CHAPTER 7

SUMMARY OF THE WORK AND FUTURE SCOPE

7.1 Summary of the Work

The present thesis consists of some important features of some recent generalized thermoelasticity theories which are based on non-Fourier heat conduction models. The major characteristics of the present study can be summarized into the following two parts:

Part I: Establish some important theorems on recent thermoelasticity theories

A domain of influence theorem for a potential-temperature disturbance under Green-Naghdi thermoelasticity theory of type II is established in section 2.1 of Chapter 2. This theorem implies that for a given bounded support of thermo-mechanical loading, the thermoelastic disturbance generated by the pair of temperature and potential of the system vanishes outside a well-defined bounded domain. This domain is shown to depend on the support of the load, i.e., on the initial and boundary data. It is also shown that under Green-Naghdi model-II, the thermoelastic disturbance propagates with a finite speed that is dependent on the thermoelastic parameters. In section 2.2 of this chapter, we have considered a more general problem and established the domain of influence theorem for a stress-heat-flux disturbance under Green and Naghdi thermoelasticity theory of type-II. Here, we consider a mixed problem of natural type represented as stress- heat-flux disturbance in the context of Green-Naghdi thermoelasticity theory of general type. We establish a general energy identity for the problem in term of stress and heat flux. Then we establish the domain of influence theorem for natural stress-heat-flux disturbance in the context of Green-Naghdi model of thermoelasticity of type-II. We prove that for a finite time, the pair of stress and the heat-flux fields generate no disturbance outside a bounded domain and the domain of influence is shown to be dependent upon the thermoelastic coupling constant and the material parameters. Next, we consider a very recently proposed heat conduction model with a single delay term and extended it for corresponding thermoelastic model. This model is proposed as a reformulation of dual-phase lag and three-pase-lag thermoelastic model with more physical and mathematical consistency. Hence, we pay attention to this model and aim to establish some basic theorems in this context. A uniqueness theorem, a convolution type variational principle and reciprocity relation for an anisotropic and inhomogeneous material are established in Chapter 4. Further, by using the representation of Galerkin type solution, we also obtained the fundamental solutions (the solution which examines the effects of concentrated loads and heat sources in an unbounded medium) for linear theory of thermoelasticity model given by Quintanilla (2011) with a single delay term in the Chapter 6. The fundamental solutions of the field equations in case of steady vibrations are also established in Chapter 6.

Part II: Some investigations under recently developed thermoelasticity theories

In a problem in Chapter 3, the effects of temperature dependency of material properties on thermo-mechanical responses of an annular cylinder whose inner surface is subjected to time dependent temperature distribution have been analyzed by employing thermoelasticity theory of type GN-II. The governing equations are derived by employing this theory and considering temperature dependent physical properties. Governing equations are obtained as non-linear coupled partial differential equations and we apply finite difference method for solving the coupled system for the present problem. We showed that the present problem can be efficiently solved by finite difference method.

We obtain the orders of truncation error for displacement and temperature variable.

Our results highlight the significant effects of temperature dependent material properties on thermoelasticity. We considered two different types of temperature distributions prescribed at the inner boundary surface of the cylinder where as the outer surface is kept at reference temperature. The effects of temperature dependent properties are shown to be of different nature in two cases. However, in both the cases it has been observed that under the present context, the difference in the numerical results with temperature dependent properties and temperature independent properties are very much pronounced. The region of influence is observed to be finite for all field variables. We note a significant difference in the prediction of results for exponential temperature distribution prescribed at the inner boundary of the cylinder with the corresponding results under thermoelasicity with one relaxation parameter. The region of influence if much smaller in case of GN-II model as compared to the same under LS model. For more accurate analysis of thermoelastic behavior of structural elements, the temperature dependency needs to be considered. Hence, this study is believed to be useful in characterizing the thermoelastic responses of structural element with temperature dependent properties under different thermoelastic models.

We analyze the thermoelastic interactions in an unbounded elastic medium with a spherical cavity under the very recent heat conduction model with single delay term introduced by Quintanilla (2011) in section 5.1 of Chapter 5. This model is an alternative formulation of the three phase-lag model, given by Roychoudhuri (2007). We have studied the thermoelastic interactions in an isotropic elastic medium with a spherical cavity subjected to three types of thermal and mechanical loads named as problem-1, problem-2 and problem-3 in the contexts of two versions of this New model (we refer them as New model-I and New model-II). We find analytical as well as numerical solutions for the distributions of the field variables displacement, temperature and stresses with the help of the integral transform technique. We also compare numerical results with the corresponding results of Green Naghdi thermoelasticity theory of type-III. We observed following significant facts while analysing the numerical and analytical results:

(1) When the boundary of the cavity is subjected to a ramp type heating (problem-1), we observe that elastic waves are propagating with finite speed. However, in the context of New model-I it propagates without any attenuation but under New model-II it decays exponentially. In this problem, the speed of elastic waves do not depend on the delay parameter as well as any material parameter. On the other hand, thermal waves propagate with infinite speed like classical thermoelasticity theory. In this case, all the fields are continuous in nature and the results are of similar nature like Green Naghdi model of type -III.

(2)When the boundary of the cavity is subjected to thermal shock (problem-2), we observe that elastic wave propagate with finite speed but attenuation coefficient of elastic wave for New model-II is totally depend on the delay time and material parameter. We also find that temperature and stress fields show prominent differences under different models and there is a significant role of the delay time parameter.

(3)When boundary of the cavity is subjected to a normal load (problem-3), we investigate that there is no prominent effect of delay parameter on displacement and stress fields. However, it affects the temperature very prominently.

(4) In problem-2 and problem-3, we also observe that there is a finite discontinuity at elastic wave front for radial and circumferential stress distribution under for both the models.

We investigate the behavior of harmonic plane wave under the heat conduction model with a single delay term (τ) introduced by Quintanilla (2011) in section 5.2 of Chapter 5. We consider two types of Taylor series expansion of this new heat conduction model (New model-I and New model-II). Asymptotic results of the dispersion relation solutions for both high and low frequency values are presented here for both the models. Also the asymptotic expressions for the important wave components like phase velocity, specific loss, penetration depth and amplitude ratio are derived for both New model-I and New model-II with variation of frequency and the thermal conductivity rate. We present both analytical and numerical results for a thorough comparison. We show that the analytical results are in perfect match with numerical results. We further compare our results with the corresponding results of Green-Naghdi thermoelasticity theory of type-III. We observed various significant facts as mentioned below:

(1) Under all the models, only longitudinal wave is observed to be coupled with the thermal field and there are two different modes of the longitudinal wave. One is elastic and the other one is thermal in nature.

(2) For lower frequency values, we observe from analytical and numerical results that all the models show the same nature for all important wave components. But for different values of thermal conductivity rate, there is a significant difference in the results under each model. In case of elastic wave, each model attains its local maxima and in case of thermal wave, it attains a local minima before tending to its constant limiting value.

(3) For higher frequency value, there is variation in results under New model- I and New model- II. However, New model- II shows much closer values like GN- III model.

(4) The effects of the material constant k^*(conductivity rate) are more prominent for lower values of frequency and the effects of the term τ^2 is more significant for higher values of frequency.

(5) For higher frequency values, we find that velocity of elastic wave approaches to constant value where as velocity of thermal wave approaches to infinite value under all the three models.

(6) The theoretical as well as numerical results indicate that in case of New model-I, the specific loss of thermal mode wave is an increasing function of ω and approaches to infinity as $\omega \to \infty$. However, this field approaches to a constant limiting value under New model-II and GN-III models.

(7) New model-II and GN-III model predict more similar results for prenetration depth of elastic and thermal mode waves.

7.2 Future Scopes

Generalized thermoelasticity theories have been developed with the objective of removing the paradox of infinite speed of heat propagation inherent in the conventional coupled dynamical theory of thermoelasticity. In recent years, a great attention is devoted to understand the thermoelastic responses in case of rapid heating of metallic films through short-pulse laser techniques required for advanced applications in modern microfabrication technologies. In this way, different generalized thermoelastic theories have been developed like Green-Naghdi theory, dual phase-lag theory, three-phase lag thermoelasticity theory and thermoelasticity with single delay term. These thermoelasticity theories have drawn the significant attention of researchers to obtain distinct characteristics of various models. Critical analysis on these models are also discussed. It is beleived that the best suitable model to account for the coupling effects of thermal and mechanical fields need to be identified through further research in theoretical as well as in experimental direction. Although, some works are reported in this direction but they are very few in number to consider the fully coupled system of equations. Therefore, it is worth pursuing the research in this area. In the analysis of thermal stresses in elements the actual properties of the material cannot be described precisely without taking the following properties (i) temperature dependent of material properties (ii) time dependent strain and stress response of materials (iii) considering the decrease in yield stress of the material with rising temperature. It is believed that further research will be required to study various models of thermoelasticity by considering these mentioned properties.The research on heat conduction problems is treated with general, analytic and numerical view points which gives an opportunity to the researcher and offers the insight which is gained from general and analytical solutions providing an important tool for numerical analysis. This must be often put into practical use. The thesis is therefore being completed with a great expectation that the above mentioned points will be taken under consideration in future.