CHAPTER 3

THERMO-MECHANICAL RESPONSES OF AN ANNULAR CYLINDER WITH TEMPERATURE DEPENDENT MATERIAL PROPERTIES UNDER THERMOELASTICITY WITHOUT ENERGY DISSIPATION

3.1 Introduction

This chapter is concerned with thermoelasticity theory without energy dissipation (TEWOED) for a problem of an infinitely long annular cylinder. While studying the problems of thermoelasticity, the material properties of the medium are in general considered to be constant. However, the structural elements are often subjected to thermal loads due to ultra-high temperature, ultra-high temperature gradient, cyclical changes of ultra-high temperature etc. as reported by Noda (1986,1991). The material parameters in these circumstances remain no longer constant and they depend on temperature. Hence, in order to perform more accurate analysis of thermoelastic behavior of the structural elements, temperature dependency of material properties needs to be considered. It is to be noted that Suhara (1918) studied a thermoelastic problem of hollow cylinder by considering temperature dependent shearing modulus. Subsequently, several investigations have been reported on the thermal stress analysis in elastic and inelastic materials with temperature dependent properties. The survey/review articles by Noda (1986, 1991) and the references there-in may be recalled in this context. We also recall some investigations on thermoelastic deformation of several basic structures like disk, cylinders, tubes etc. with temperature dependent properties

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as reported by Ezzat *et al.* (2011), Othman (2002), Ezzat and Othman (2002), Othman (2011), Eraslan and Kartal (2004), Ezzat *et al.* (2004), Youssef and Abbas (2007), Argeso and Eraslan (2008), Mukhopadhyay and Kumar (2005), Othman *et al.* (2013, 2015), Kalkal and Deswal (2014), Wang *et al.* (2013), Abbas (2014) etc. In these studies the temperature dependent properties of the medium have been considered. Recently, Abbas and Youssef (2012), Zenkour and Abbas (2014a, 2014b), He and Shi (2014) employed generalized thermoelasticity theory by Lord and Shulman (1967) to investigate the effects of temperature dependent material properties on the numerical solution of thermoelastic problems obtained by finite element method. Mukhopadhyay and Kumar (2009) investigated the effects of temperature dependent material properties on thermoelastic interactions in the context of Lord-Shulman model.

In the present work, we aim to investigate a problem of an infinitely long annular cylinder in the context of thermoelasticity theory without energy dissipation (TEWOED). We assume that the material properties like, modulus of elasticity and thermal conductivity vary with temperature. Hence, by considering temperature dependency of material properties, we formulate the governing equations under TEWOED theory as introduced by Green and Naghdi (1993). The governing equations in this case are derived as coupled non-linear partial differential equations because of varying material parameters. The outer boundary of the annulus is assumed to be stress free and is maintained at reference temperature, while the inner surface is subjected to two different types of variation in temperature together with zero stress. Using the finite difference method, the governing equations are transformed into a system of coupled difference equations and the numerical solution of the problem is obtained. The values of the field variables inside the annulus for copper material are simulated directly in space-time domain. Results are displayed graphically. We analyze the effects of temperature dependency of material parameters. We also compare the results with the corresponding results obtained for temperature-independent material properties. A thorough comparision between the results predicted by the present model with corresponding results under thermoelasticity with one thermal relaxation parameter reported by Mukhopadhyay and

Kumar (2009) is further presented. This study brings to light several points highlighting the effects of temperature dependency of material properties under thermoelasticity without energy dissipation theory that accounts for the finite speed for the thermal disturbance.

3.2 Basic Governing Equations

We consider an isotropic, linear and thermally conducting elastic medium with temperature dependent mechanical properties. The governing equations for thermoelastic model without dissipation of energy by Green Naghdi (1993) in the absence of external body forces and heat sources can therefore be written as follows:

Stress-strain temperature relation:

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e - \beta\theta)\delta_{ij} \tag{3.1}$$

Strain-displacement relations:

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3.2}$$

Equation of motion in absence of body forces:

$$\sigma_{ij,j} = \rho\ddot{u}_i \tag{3.3}$$

Heat conduction equation in absence of heat sources:

$$(k^*\theta_{,i})_{,i} = k\eta\frac{\partial^2\theta}{\partial t^2} + \beta\theta_0\frac{\partial^2 e}{\partial t^2} \tag{3.4}$$

Here η is the thermal diffusivity, where $\eta = \frac{\rho c_E}{k}$.

Our goal is to investigate the effects of temperature dependent material properties on thermoelastic behavior.

Therefore, we assume that, $\lambda = \lambda_0 f(\theta),\ \mu = \mu_0 f(\theta),\ k = k_0 f(\theta\,),\ k^* = k_0^* f(\theta),\ \beta = \beta_0 f(\theta)$.

where λ_0, μ_0, k_0, k_0^* and β_0 are considered to be constant material properties at reference temperature θ_0, and $f(\theta)$ is a given function of temperature. It is to be noted that the case of temperature-independent material property corresponds to the case when $f(\theta) = 1$, i.e., $\lambda = \lambda_0$, $\mu = \mu_0$, $k = k_0$, $k^* = k_0^*$, $\beta = \beta_0$. In general, different material properties vary in different manner with the increase of temperature. For example, Young's modulus, shearing modulus, density, thermal conductivity etc. usually decrease with the rising of temperature, while the coefficient of Poisson ratio, and linear thermal expansion usually increase with the increase of temperature. However, the dependency of some properties like, Poisson ratio is less than the that of other material properties (Noda (1991)). Hence, for simplicity of the present problem it is assumed that λ, μ, k, k^* and β vary as per above law and the other properties are assumed to be independent of temperature for our present analysis.

In view of the above assumptions, equations (3.1), (3.3) and (3.4) yield

$$\sigma_{ij} = \left[2\mu_0 e_{ij} + (\lambda_0 e - \beta_0 \theta)\,\delta_{ij}\right] f(\theta) \tag{3.5}$$

$$\rho \ddot{u}_i = \left[2\mu_0 e_{ij} + (\lambda_0 e - \beta_0 \theta)\,\delta_{ij}\right]_{,j} f(\theta) + \left(f(\theta)\right)_{,j} \left[2\mu_0 e_{ij} + (\lambda_0 e - \beta_0 \theta)\,\delta_{ij}\right] \tag{3.6}$$

$$\left[k_0^* f(\theta)\,\theta_{,i}\right]_{,i} = k_0 \eta f(\theta) \frac{\partial^2 \theta}{\partial t^2} + \beta_0 \theta_0 f(\theta) \frac{\partial^2 e}{\partial t^2} \tag{3.7}$$

3.3 Problem Formulation

We consider an infinitely long annular cylinder of isotropic elastic material. It is assumed that the material properties, except density and specific heat, of the cylinder are temperature dependent. (r, ϕ, z) are taken as cylindrical polar coordinates with the origin at the center of the system and z-axis is taken to be along the axis of the cylinder. We consider axi-symmetric plane strain problem and the physical quantities are assumed to be the functions of radial coordinate r and time t. Since modulus of rigidity and many other properties decreases monotonically with the rise of temperature (see Rishin *et al.* (2005)), we assume $f(\theta) = e^{-\alpha T}$. However, for simplicity and without any loss of generality we approximate the function $f(\theta)$ as $f(\theta) = 1 - \alpha\theta$, where α is an empirical material parameter of the dimension K^{-1} (Noda (1991)).

Therefore, with the help of (3.5), we get the non-zero stress components as

$$\sigma_{rr} = \left[(\lambda_0 + 2\mu_0)\frac{\partial u}{\partial r} + \lambda_0\frac{u}{r} - \beta_0\theta\right](1 - \alpha\theta) \tag{3.8}$$

$$\sigma_{\phi\phi} = \left[(\lambda_0 + 2\mu_0)\frac{u}{r} + \lambda_0\frac{\partial u}{\partial r} - \beta_0\theta\right](1 - \alpha\theta) \tag{3.9}$$

We use the equation (3.3) to get the equation of motion in the cylindrical co-ordinates as

$$\frac{\partial\sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\phi\phi}) = \rho\frac{\partial^2 u}{\partial t^2} \tag{3.10}$$

By using (3.8)-(3.10), we obtain

$$\begin{aligned}(\lambda_0 + 2\mu_0)\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right](1-\alpha\theta) - \\ \left[\alpha\left\{(\lambda_0 + 2\mu_0)\frac{\partial u}{\partial r} + \lambda_0\frac{u}{r} - \beta_0\theta\right\} + \beta_0(1-\alpha\theta)\right]\frac{\partial\theta}{\partial r} = \rho\frac{\partial^2 u}{\partial t^2}\end{aligned} \tag{3.11}$$

Equation (3.7) then yields

$$\frac{k_0^*}{k_0}\left(\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r}\right) - \frac{k_0^*}{k_0}\frac{\alpha}{(1-\alpha\theta)}\left(\frac{\partial\theta}{\partial r}\right)^2 = \eta\frac{\partial^2\theta}{\partial t^2} + \frac{\beta_0\theta_0}{k_0}\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) \tag{3.12}$$

For our convenience, we now introduce the following non dimensional variables and notations:

$r' = c_0\eta r,\ u' = c_0\eta r,\ t' = c_0^2\eta t,\ \theta' = \frac{\theta-\theta_0}{\theta_0},\ \sigma'_{ij} = \frac{\sigma_{ij}}{(\lambda_0+2\mu_0)},\ \lambda_1 = \frac{\lambda_0}{(\lambda_0+2\mu_0)},\ c_0^2 = \frac{(\lambda_0+2\mu_0)}{\rho},$

$a_0 = \frac{k_0^*}{k_0 c_0^2 \eta}$, $a_1 = \frac{\beta_0 \theta_0}{(\lambda_0 + 2\mu_0)}$, $a_2 = \frac{\beta_0}{k_0 \eta}$, $\gamma = \alpha\theta_0$.

Therefore, the dimensionless forms of equations (3.8)-(3.12) are obtained as follows (after dropping the primes for convenience):

$$\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right]\{1 - \gamma(\theta + 1)\} - \left[a_1\{1 - 2\gamma(\theta + 1)\} + \gamma\left(\frac{\partial u}{\partial r} + \lambda_1 \frac{u}{r}\right)\right] = \frac{\partial^2 u}{\partial t^2} \tag{3.13}$$

$$a_0\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r}\right) - \frac{\gamma a_0}{\{1 - \gamma(\theta + 1)\}}\left(\frac{\partial \theta}{\partial r}\right)^2 = \frac{\partial^2 \theta}{\partial t^2} + a_2 \frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) \tag{3.14}$$

$$\sigma_{rr} = \{1 - \gamma(\theta + 1)\}\left[\frac{\partial u}{\partial r} + \lambda_1 \frac{u}{r} - a_1 \theta\right] \tag{3.15}$$

$$\sigma_{\phi\phi} = \{1 - \gamma(\theta + 1)\}\left[\lambda_1 \frac{\partial u}{\partial r} + \frac{u}{r} - a_1 \theta\right] \tag{3.16}$$

3.3.1 Initial and Boundary Conditions

We assume that initially the annulus has no deformation and have the reference temperature θ_0 and also has the zero rate of change of temperature. Therefore, initial conditions are expressed to be homogeneous i.e, we have

$$u(r, 0) = \frac{\partial u(r, 0)}{\partial t} = 0, \quad \theta(r, 0) = \frac{\partial \theta(r, 0)}{\partial t} = 0, \qquad a \le r \le b. \tag{3.17}$$

where a and b are the dimensionless inner and outer radii of the cylinder.

It is assumed that both the inner and outer curved surfaces of the annulus are stress free and the inner surface is subjected to a temperature which is varying as $f(t)$ with time t, where as the outer surface is maintained at the reference temperature. The boundary conditions are therefore taken to be as follows:

$$\sigma_{rr} = 0, \quad \theta = f^*(t) \qquad \text{at} \quad r = a, \quad t > 0 \tag{3.18}$$

$$\sigma_{rr} = 0, \quad \theta = 0 \qquad \text{at} \quad r = b, \quad t > 0 \tag{3.19}$$

3.4 Solution of the Problem (Numerical scheme)

The governing equations obtained in the last section are non linear partial differential equations. For the solution of the problem we therefore use the finite difference method. We assume that the solution domain $a \leq r \leq b$, $0 \leq t \leq t_0$ is replaced by a grid described by the set of node points (r_m, t_n), in which $r_m = a + mp$, $m = 0, 1, ..., N$ and $t_n = nl$, $n = 0, 1, ..., P$. Therefore, $p = \frac{(b-a)}{N}$ is taken as mess width and $l = \frac{t_0}{P}$ is assumed to be the time-step. Here, we assume that t_0 is the final value of time. In the following equations we use the notation u_m^n in place of $u(r_m, t_n)$, $m = 0, 1, \ldots, N$ and $n = 0, 1, \ldots, P$. The finite difference approximations for the partial differential coefficients with respect to the independent variables r and t are obtained as follows:

$$\frac{\partial y}{\partial r} = \frac{y_{m+1}^n - y_{m-1}^n}{2p} + o(p^2),$$
$$\frac{\partial^2 y}{\partial r^2} = \frac{y_{m+1}^n - 2y_m^n + y_{m-1}^n}{p^2} + o(p^2), \quad \frac{\partial y}{\partial t} = \frac{y_m^{n+1} - y_m^{n-1}}{2l} + o(p^2) \tag{3.20}$$

In view of equation (3.20), after detailed manipulations, the equations (3.13) and (3.14) are then replaced by the explicit forms of finite difference equations as follows:

$$\begin{aligned} u_m^{n+1} &= 2u_m^n - u_m^{n-1} \\ &+v\left\{1-\gamma\left(\theta_m^n+1\right)\right\}\left[\left(u_{m+1}^n - 2u_m^n + u_{m-1}^n\right) + \frac{p}{2r_m}\left(u_{m+1}^n - u_{m-1}^n\right) - \frac{p^2}{r_m^2}u_m^n\right] \\ &- -\frac{v}{4}\left(\theta_{m+1}^n - \theta_{m-1}^n\right)\left[2a_1 p\left\{1-2\gamma\left(T_m^n+1\right)\right\} + \gamma\left(u_{m+1}^n - u_{m-1}^n\right) + \frac{2p\lambda_1}{r_m}u_m^n\right] \end{aligned} \tag{3.21}$$

$$
\begin{aligned}
\theta_m^{n+1} &= 2\theta_m^n - \theta_m^{n-1} \\
+a_0 \upsilon \left[\left(\theta_{m+1}^n - 2\theta_m^n + \theta_{m-1}^n \right) + \frac{p}{2r_m} \left(\theta_{m+1}^n - \theta_{m-1}^n \right) \right] \\
-\frac{\gamma a_0 \upsilon}{4\left\{1-\gamma\left(T_m^n+1\right)\right\}} \left(\theta_{m+1}^n - \theta_{m-1}^n \right)^2 \\
-\frac{a_2}{2p} \left(u_{m+1}^{n+1} - 2u_{m+1}^n + u_{m+1}^{n-1} \right) - \left(u_{m-1}^{n+1} - 2u_{m-1}^n + u_{m-1}^{n-1} \right) \\
-\frac{a_2}{2p} \left[\frac{2p}{r_m} \left(u_m^{n+1} - 2u_m^n + u_m^{n-1} \right) \right]
\end{aligned} \tag{3.22}
$$

where, we have used the notation $\upsilon = \frac{l^2}{p^2}$.

Further, equations (3.15) and (3.16) reduce to

$$
[\sigma_{rr}]_m^n = \left\{1-\gamma\left(\theta_m^n+1\right)\right\} \left[\frac{u_{m+1}^n - u_{m-1}^n}{2p} + \lambda_1 \frac{u_m^n}{r_m} - a_1 \theta_m^n \right] \tag{3.23}
$$

$$
[\sigma_{\phi\phi}]_m^n = \left\{1-\gamma\left(\theta_m^n+1\right)\right\} \left[\lambda_1 \frac{u_{m+1}^n - u_{m-1}^n}{2p} + \frac{u_m^n}{r_m} - a_1 \theta_m^n \right] \tag{3.24}
$$

From the initial condition (3.17) and by using equation(3.20), we get

$$
\frac{\partial u_m^0}{\partial t} = \frac{u_m^1 - u_m^{-1}}{2l} = 0, \quad \frac{\partial \theta_m^0}{\partial t} = \frac{\theta_m^1 - \theta_m^{-1}}{2l} = 0 \tag{3.25}
$$

Now, using equation (3.25) we can eliminate u^{-1} um and θ^{-1} from equations (3.21) and (3.22) to get the equations satisfied by u_m^n and T_m^n for the first level of t (*i.e.*, $n = 0$) as

$$
\begin{aligned}
u_m^1 = u_m^0 + \frac{\upsilon}{2} \left\{1-\gamma\left(\theta_m^0+1\right)\right\} \left[\left(u_{m+1}^0 - 2u_m^0 + u_{m-1}^0 \right) + \frac{p}{2r_m} \left(u_{m+1}^0 - u_{m-1}^0 \right) - \frac{p^2}{r_m^2} u_m^0 \right] \\
-\frac{\upsilon}{8} \left(\theta_{m+1}^0 - \theta_{m-1}^0 \right) \left[2a_1 p \left\{1-2\gamma\left(\theta_m^0+1\right)\right\} + \gamma \left(u_{m+1}^0 - u_{m-1}^0 \right) + \frac{2p\lambda_1}{r_m} u_m^0 \right]
\end{aligned} \tag{3.26}
$$

$$\theta_m^1 = \theta_m^0 + \frac{a_0 v}{2}\left[\left(\theta_{m+1}^0 - 2\theta_m^0 + \theta_{m-1}^0\right) + \frac{p}{2r_m}\left(\theta_{m+1}^0 - \theta_{m-1}^0\right)\right]$$
$$-\frac{\gamma a_0 v}{8\left\{1-\gamma\left(\theta_m^0+1\right)\right\}}\left(\theta_{m+1}^0 - \theta_{m-1}^0\right)^2$$
$$-\frac{a_2}{2p}\left[\left(u_{m+1}^1 - u_{m+1}^0\right) - \left(u_{m-1}^1 - u_{m-1}^0\right) + \frac{2p}{r_m}\left(u_m^1 - u_m^0\right)\right] \quad (3.27)$$

In view of boundary condition (3.18) and equation (3.23) , we get for the line $r = a$ as

$$\frac{u_1^n - u_{-1}^n}{2p} + \lambda_1 \frac{u_0^n}{r_0} - a_1\theta_0^n \text{ and } \theta_0^n = f^*(t_n) \quad (3.28)$$

Now, substituting the expression for u_{-1}^n from equation (3.28) into equation (3.21), we get the equation satisfied by u_m^n for $r = a$ (i.e. for the level $m = 0$) as

$$u_0^{n+1} = 2u_0^n - u_0^{n-1} + 2v\left\{1-\gamma\left(\theta_0^n+1\right)\right\}\left[\left\{u_1^n - u_0^n + p\left(\lambda_1\frac{u_0^n}{r_0} - a_1\theta_0^n\right)\right\}\right.$$
$$\left.-\frac{p^2}{2r_0}\left(\lambda_1\frac{u_0^n}{r_0} - a_1\theta_0^n\right) - \frac{p^2}{2r_0^2}u_0^n\right] - \frac{pv}{2}\left(-3\theta_0^n + 4\theta_1^n - \theta_2^n\right)\left[a_1\left\{1-2\gamma\left(\theta_0^n+1\right)\right\}\right.$$
$$\left.-\gamma\left(\lambda_1\frac{u_0^n}{r_0} - a_1\theta_0^n\right) + \frac{\lambda_1}{r_0}u_0^n\right] \quad (3.29)$$

Similarly, by using equation (3.19) we get for the line $r = b$ as

$$\frac{u_{N+1}^n - u_{N-1}^n}{2p} + \lambda_1\frac{u_N^n}{r_N} - a_1\theta_N^n = 0 \text{ and } \theta_N^n = 0 \quad (3.30)$$

Therefore, substituting u_{N+1}^n from equation (3.30) into equation (3.21), we obtain the equation for the level $m = N$ as follows:

$$u_N^{n+1} = 2u_N^n - u_N^{n-1} + 2v\left\{1 - \gamma\left(\theta_N^n + 1\right)\right\}\left[\left(-u_N^n + u_{N-1}^n + p\lambda_1 \frac{u_N^n}{r_N}\right)\right.$$
$$\left. - - \frac{p^2\lambda_1}{2r_N^2}u_N^n - \frac{p^2}{2r_N^2}u_N^n\right] - \frac{pv}{2}\left(3\theta_N^n - 4\theta_{N-1}^n + \theta_{N-2}^n\right)\left[a_1\left\{1 - 2\gamma\left(\theta_N^n + 1\right)\right\}\right.$$
$$\left. -\gamma\lambda_1 \frac{u_N^n}{r_N} + \frac{\lambda_1}{r_N}u_N^n\right] \tag{3.31}$$

The equations (3.21)-(3.31) therefore constitute the model of finite difference scheme for the present problem to determine the values of the physical field variables u, θ, σ_{rr} and $\sigma_{\phi\phi}$ at different points of the solution domain $a \le r \le b,\ \ 0 \le t \le t_0$.

3.4.1 Truncation Error

Now, we expand the finite difference equations (3.21) and (3.22) by using Taylor series expansion and subtract from the equations (3.13) and (3.14), respectively. Therefore, we find the truncation error associated with finite difference equations (3.21) and (3.22) as follows:

$$T.E.^u = l^4\left[\frac{1}{12}\frac{\partial^4 u}{\partial t^4} + \frac{l^2}{360}\frac{\partial^6 u}{\partial t^6} + ...\right]$$
$$-l^2p^2\left(1 - \gamma - \gamma\theta_m^n\right)\left[\left(\frac{1}{12}\frac{\partial^4 u}{\partial r^4} + \frac{p^2}{360}\frac{\partial^6 u}{\partial r^6} + ...\right) + \frac{1}{r_m}\left(\frac{1}{6}\frac{\partial^3 u}{\partial r^3} + \frac{p^2}{120}\frac{\partial^5 u}{\partial r^5} + ...\right)\right]$$
$$-\gamma h^4\left(\frac{1}{6}\frac{\partial^3 \theta}{\partial r^3} + \frac{p^2}{120}\frac{\partial^5 \theta}{\partial r^5} + ...\right)\left(\frac{1}{6}\frac{\partial^3 u}{\partial r^3} + \frac{p^2}{120}\frac{\partial^5 u}{\partial r^5} + ...\right) \tag{3.32}$$

$$T.E.^\theta = l^4\left[\frac{1}{12}\frac{\partial^4 \theta}{\partial t^4} + \frac{l^2}{360}\frac{\partial^6 \theta}{\partial t^6} + ...\right]$$
$$-l^2p^2a_0\left[\left(\frac{1}{12}\frac{\partial^4 \theta}{\partial r^4} + \frac{p^2}{360}\frac{\partial^6 \theta}{\partial r^6} + ...\right) + \frac{1}{r_m}\left(\frac{1}{6}\frac{\partial^3 \theta}{\partial r^3} + \frac{p^2}{120}\frac{\partial^5 \theta}{\partial r^5} + ...\right)\right]$$
$$+\frac{l^2p^2a_0\gamma}{\left(1 - \gamma - \gamma\theta_m^n\right)}\left[\frac{1}{3}\frac{\partial \theta}{\partial r}\frac{\partial^3 \theta}{\partial r^3} + p^2\left\{\frac{1}{36}\left(\frac{\partial^3 \theta}{\partial r^3}\right)^2 + \frac{1}{60}\frac{\partial \theta}{\partial r}\frac{\partial^5 \theta}{\partial r^5}\right\} + ...\right]$$
$$+\frac{l^4a_2}{r_m}\left[\frac{1}{12}\frac{\partial^4 u}{\partial t^4} + \frac{l^2}{360}\frac{\partial^6 u}{\partial t^6} + ...\right] \tag{3.33}$$

The truncated errors given by equations (3.32) and (3.33), indicate that $\lim_{(p,l)\to(0,0)} T.E.^{u} = 0$ and $\lim_{(p,l)\to(0,0)} T.E.^{\theta} = 0$. This implies that the difference equations (3.21) and (3.22) are consistent. Thus, the finite differences (3.21)and (3.22) has the accuracy of orders $o(p^4, p^2l^2, l^4)$ and $o(p^2l^2, l^4)$ respectively.

3.5 Numerical Results and Discussion

We consider following two types of problems by taking two types of variations in the prescribed temperature distribution $f^*(t)$ at the inner surface of the cylinder:

Case-I: $f^*(t) = \mathrm{e}^{-\omega t}$

This implies that the temperature at the inner surface of the cylinder decreases exponentially with time and ω is the decaying exponent.

Case-II: $f^*(t) = \sin(\omega t)$

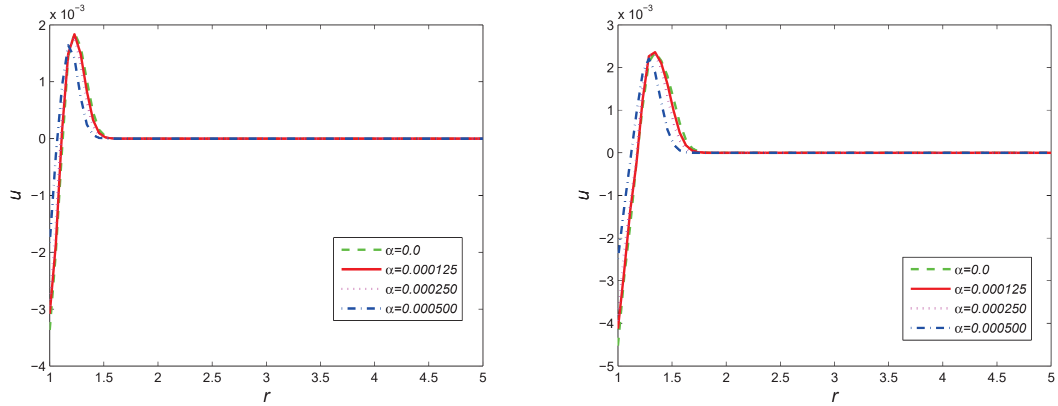
Here, we assume that the temperature at the inner surface of the cylinder varies like sine function.

For our numerical work, we consider copper material and the physical data for which is taken as follows (see Sherief and Salah (2005)):

$\theta_0 = 819$K, $\lambda_0 = 7.76 \times 10^{10}$Nm^{-2}, $\mu_0 = 3.86 \times 10^{10}$Nm^{-2}, $\rho = 8954$ kgm^{-3},

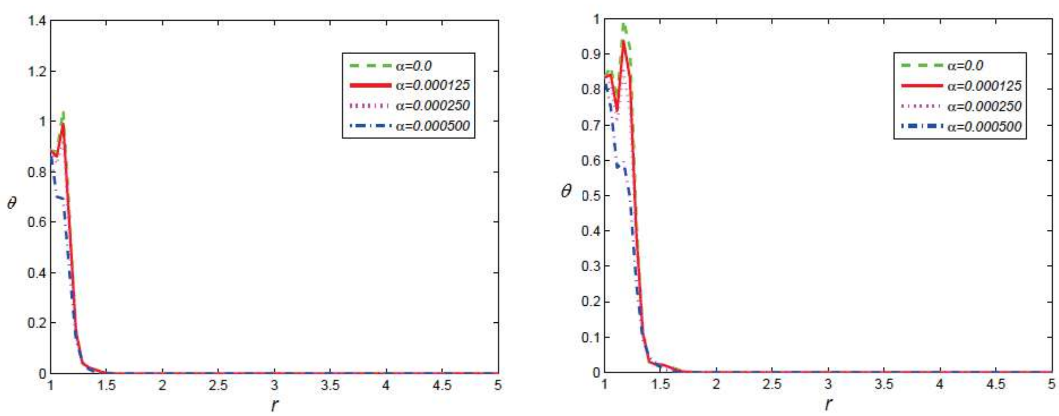
$\alpha_t = 1.78 \times 10^{-5}$K^{-1}, $\eta = 8849.6$ m^{-2}s. We assume $\omega = 0.1$.

The inner radius and the outer radius of the cylinder are taken as 1.0 and 5.0, respectively and we assume $t_0 = 1.0$ and $\upsilon = 0.0156$. Now, by using the equations (3.21), (3.22) and (3.26)-(3.31), the numerical (discrete) values of dimensionless displacement u and dimensionless temperature θ are computed simultaneously for different values of specified domain. Then, the values of stresses are computed from equations (3.23) and (3.24). We get the nature of variations of different field variables like, displacement, temperature and stresses inside the medium with the help of computer programming. In order to see the effects of temperature dependency of the material parameters, the computations



(a) (b)

Fig. 3.1(a,b)*:* Variation of displacement, u with r at times, *t=0.4* and *t=0.8,* respectively for Case-I.

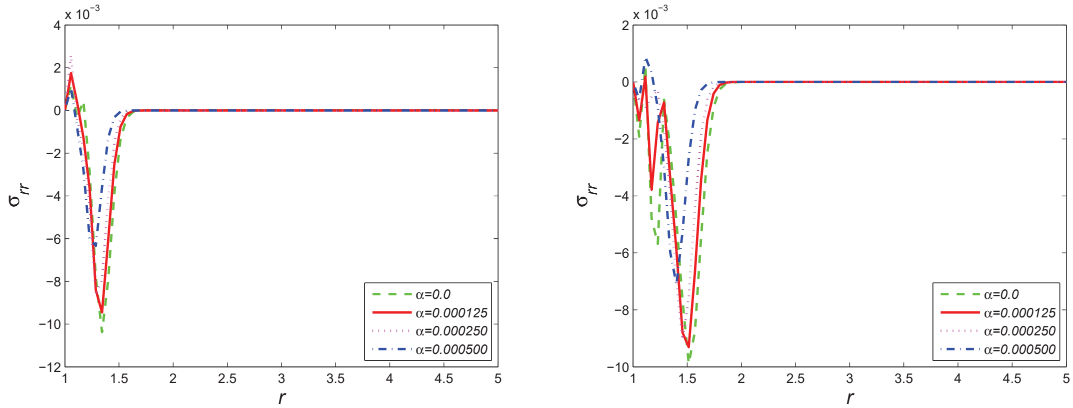


(a) (b)

Fig. 3.2(a,b): Variation of temperature, θ with t at times, *t=0.4* and *t=0.8,* respectively for Case-I.

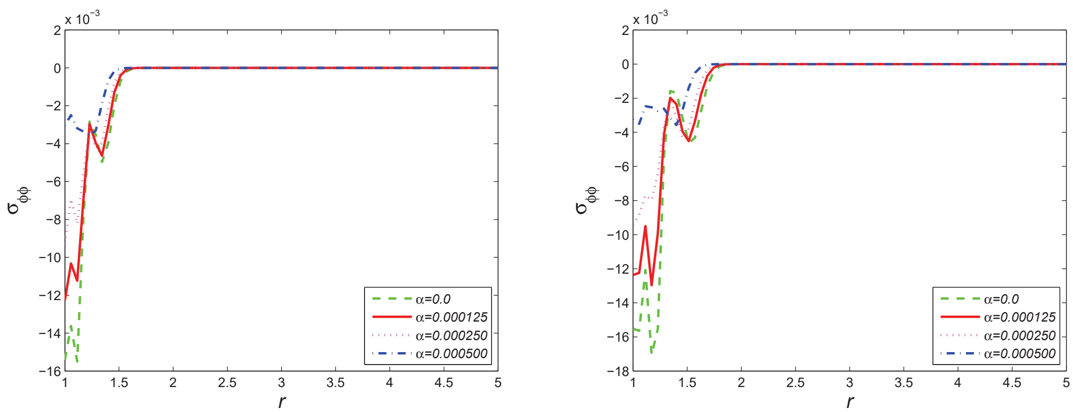
are done for different values of the parameter α. Clearly, $\alpha = 0$ indicates the case of temperature independent material properties. The results are displayed in different Figures to show the variations of different fields with respect to radial coordinates.

Figures 3.1(a,b), 3.2(a,b), 3.3(a,b) and 3.4(a,b) show the variations of displacement u, temperature θ, radial stress σ_{rr} and circumferential stress $\sigma_{\phi\phi}$, respectively at two different times $t = 0.4$ and $t = 0.8$, when the inner boundary temperature is varying exponentially (case-I) and Figs. 3.5(a,b), 3.6(a,b), 3.7(a,b) and 3.8(a,b), show the variations of u, θ, σ_{rr} and $\sigma_{\phi\phi}$, respectively at two different times $t = 0.4$ and $t = 0.8$, when the inner boundary temperature is varying as a function of sine (case-II). The



(a) (b)

Fig. 3.3(a,b): Variation of radial stress, σ_{rr} with r at times, t=*0.4* and t=*0.8*, respectively for Case-I.



(a) (b)

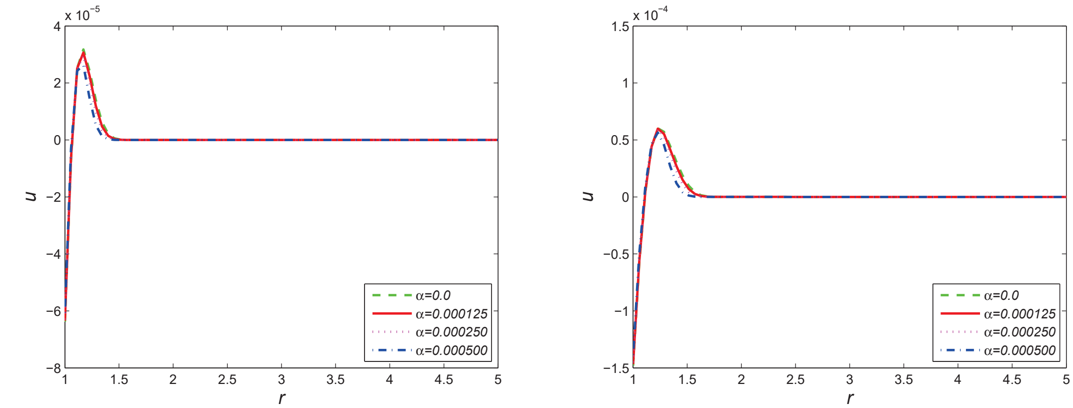
Fig. 3.4(a,b): Variation of transverse stress, $\sigma_{\phi\phi}$ with r at times, t=*0.4* and t=*0.8*, respectively for Case-I.

nature of variations of various fields observed in different figures indicate that our system of difference equations (3.21)-(3.31) efficiently compute the numerical solutions of the problem and the solutions obtained are in complete agreement with the theoretical boundary conditions of the problem. We observe several important facts evident from the graphical results as mentioned below.

Figures 3.1(a,b) and 3.5(a,b), showing the distributions of displacement indicate that the nature of variation of displacement inside the annulus is similar in both the cases: whether the inner boundary surface of the annulus is subjected to exponentially varying temperature or to sinusoidal varying temperature. However, there is a significant effect of the temperature dependency on the material parameters. The numerical values of displacement is maximum in temperature independence case and the values of u decreases with the increase of parameter, α. The differences of displacement profiles for the cases of temperature dependent material properties and the case of temperature independent properties increases with the increase of time and the region of influence increases with time. Figures 3.1(a,b) and 3.5(a,b) further indicate that the effect of temperature dependency of material parameters on displacement is more significant in case of exponentially varying temperature applied at the inner surface of the cylinder as compared to the case of sinusoidal temperature distribution applied at the inner boundary. The displacement field achieves zero values in all the cases after some distance from the inner boundary which proves the fact that the theory of thermoelasticity without energy dissipation accounts for finite speeds of elastic as well as thermal disturbances.

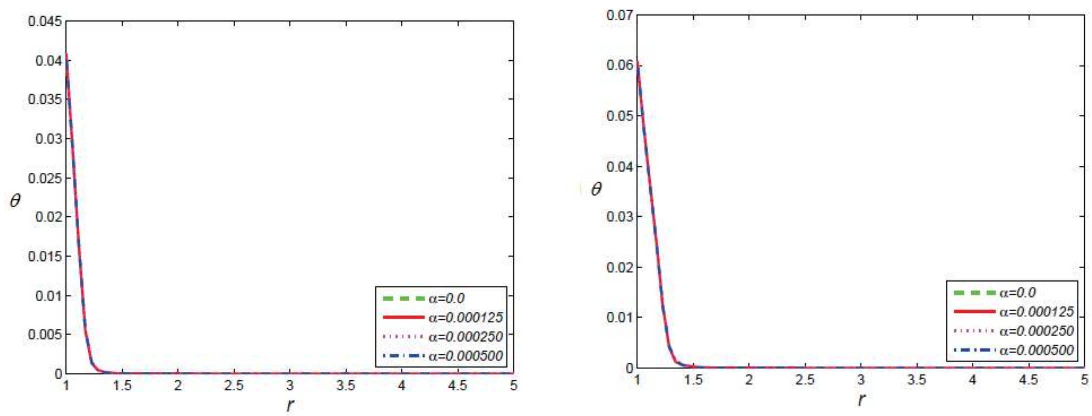
The variation of temperature can be observed from Figs. 3.2(a,b) and 3.6(a,b). Figures 3.2(a, b) show the variation of this field when exponentially decaying temperature is applied at the inner surface of the cylinder and Figs. 3.6(a,b) depicts the case when the inner boundary is subjected to sinusoidal temperature distribution. The nature in variation in temperature in these two cases are very much different, specially near the inner boundary. The variation is oscillatory in nature near the boundary in the case-I as compared to the case-II. Temperature shows larger value in the case when we consider that material parameters are independent of temperature. Furthermore, the effect of temperature dependency of material parameters is very much significant in the first case, while it is almost negligible in the second case of prescribed boundary temperature. The region of influence increases with the increases of time for this field too.

Figures 3.3(a,b) and 3.7(a,b) show the variation of radial stress and we observe a significant difference in the nature of variation of radial stress in cases of two types of boundary temperatures prescribed at the inner boundary of the annulus. In the second case, the radial stress is fully compressive, while it is tensile for some region after some



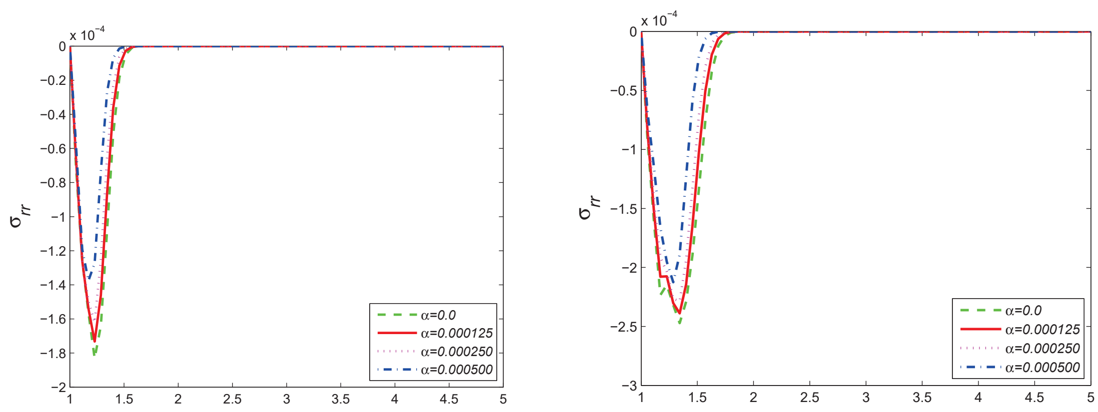
(a) (b)

Fig. 3.5(a,b): Variation of displacement, u with r at times, t=*0.4* and t=*0.8*, respectively for Case-II.



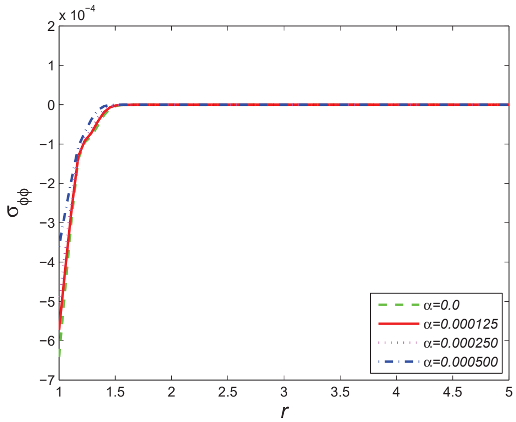
(a) (b)

Fig. 3.6(a,b): Variation of temperature, θ with r at times, t=*0.4* and t=*0.8*, respectively for the Case-II.

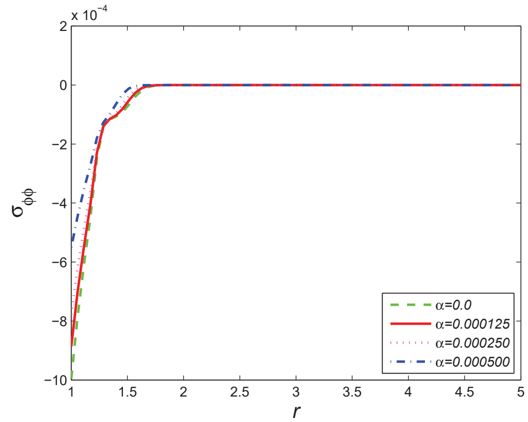


(a) (b)

Fig. 3.7(a,b): Variation of radial stress, σ_{rr} with r at times, t=*0.4* and t=*0.8*, respectively for Case-II.



(a)



(b)

Fig. 3.8(a,b): Variation of transverse stress, $\sigma_{\phi\phi}$ with r at times, t=*0.4* and t=*0.8*, respectively for Case-II.

distance from the inner surface and thereafter becoming compressive. Like temperature field, radial stress also shows oscillatory nature in the first case. However, the effect of α is very much pronounced in both the cases. Numerical values of radial stress is maximum in the case of temperature independent physical parameters and the value of radial stress decreases with the increase of α. Circumferential stress distributions for two different cases can be observed from Figs. 3.4(a,b) and 3.8(a,b). Like the case of radial stress, circumferential stress also shows significantly different trend of variation in two different types of temperatures prescribed at the inner surface of the cylinder. The nature is more oscillatory near the inner boundary in the case when exponentially decaying temperature is prescribed. The effect of α is more pronounced in the first case as compared to the second case and the absolute value of this stress is maximum in the case of temperature independent physical properties. The difference in the trend of variation increases with the increases of time as well as with α.

It is interesting to make a comparison between the results in case-I of Green-Naghdi model and the corresponding results predicted by Lord-Shulman model as discussed by Mukhopadhyay and Kumar (2009) to demonstrate the effects of temperature dependent properties. Figure 3.1(a) and the corresponding figure for displacement of Mukhopadhyay and Kumar (2009) show that u has a maximum value nearer to the boundary of annulus cylinder in case of both the models while this maximum value is slightly smaller in case of GN-II model. Further, the displacement decreases as the value of α increases which agree with the corresponding figure of LS model. From Fig. 3.2(a), the temperature distribution shows a oscillatory behaviour in the region nearer to the inner boundary of the annulus in case of GN-II model and there are extreme points. However, the corresponding figure of LS model exhibits that temperature distribution shows a smooth decrements without any local extreme points. The temperature also decreases with α under both the models. Fig. 3.3(a) shows that in the context of GN-II model, the radial stress is tensile for some region nearer to the inner surface and thereafter becoming compressive, but in the case of LS theory, the radial stress, σ_{rr} was shown to be fully compressive (Mukhopadhyay and Kumar (2009)). Furthermore, it is clear that σ_{rr} is inversely proportional to α for GN-II model while it is directly proportional to α in case of LS model which found to be a notable difference in two different thermoelasticity theories. On comparision of the Fig. 3.4(a) with the corresponding figure under LS model, the effect of α is similar in nature for both the models. Furthermore, both the stresses show oscillatory type variation through the radial direction in case of GN-II model while LS model shows smooth variation in stresses. The region of influence for each phyical field is observed to be finite under both the theories supporting the fact that thermal wave propagate with finite speed in the context of LS-model as well as in case of GN-II model. However, the influence area for all physical quantities is much much narrow under GN-II model (see Figs. 3.1(a), 3.2(a), 3.3(a), 3.4(a)) as compared to LS model (Mukhopadhyay and Kumar (2009)).

3.6 Concluding Remarks

The effects of temperature dependency of material properties on thermo-mechanical responses of an annular cylinder whose inner surface is subjected to time dependent temperature fields has been analyzed by employing thermoelasticity theory of type GN-II. The governing equations are derived by employing this theory and considering temperature dependent physical properties. Governing equations are obtained as non-linear coupled partial differential equations and we apply finite difference method for solving the coupled system for the present problem. We showed that the present problem can be efficiently solved by finite difference method. We obtain the orders of truncation error for displacement and temperature.

Our results highlight the significant effects of temperature dependent material properties on thermoelasticity. We considered two different types of temperature distributions prescribed at the inner boundary surface of the cylinder where as the outer surface is kept at reference temperature. The effects of temperature dependent properties are shown to be of different nature in two cases. However, in both the cases it has been observed that under the present context, the difference in the numerical results with temperature dependent properties and temperature independent properties are very much pronounced. The region of influence is observed to be finite for all field variables. We note a significant difference in the prediction of results for exponential temperature distribution prescribed at the inner boundary of the cylinder in the present context with the corresponding results under thermoelasicity with one relaxation parameter. The region of influence if much smaller in case of GN-II model as compared to the same under LS model. For more accurate analysis of thermoelastic behavior of structural elements, the temperature dependency needs to be considered. Hence, this study is believed to be useful in characterizing the thermoelastic responses of structural element with temperature dependent properties under different thermoelastic models.