison provides satisfactory results. Furthermore, based on the analysis and findings, a CP rectenna with reduced size is proposed, and for size reduction, CMRC is used here. The CP truncated patch antenna has a maximum gain of 3.07 dBi at the fundamental resonant frequency, i.e., 2.45 GHz. The circular polarization 3 dB axial ratio bandwidth is from 2.419 GHz to 2.461 GHz frequency that is found to be 42 MHz, with the minimum axial ratio of 1.23 dB. The RF-DC conversion efficiency reaches 36.75 % at 10 dBm input power.