CHAPTER 8

SUMMARY OF THE THESIS AND SCOPE FOR FURTHER WORK

8.1 Summary of the Work

Thermoelasticity is the branch of science dealing with the influence of the temperature of an elastic solid upon the distribution of stress and strain, and the inverse effects of deformation upon the temperature distribution.This subject has been stimulated by the advances in several branches of engineering sciences. Specially, thermal stress play a primary role in the field of aircraft and machine structure. The present thesis is concerned with the mathematical modeling on various problems involving thermoelastic interactions in the contexts of some recently developed thermoelasticity theories. The thesis is basically divided into two different parts concerning with mathematical modeling on various types of coupled thermo-mechanical problems in the contexts of these recent models. It is aimed at analyzing various aspects of these recently proposed thermoelasticity theories by investigating various problems involving thermoelastic interactions inside thermoelastic medium due to various types of thermo-mechanical loads. Specially, we concentrate on the two-temperature thermoelastic model with two relaxation parameters and the thermoelastic model with a single delay term. Problems on homogeneous as well as non homogeneous medium are considered and different methodologies are applied to solve the problems. The thesis brought some light into the effects of applying the recently developed non-Fourier heat conduction models to understand thermo-mechanical responses under some situations.

For investigating the two-temperature thermoelastic model with two relaxation parameters, we studied three different problems (**Chapters-2, 3 and 4).** The first two problems are based on isotropic and homogeneous medium, where as the third problem is considered for a non-homogeneous medium. **In the first problem**, an attempt has been made to investigate the thermoelastic interactions in an isotropic homogeneous elastic medium with a cylindrical cavity in the context of this two-temperature theory of thermoelasticity with two relaxation time parameters. We investigate the thermoelastic interactions inside the medium due to thermal shock applied at the stress free boundary of the cavity and present analytical results for different physical fields. We further present a detailed comparative analysis of the results in the present context with the corresponding results under Lord–Shulman model, Green–Lindsay (GL) model and also under two-temperature model with one relaxation parameter. Some distinct predictions of the two-temperature model as compared to the conventional one-temperature model are highlighted through this study which indicate some significant features of the two-temperature thermoelastic model. It has been shown that the two-temperature model does not predict a finite speed of the thermal signal and a predominantly thermal wave is not found as a contribution to the solution of the physical fields, although these theories include the thermal relaxation parameters. This is obviously a significant feature of the two-temperature model and these models therefore may not be referred to as generalized thermoelastic models which have a significant feature of admitting a finite speed of the thermal wave.

In the second problem under the two-temperature model, the propagation of plane harmonic waves in a homogeneous and isotropic unbounded medium is interpreted. Here, we investigate the effects of two thermal relaxation time parameters on plane harmonic wave under two-temperature theory for the purpose of comparing the results predicted by TGL model with the corresponding results of TLS, LS and GL models. We obtain the dispersion relation solutions of longitudinal plane waves and find the asymptotic expansions of several qualitative characterizations of the wave field, such as, phase velocity, specific loss and penetration depth for the high and low frequency values for this model. In order to verify these analytical results predicting the limiting behavior of the wave characteristics, numerical values of the above mentioned quantities for intermediate values of frequency are also computed directly by applying computational tool. We observe some important facts comparing our analytical as well as numerical results with the corresponding results of all four models as mentioned.

Under both the LS and GL models, the phase velocity of thermal mode wave tends to a constant limiting value as the frequency increases, where as under the two-temperature theories the phase velocity of thermal wave increases with the increase of frequency and tends to infinity as frequency tends to infinity. This is an unrealistic prediction by two-temperature theories like the classical coupled theory. As a significant difference, the specific loss of thermal wave tends to infinity as frequency, $\omega \to \infty$ for TLS model, but this field tends to zero under TGL model. However, the specific loss of thermal wave tends to zero as $\omega \to \infty$ for GL and LS models. This is also a significant difference predicted by the generalized and the two-temperature theories including thermal relaxation parameters. The penetration depth profile also shows a significant difference between the generalized theories and the two models of two-temperature theories. The penetration depth of thermal wave tends to a constant limiting value as $\omega \to \infty$ for TLS and TGL models but it tends to zero as $\omega \to \infty$ in case of generalized theories (LS and GL models). This is also an unrealistic feature predicted by the two-temperature models. Furthermore, there is a prominent difference between the results predicted by LS and GL, TLS and TGL models for the penetration depth for elastic wave. The trend is same for LS and GL models. The penetration depth for elastic wave tends to a constant value as $\omega \to \infty$ in both the cases. A similar nature is also observed in the context of TGL model, although the constant limiting value is nearer to 120 in cases of LS and GL models, and it is unity in case of TGL model. However, the penetration depth of elastic wave tends to infinity as $\omega \to \infty$, which is an unrealistic prediction by TLS model and this feature is not observed in case of TGL model. The models of two-temperature theories includes thermal relaxation time parameters like the generalized theories of thermoelasticity. However, they suffer the similar drawbacks like the classical theory. Furthermore, all the wave characteristics for both the waves show almost similar results in the contexts of LS and GL theories. However, the nature of wave components predicted by TGL and TLS models are significantly different and it is more prominent for the the wave characterizations of the elastic mode wave.

The third problem under this model analyzed the effects of temperature dependent thermal conductivity on thermoelastic interactions inside a medium with a spherical cavity. The thermal conductivity of the material is assumed to be dependent on temperature linearly. Initially, the temperature at the boundary of the spherical cavity is assumed to be subjected to a thermal shock and it is assumed that there is no stress on the surface of the cavity. A detailed analysis of the obtained results due to temperature dependent material properties and effects of employing two-temperature model is presented. We highlight some important features of the present two-temperature model in the context of temperature dependent thermal conductivity.

The effect of the temperature dependent parameter and time on displacement, temperature and stress components under TGL model is significant and the influence region is dominant near the boundary of the spherical cavity and it is insignificant when we move away from boundary of the cavity.

There is no prominent difference in the field variables predicted by GL and LS models or between TGL and TLS thermoelasticity theories. However, the difference is notable under a two-temperature thermoelasticity theory and without a two-temperature thermoelasticity theory.

Next, we investigate the newly introduced thermoelastic model

with a single delay term given by Quintanilla (2011). This model considers all the micro-structural effects in the heat transport phenomenon like dual-phase-lag and three-phase-lag model. We investigate this model by considering **three different problems (Chapters-5, 6 and 7). Firstly,** we make an attempt to investigate a problem of thermoelastic interactions in the context of this model. The state-space approach is employed to formulate the problem and the formulation is then is applied to solve a boundary value problem of an isotropic elastic half space with its plane boundary subjected to sudden increase in temperature and zero stress. The short-time approximated solution for the field variables is obtained analytically. A detailed analysis of analytical results is provided. We observe that each of the distributions of the physical fields like, temperature, displacement and stresses consists of two main parts. In the first part, the term involving corresponds to a wave propagating with finite speed (unity) which can be identified as predominantly elastic wave. Under this new theory of thermoelasticity, the modified elastic wave propagates without any attenuation. This is a distinct feature predicted by the theory and it has a similarity with the thermoelasticity without energy dissipation (GN-II) model. The second part of the solutions of the field variables does not indicate to be a contribution of a wave, instead it is diffusive in nature indicating an exponential decay with distance with an attenuating coefficient. This represents that under this present model, the thermal wave do not propagate with finite wave speed like, other generalized thermoelasticity theories: LS model, GL model, GN-II model, dual-phase-lag model or three-phase-lag model. However, a similar nature is observed under GN-III model . Furthermore, we note that the solutions for temperature and displacement are continuous in nature. However the analytical results obtained for stress distribution has discontinuity with finite jumps at the elastic wave front. An attempt has also been made to illustrate the problem and numerical values of field variables are obtained for a particular material. Results are analyzed with different graphs and a comparison of the results with others existing models of thermoelasticity is made. It is observed that behavior of solutions in the context of the present new model has similarity with the corresponding results under GN-III model. It is also revealed from our numerical results that the new thermoelastic model based on the exact heat conduction model with a delay predicts almost similar results like GN-III model as compared to GN-II and LS models.

In the next problem, we aim at the investigation of thermoelastic interactions in a temperature dependent spherical shell under the same model (Quintanilla (2011)). The thermal properties of the medium under the present thermoelasticity theory is taken as linear function of temperature. We consider the problem to be studied under three different kinds of boundary conditions and we compute the numerical results of the field variables in each case. Results are shown in the different graphs to study the influence of temperature dependent thermal parameters in the context of new model.

In the first case, i.e., when we consider that the inner and outer boundaries of the spherical shell are traction free and are subjected to a unit step increase in temperature, we find that the variation in displacement is prominently affected only near the boundaries and through the middle region of the shell the effect of time and temperature dependent property on displacement is negligible. The amplitude of displacement increases with time. It is further evident that at higher time, the absolute value of displacement decreases with larger numerical values of temperature dependent parameter and for smaller time, the dependency of thermal parameters on temperature is negligible for displacement. In this case, the temperature field is more sensitive to the temperature dependent material properties. The temperature increases with the increase of the temperature dependency parameter. Both radial stress and shear stress components show significant variation near the boundaries and in the middle region of the shell. The stresses increase with higher negative value of temperature dependent parameter at any time. One important fact is observed that the effect of temperature dependency of stresses is independent of time, implying that at any time, a similar effect of temperature dependency is observed for the stress components in this case.

In second case, the inner boundary is subjected to an exponential variation in temperature and the outer boundary is maintained to be insulated. It is observed that there is no adequate effect of temperature dependent material properties on any physical quantities except the field temperature. At any time, the same effect of temperature dependency is noted for all the field variables. It can be seen that the effective value of displacement increases with increase of time. Displacement tends to zero through the radial distance for all times. The values of temperature increases with respect to the time and also w.r.t. temperature dependent parameter. It could be observed that the effective regions of stress components increases with increase of time.

In the last case the sinusoidal varying temperature and zero displacement at the boundary of the spherical shell is taken. It is observed that like case-I, the influence of temperature dependent properties is prominent in this case. The displacement is positive throughout the distance and gets a local maximum value within middle region of the shell. The effect of temperature dependent property is more prominent at higher times. The displacement increases with time implying that the region of influence for displacement increases with the time. However, it decreases with larger numerical value of temperature dependent parameter. It is depicted that the temperature is influenced significantly by the temperature dependent property of the material. The temperature field has an increasing trend with time and also with temperature coefficient. The stresses are compressive in nature throughout region of the medium and the influence of temperature dependent properties on stress distributions is much more prominent in this case. This effect on stresses is prominent near the inner boundary of the shell.

Lastly, in Chapter-7, we make an attempt to investigate a problem of spherical shell with functionally graded material in the context of this recent heat conduction model (Quintanilla, 2011). We employ Galerkin type finite element method along with Laplace transform technique to solve a system of non-linear coupled partial differential equations arising out in the coupled problem. Considering a metal-ceramic FGM, the inner boundary of the shell is assumed to be stress free and is subjected to thermal shock while the other boundary is insulated with temperature and fixed with some rigid support. We consider a law of mixture of material constituents so that the effective material properties follow a rule of volume-fraction. We show that the present problem can be solved by trans-finite element method. We carry out our computational work in MATLAB programming code and obtain the numerical solution of the problem. The results are displayed in different graphs to display the distributions of the field variables inside the shell at different time and various types of non-homogeneity condition. A detailed comparative study on the obtained results is discussed to focus on the functionally graded material and the effect of non-homogeneity. The present work specially considers to highlight the necessity of the finite element method for the solution of problems on coupled thermoelasticity for non-homogeneous medium and to display a compact analysis of different coupled thermoelasticity theories for functionally graded materials. We observe that the effect of non-homogeneity is prominent. Further, the predictions of different models for the present problem are significantly different and the new model shows the highest disturbance range due to thermal loading as compared to the other models. The GN-II model has the lowest region of influence for each field at any time.

8.2 Future Scopes

Some heat conduction models have became the center of active research for last few decades. However, it is believed that there are mathematical issues and challenges with these non-Fourier heat conduction models to clarify the possibilities of these equations to define a stable theory. There is a need of establishing the well-posedness of any problem under a non-Fourier heat conduction model so that mathematical consistency and physical relevance of the model can be well understood. It is also worth pursuing research to analyze the non-Fourier heat conduction models by applying them to some real world problems, specially, thermo-mechanical and bio-thermo-mechanical problems where heat conduction equation plays very important role. Various generalizations are proposed to overcome the limitations/drawbacks in uncoupled thermoelasticity and the classical theory of thermoelasticity. The present thesis analyzed the predictions of some recently proposed theories and highlighted the drawbacks in some newly proposed models. There is a need of further research in this respect to understand the coupling effects of thermal and mechanical fields. It is believed that the best suitable model may be identified through further research in theoretical as well as in experimental direction. Heat and mass transport is fundamental to many biological processes. Area like biomedical device development requires understanding how energy and mass can be transported through bio materials and dealing of bio-thermo-mechanical problems requires understanding of multi-phase transport mechanism inside tissue during laser surgery. Hence it is worth pursuing mathematical modeling on this issue by taking into account the coupling effects of thermal fields with other fields and to understand whether non-Fourier heat conduction models have significant applications in this direction. FGMs are composite media that have continuously changing material properties and they constitute a new branch of materials developed with the purpose of design of structures to withstand suddenly applied loads as compared with traditional laminated composites. It is very important to highlight the thermo-mechanical responses of structural elements with FGM properties. Work carried out in this direction is rare as the mathematical formulation on coupling effects of FGMs with thermal field is a challenging task. Hence, it is worth pursuing further research on this topic. We conclude the present thesis with a great expectation that the above mentioned points will be taken under consideration in near future.