APPENDIX-A

The method for the numerical inversion of Laplace transform (Honig and Hirdes (1984)) which has been followed in section 5.1, is outlined here.

The inversion of the Laplace transform is defined by the formula:

$$f(t) = \frac{1}{2\pi \mathrm{i}} \int_{c-\mathrm{i}\infty}^{c+\mathrm{i}\infty} e^{pt} \bar{f}(p) dp \qquad \textbf{(A.1)}$$

where c is an arbitrary real constant greater than all the real parts of the singularities of $\bar{f}(p)$.

Taking $p = c+i\omega$, we get from (A.1)

$$f(t) = \frac{\mathrm{e}^{\mathrm{ct}}}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}t\omega} \, \bar{f}(c+i\omega) \, \mathrm{d}\omega \qquad \textbf{(A.2)}$$

Expanding the function $h(t) = \exp(-ct) f(t)$ in a Fourier series in the interval [0, $2T$], we obtain the following approximate formula

$$f(t) = f_\infty(t) + E_D$$

where $$f_\infty(t) = \frac{1}{2} c_0 + \sum_{k=1}^{\infty} c_k \quad \text{for } 0 \le t \le 2T \qquad \textbf{(A.3)}$$

with $$c_k = \frac{e^{ct}}{T} \mathrm{Re}\left[e^{ik\pi t/T} \bar{f}(c + ik\pi/T) \right]$$

E_D, the discretization error can be made arbitrarily small by choosing 'c' large enough. As the infinite series in equation (A.3) can only be summed to a finite number (say, N) of terms, the approximate value of $f(t)$ becomes

$$f_N(t) = \frac{1}{2} c_0 + \sum_{k=1}^{N} c_k \,, \qquad \text{for } 0 \le t \le 2T \qquad \textbf{(A.4)}$$

Using the above formula to evaluate $f(t)$, we introduce a truncation error E_T that also needs to be added to the discretization error to produce the total approximation error. The

'Korrecktur' method is used to reduce the discretization error and ε-algorithm is used to reduce the truncation error and hence to accelerate the convergence.

The Korrecktur method uses the following formula to evaluate the function $f(t)$:

$$f(t) = f_{\infty}(t) - e^{-2dT} f_{\infty}(2T + t) + E'_D$$

where the discretization error $|E'_D|$ << $|E_D|$. Therefore the approximate value of $f(t)$ becomes

$$f_{NK}(t) = f_N(t) - e^{-2dT} f_{N'}(2T + t)$$

where N' is an integer such that $N' < N$.

We shall now describe the ε-algorithm that is used to accelerate the convergence of the series in equation (A.4). Let $N = 2q+1$ where q is a natural number, and let $s_m = \sum_{k=1}^{m} c_k$ be the sequence of partial sums of equation (A.4). We define the ε-sequence by

$$\varepsilon_{0,m} = 0 \qquad \varepsilon_{1,m} = s_m$$

$$\varepsilon_{p-1,\, m} = \varepsilon_{p-1,\, m-1} + 1/\{\varepsilon_{p,\, m+1} - \varepsilon_{p,\, m}\} \qquad p = 1, 2, 3 \ldots$$

It can be shown that the sequence

$\varepsilon_{1,1}$, $\varepsilon_{2,1}$, ………, $\varepsilon_{N,1}$ converges to $f(t) + E_D - c_0/2$ faster than the sequence of partial sums s_m for $m = 1, 2, 3$ ….