PREFACE

Fourier law of heat conduction has been widely and successfully applied to the conventional engineering heat conduction problems. However, it has been realized through active research in recent years that Fourier law is applicable to those problems that involve large spatial dimension and when the focus is on long time behavior, whereas it yields unacceptable results in situations involving temperature near absolute zero, extreme thermal gradients, high heat flux conduction and short time behavior, such as laser-material interactions. Furthermore, due to the advancement of modern technology of material processing by pulsed sources, Fourier law has been shown to be inadequate in modeling laser processing of materials and high frequency response of materials. Hence, intense efforts are put forth since 1950s to better understand the limitations of Fourier law and for more accurate predictions of temperature. Accordingly, some non-Fourier heat conduction models accounting for finite speed of heat propagation are proposed. Some of the non-Fourier heat conduction models which are worth to be mentioned are Catteneo-Vernotte model, low temperature model, dual-phase-lag model, two-temperature heat conduction model etc. It has been reported by both experimental and theoretical results that these heat conduction models yield qualitatively different results as compared to the conventional Fourier model. Some heat conduction models have become the center of active research for last few decades.

Changes in temperature causes thermal effects on most of the materials. If the thermal expansions or contraction are not freely admitted due to temperature change, thermal stress is developed inside the material. Further, if the material is compressed or expanded the volume change is accompanied by heating and cooling. The subject "thermoelasticity" deals with the influence of the temperature of an elastic solid upon the distribution of stress and strain, and the inverse effects of mechanical deformation upon the temperature distribution. Thermoelasticity is therefore considered to be the branch of science dealing with the mutual interactions between the mechanical and thermal fields and relates two separate and independently developed sciences: the theory of elasticity and the theory of heat conduction. This subject has been stimulated by the advances in several branches of engineering sciences. Specially, thermal stress play a primary role in the field of aircraft and machine structure. At initial time of research, the investigations in this area were based on the "uncoupled theory of thermoelasticity" by ignoring the influence of the strain and stresses on the temperature field. It must be mentioned here that introducing the strain-rate term in the uncoupled heat conduction equation, Biot (1956) extended the analysis to incorporate coupled thermoelasticity for the first time. However, this theory was based on Fourier law. Hence shortcoming of uncoupled thermoelasticity is although eliminated in this theory, but there remained the parabolic type partial differential equation of heat conduction predicting infinite velocity of the thermal wave. Subsequently, generalized thermoelasticity theory is developed to eliminate this inadequacy of the classical

Biot's model.

The concept of generalized thermoelasticity covers a wide range of extensions of classical dynamical coupled thermoelasticity. This newly emerged theory admits finite speed of heat propagation and is therefore referred to as the hyperbolic thermoelasticity theory. The first generalized theory of thermoelasticity is due to Lord and Shulman (1967) in which coupled elasticity theory and theory of heat conduction are coupled with a way in which temperature can travel with finite wave speed. The second generalization to the coupled theory is known as the generalized theory with two relaxation times. Muller (1971) introduced the theory of generalized thermoelasticity with two relaxation times. A more explicit version was then developed by Green and Lindsay (1972). In this theory, the temperature rates are considered among the constitutive variables and this theory also predicts finite speed of propagation for heat and elastic waves. As mentioned by Chandrasekharaiah (1986), this wave-like thermal disturbance in generalized thermoelasticity is now termed as "second sound" effect.

Later on, Green and Naghdi (1991; 1992; 1993) proposed a new thermoelasticity theory based on entropy equality rather than the usual entropy inequality. The constitutive assumptions for the heat flux vector are different in their theory. This theory is considered to be an alternative thermoelasticity theory that is divided into three parts which are subsequently called as theory of thermoelasticity of type-I, II and III or theories of type GN-I, II and III. The linearized form of GN-I exhibits the paradox of infinite heat propagation speed. In case of GN-II theory, the internal rate of production of entropy is taken to be identically zero, implying no dissipation of thermal energy (1993). GN-III includes the previous two models as special cases and admits dissipation of energy.

The concept of heat conduction with phase-lag is introduced by Tzou (1992). Subsequently, Tzou (1995b) introduced the dual-phase-lag heat conduction by including two phase-lags in the Fourier's law of heat conduction in order to take into account the micro structural effects that arise in high rate heat transfer. Later on, two phase-lag thermoelastic models are developed by Chandrasekharaiah (1998) by considering this dual-phase-lag heat conduction law given by Tzou (1995b). Roychoudhari (2007) has developed another thermoelasticity theory with the concept of a new heat conduction equation. He has established the three-phase-lag constitutive model by introducing the phase-lags of the heat flux, temperature gradient and thermal displacement gradient in heat conduction equation proposed in type-III model by Green and Naghdi. Recently, Quintanilla (2011) has reformulated the three-phase-lag model in an alternative way by defining a relation between the phase-lag parameters. When we combine this heat conduction equation with the energy equation then we obtain a new form of heat conduction equation. Subsequently, Leseduarte and Quintanilla (2013) have established the mathematical consistency in the system of equations in this new model and examined this reformulated model in a precised way.

In the mechanics of continuous media, a material is said to have memory effect or hereditary characteristics if the behavior of the material at time, t is specified in terms of the experience of the body up to the time, t. Coleman (1964) formulated a theory of materials with memory. An alternative thermoelasticity theory called as two-temperature thermoelasticity theory is proposed by Gurtin and Williams (1966), Chen and Gurtin (1968), Chen and Williams (1968) and Chen *et al.* (1969). The two-temperature thermoelasticity theory proposes that the heat conduction on a deformable body depends on two different temperatures- the conductive temperature and the thermodynamic temperature. According to this theory, the entropy contribution due to heat conduction is governed by thermodynamic temperature and that of the heat supply by the conductive temperature. Uniqueness and reciprocity theorems for the two-temperature thermoelasticity theory in case of a homogeneous and isotropic solid have been reported by Iesan (1970). Recently, this two-temperature model of thermoelasticity has drawn the serious attention of researchers. Puri and Jordan (2006) reported a detailed investigation on a plane harmonic wave under this theory. Youssef (2006b) extended this theory in the frame of the generalized theory of heat conduction and formulated two versions of two-temperature theory with relaxation parameters by providing the uniqueness theorem. Subsequently, some investigations have been carried out on the basis of this two-temperature thermoelastic model with one relaxation time and have highlighted some interesting results on this two-temperature theory.

Main Objective of the present thesis is concerned with the mathematical modeling on various problems involving thermoelastic interactions. The thesis aims to study the physical behavior of field variables of various thermoelastic systems under different thermoelastic models and thereby to understand the basic differences among these models with respect to the responses of the field variables due to thermoelastic interactions. We concentrate on the two-temperature thermoelastic model with two relaxation parameters and the thermoelastic model with a single delay term. Different types of coupled problems are solved. Problems on homogeneous as well as non homogeneous medium are considered and different methodologies are applied to solve the problems.

The thesis consists of eight chapters. The **Chapter-1** is the introductory in which the basics of coupled thermoelasticity theory along with its recent development is described. The relevant literature review in the concerned area of the topic of the thesis is also provided in this chapter.

In Chapter-2, an attempt has been made to investigate the thermoelastic interactions in an isotropic homogeneous elastic medium with a cylindrical cavity in the context of the two-temperature theory of thermoelasticity with two relaxation time parameters. Chandrasekharaiah and Keshavan (1992) have studied axisymmetric thermoelastic interactions in an unbounded body with a cylindrical cavity by using the classical coupled thermoelastic model, the Lord–Shulman model and Green–Lindsay (GL) model (1972) in a unified way. The results of the present work are compared with the corresponding results reported in Chandrasekharaiah and Keshavan (1992) and Mukhopadhyay and Kumar (2009). Some distinct predictions of the two-temperature model as compared to the conventional one-temperature model are highlighted through this study which indicate some significant features of the two-temperature thermoelastic model.

In Chapter-3, the propagation of plane harmonic waves in a homogeneous and isotropic unbounded medium in the reference of the linear theory of two-temperature thermoelasticity with two relaxation parameters (TGL) is interpreted. The propagation of plane waves in a thermoelastic medium has been investigated earlier by Kumar and Mukhopadhyay (2010b) for two-temperature thermoelasticity with one thermal relaxation parameter (TLS). We investigate the effects of two thermal relaxation time parameters on plane harmonic wave under two-temperature theory for the purpose of comparing the results predicted by TGL model with the corresponding results of TLS, LS and GL models. For this, we formulate the problem in the context of four models of thermoelasticity, namely: thermoelasticity with one relaxation parameter (LS model), thermoelasticity with two thermal relaxation parameters (GL model), two-temperature thermoelasticity with one relaxation parameter (TLS) and the two-temperature thermoelasticity with two relaxation parameters (TGL model) in a unified way. After mathematical formulation of the present problem, we obtain the dispersion relation solutions of longitudinal plane waves. We find the asymptotic expansions of several qualitative characterizations of the wave field, such as, phase velocity, specific loss and penetration depth for the high and low frequency values for this model. In order to verify these analytical results predicting the limiting behavior of the wave characteristics, numerical values of the above mentioned quantities for intermediate values of frequency are also computed directly by applying computational tool. Finally, we analyze the results in a detailed way by comparing our analytical as well as numerical results with the corresponding results of all four models as mentioned. Several important points regarding the predictions of various theories for the harmonic plane waves are highlighted.

The Chapter-4 of the thesis analyzes the effects of temperature dependent thermal conductivity on thermoelastic interactions inside a medium with a spherical cavity under two-temperature generalized thermoelastic theory that involves two thermal relaxation parameters. The thermal conductivity of the material is assumed to vary with temperature linearly. Initially, the temperature at the boundary of the spherical cavity is assumed to be subjected to a thermal shock and it is assumed that there is no stress on the surface of the cavity. We solve the problem by using Kirchhoff transformation along with Laplace transform technique. Various graphs are plotted to display the distributions of different field variables like, conductive temperature, thermodynamic temperature, displacement and two non-zero components of stress. An attempt is also made to compare the results in the present context with the corresponding results predicted by other thermoelasticity theories. A detailed analysis of the obtained results due to temperature dependent material properties and effects of employing two-temperature model is presented. We highlight some important features of the present two-temperature model in the context of temperature dependent thermal conductivity.

In Chapter-5, we consider the newly introduced thermoelastic model with a single delay term given by Quintanilla (2011). This model considers all the micro-structural effects in the heat transport phenomenon like dual-phase-lag and three-phase-lag model. We make an attempt to investigate a problem of thermoelastic interactions in the context of this model. The state-space approach is employed to formulate the problem and the formulation is then applied to solve a boundary value problem of an isotropic elastic half space with its plane boundary subjected to sudden increase in temperature and zero stress. The Laplace transform is applied to obtain the solution of the problem. The short-time approximated solution for the field variables is obtained analytically. A detailed analysis of analytical results is provided. An attempt has also been made to illustrate the problem and numerical values of field variables are obtained for a particular material. Results are analyzed with different graphs and a comparison of the results with others existing models of thermoelasticity is made. To the best of the author's knowledge, this thermoelastic model is not yet received much attention of researchers. Hence, this problem is considered to understand the basic feature of this new model with respect to other well established models of thermoelasticity.

The Chapter- 6 of the thesis is aimed at the investigation of thermoelastic interactions in a temperature dependent spherical shell under the exact heat conduction model with a single delay term. The thermal properties of the medium under the present thermoelasticity theory is taken as linear function of temperature. We consider the problem to be studied under three different kinds of boundary conditions. Due to the consideration of varying material properties, the governing equations reduce to non-linear differential equations. We apply Kirchhoff transformation along with integral transform technique to solve the problems. Inversion of Laplace transforms carried out by a numerical approach gives the final solution for different field variables inside the medium. The numerical results of the field variables are shown in different graphs to study the influence of temperature dependent thermal parameters in the context of new model.

In Chapter-7, we make an attempt to investigate a problem of spherical shell with functionally graded material in the context of the recent heat conduction model given by Quintanilla (2011). We employ Galerkin type finite element method along with Laplace transform technique to solve a system of non-linear coupled partial differential equations arising out in the coupled problem. Considering a metal-ceramic FGM, the inner boundary of the shell is assumed to be stress free and is subjected to thermal shock while the other boundary is insulated with temperature and fixed with some rigid support. We consider a law of mixture of material constituents so that the effective material properties follow a rule of volume-fraction. We show that the present problem can be solved by trans-finite element method, i.e., firstly, we use the Laplace transform technique and then, in order to tackle the non-linearity of governing equations generated due to dependency of thermal parameters on spacial coordinate, we apply finite element method by considering that the geometry of the shell is divided into discrete elements of equal length along the radius of the shell. We carry out our computational work in MATLAB programming code and obtain the numerical solution of the problem. The results are displayed in different graphs to display the distributions of the field variables inside the shell at different time and various types of non-homogeneity condition. A detailed comparative study on the obtained results is discussed to focus on the effects of non-homogeneity for the functionally graded material. The present work specially considers to highlight the necessity of the finite element method to the coupled thermoelasticity and to display a compact analysis of different coupled thermoelasticity theories for functionally graded materials.

The Chapter- 8 summarizes the work done in the thesis and highlights the significant contributions of the work and also the scope for future work in the concerned area. The references cited in the thesis are appended at the end of thesis.