CHAPTER 6

INVESTIGATION ON THE EFFECTS OF TEMPERATURE DEPENDENCY OF MATERIAL PARAMETERS ON A THERMOELASTIC LOADING PROBLEM[1]

6.1 Introduction

Earlier in thermoelasticity theory it was assumed that all the thermal parameters are free from temperature. But at high temperature, considering many practical and theoretical results of the materials, Noda (1986) reported a detail review on temperature dependent material properties and showed that thermal conductivity of the materials decreases exponentially with temperature. In recent years, different researchers consider the generalized thermoelasticity theories by taking into account that thermal parameters vary with temperature. It must be mentioned here that earlier in 1918, considering only shear modulus depending on temperature, a thermoelastic model was solved by Suhara (1918) and the effects of temperature dependency of shear modulus was investigated. Youssef and Abbas (2007) discussed the dependency of modulus of elasticity and thermal conductivity of the material on temperature in generalized thermoelasticity theory for an unbounded medium with a spherical cavity. Othman (2002; 2003; 2013;

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2015) investigated the thermoelastic interactions in two dimensional thermoelastic problems with temperature dependent elastic moduli. Zenkour and Abbas (2014) analyzed a problem with density and thermoelastic properties depending on temperature and discussed some characteristic features of temperature dependent properties of the materials.

In the previous chapter, we have investigated a half space problem on generalized thermoelasticity theory with a delay term proposed by Quintanilla (2011). In the present chapter, we aimed at the investigation of thermoelastic interactions in a temperature dependent spherical shell under the same model (Quintanilla (2011)): an exact heat conduction model with a single delay term. The thermal properties of the medium under the present thermoelasticity theory is taken as linear function of temperature. We consider the problem to be studied under three different kinds of boundary conditions. Due to the consideration of varying material properties, the governing equations reduce to non-linear differential equations. We apply Kirchhoff transformation along with integral transform technique to solve the problems. Inversion of Laplace transforms carried out by a numerical approach, that gives the final solution for different field variables inside the medium. The numerical results of the field variables are shown in the different graphs to study the influence of temperature dependent thermal parameters in the context of new model.

6.2 Problem Formulation

We consider an isotropic elastic medium with temperature dependent material properties and employ the thermoelasticity theory based on the heat conduction model with a delay term (Leseduarte and Quintanilla (2013)) to consider the thermoelastic interactions in the absence of any body forces or heat sources. The basic governing equations in usual indicial notation therefore can be written as follows:

The equation of motion:

$$\mu u_{i,jj} + (\lambda + \mu)\, u_{j,ji} - \gamma \theta_{,i} = \rho \ddot{u}_i \tag{6.1}$$

The equation of heat conduction:

$$\frac{\partial}{\partial t}\left(K\theta_{,i}\right)_{,i} + \left(1 + \tau\frac{\partial}{\partial t} + \frac{\tau^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(K^*\theta_{,i}\right)_{,i} = \frac{\partial}{\partial t}\left[\rho c_E \frac{\partial \theta}{\partial t} + T_0 \gamma \frac{\partial e}{\partial t}\right] \tag{6.2}$$

The equation of stress-strain-temperature relation:

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \gamma\theta)\, \delta_{ij} \tag{6.3}$$

where τ is the delay parameter.

We consider a spherical shell of inner radius a and outer radius b, initially at uniform reference temperature T_0. Considering the center of the shell at the origin and spherical symmetry, introducing spherical polar coordinates (r, ϑ, φ), equations (6.1)-(6.3) reduce to

$$(\lambda + 2\mu)\frac{\partial e}{\partial r} - \gamma\frac{\partial \theta}{\partial r} = \rho\frac{\partial^2 u}{\partial t^2} \tag{6.4}$$

$$\frac{\partial}{\partial t}\left[K\nabla^2\theta + \frac{\partial K}{\partial \theta}\left(\frac{\partial \theta}{\partial r}\right)^2\right] + \left(1 + \tau\frac{\partial}{\partial t} + \frac{\tau^2}{2}\frac{\partial^2}{\partial t^2}\right)\left[K^*\nabla^2\theta + \frac{\partial K^*}{\partial \theta}\left(\frac{\partial \theta}{\partial r}\right)^2\right]$$
$$= \rho c_E \frac{\partial^2 \theta}{\partial t^2} + \rho\frac{\partial c_E}{\partial \theta}\left(\frac{\partial \theta}{\partial t}\right)^2 + \gamma T_0 \frac{\partial^2 e}{\partial t^2} \tag{6.5}$$

$$\sigma_{rr} = 2\mu \frac{\partial u}{\partial r} + \lambda e - \gamma \theta \tag{6.6}$$

$$\sigma_{\varphi\varphi} = \sigma_{\vartheta\vartheta} = 2\mu \frac{u}{r} + \lambda e - \gamma \theta \tag{6.7}$$

where u is the single non zero component of displacement.

We take that material properties vary with the temperature and assume that the thermal conductivity, K and conductivity rate, K^* are varying as

$$K(\theta) = K_0(1 + K_1\theta) \tag{6.8}$$

$$K^*(\theta) = K_0^*(1 + K_1\theta) \tag{6.9}$$

where K_1 is a constant and it is zero at reference temperature, K_0 and K_0^* are the thermal conductivity and thermal conductivity rate at reference temperature,T_0 respectively. Further, specific heat, c_E is assumed to be independent of temperature.

In view of equations (6.8,6.9), we find that equation (6.5) is non-linear, and therefore to tackle the non-linearity we consider a new function Φ expressing the temperature with Kirchhoff transformation as

$$\Phi = \frac{1}{K_0} \int_0^\theta K(p)dp = \frac{1}{K_0^*} \int_0^\theta K^*(p)dp = \theta + \frac{1}{2} K_1 \theta^2 \tag{6.10}$$

Hence, by using equations (6.8)-(6.10) and the fact that $\theta = T - T_0$ with $\mid \theta/T_0 \mid << 1$, equations (6.4), (6.5), (6.6) and (6.7) respectively, reduce to

$$(\lambda + 2\mu)\frac{\partial e}{\partial r} - \gamma\frac{\partial \Phi}{\partial r} = \rho\frac{\partial^2 u}{\partial t^2} \tag{6.11}$$

$$K_0\nabla^2\dot{\Phi} + K_0^*\left(1 + \tau\frac{\partial}{\partial t} + \frac{\tau^2}{2}\frac{\partial^2}{\partial t^2}\right)\nabla^2\Phi = \eta K_0\frac{\partial^2\Phi}{\partial t^2} + \gamma T_0\frac{\partial^2 e}{\partial t^2} \tag{6.12}$$

$$\sigma_{rr} = 2\mu\frac{\partial u}{\partial r} + \lambda e - \gamma\Phi \tag{6.13}$$

$$\sigma_{\varphi\varphi} = \sigma_{\vartheta\vartheta} = 2\mu\frac{u}{r} + \lambda e - \gamma\Phi \tag{6.14}$$

Now, we use the following symbols and notations to make equations (6.11)-(6.14) dimensionless:

$r' = c_0\eta r$, $u' = c_0\eta r$, $t' = c_0^2\eta t$, $\tau' = c_0^2\eta\tau$, $\Phi' = \frac{\Phi}{T_0}$, $e' = e$, $\sigma'_{ij} = \frac{\sigma_{ij}}{(\lambda_0+2\mu_0)}$, $c_0^2 = \frac{(\lambda+2\mu)}{\rho}$, $K^* = \frac{c_E(\lambda+2\mu)}{4}$ $\eta = \frac{\rho c_E}{K}$, $a_0 = \frac{K_0^*}{K_0 c_0^2\eta}$, $a_1 = \frac{\gamma T_0}{(\lambda+2\mu)}$, $a_2 = \frac{\gamma}{K_0\eta}$, $\lambda_1 = \frac{\lambda}{(\lambda+2\mu)}$.

Therefore, after dropping the primes for clarity, equations (6.11)-(6.14) change to their dimensionless forms as follows

$$\frac{\partial e}{\partial r} - a_1\frac{\partial \Phi}{\partial r} = \frac{\partial^2 u}{\partial t^2} \tag{6.15}$$

$$\nabla^2\dot{\Phi} + a_0\left(1 + \tau\frac{\partial}{\partial t} + \frac{\tau^2}{2}\frac{\partial^2}{\partial t^2}\right)\nabla^2\Phi = \frac{\partial^2\Phi}{\partial t^2} + a_2\frac{\partial^2 e}{\partial t^2} \tag{6.16}$$

$$\sigma_{rr} = (1 - \lambda_1)\frac{\partial u}{\partial r} + \lambda_1 e - a_1\Phi \tag{6.17}$$

$$\sigma_{\varphi\varphi} = \sigma_{\vartheta\vartheta} = (1 - \lambda_1)\frac{u}{r} + \lambda_1 e - a_1 \Phi \tag{6.18}$$

6.3 Solution of the Problem

Applying Laplace transform to the equations (6.15), (6.16), (6.17) and (6.18) with homogeneous initial conditions, we obtain

$$\frac{\partial \bar{e}}{\partial r} - a_1 \frac{\partial \bar{\Phi}}{\partial r} = s^2 \bar{u} \tag{6.19}$$

$$\nabla^2 \bar{\Phi} = \frac{2s^2}{b_0(s)}\bar{\Phi} + \frac{2a_2 s^2}{b_0(s)}\bar{e} \tag{6.20}$$

$$\bar{\sigma}_{rr} = (1 - \lambda_1)\frac{\partial \bar{u}}{\partial r} + \lambda_1 \bar{e} - a_1 \bar{\Phi} \tag{6.21}$$

$$\bar{\sigma}_{\varphi\varphi} = \bar{\sigma}_{\vartheta\vartheta} = (1 - \lambda_1)\frac{\bar{u}}{r} + \lambda_1 \bar{e} - a_1 \bar{\Phi} \tag{6.22}$$

where $b_0(s) = a_0 \tau^2 s^2 + 2(1 + a_0 \tau) s + 2a_0$.

Now taking divergence of equation (6.19), we get

$$\nabla^2 \bar{e} - a_1 \nabla^2 \bar{\Phi} = s^2 \bar{e} \tag{6.23}$$

We employ equations (6.20) and (6.23) to get

$$\nabla^2 \bar{e} = \frac{2a_1 s^2}{b_0(s)}\bar{\Phi} + \left(\frac{2\epsilon s^2}{b_0(s)} + s^2\right)\bar{e} \tag{6.24}$$

where $\epsilon = a_1 a_2$. Applying ∇^2 operator on equation (6.24) to get

$$\nabla^4 \bar{e} = \frac{2a_1 s^2}{b_0(s)} \nabla^2 \bar{\Phi} + \left(\frac{2\epsilon s^2}{b_0(s)} + s^2 \right) \nabla^2 \bar{e} \tag{6.25}$$

With the help of equations (6.23) and (6.25), we find

$$\left[\nabla^4 - b_1(s) \nabla^2 + b_2(s) \right] \bar{e} = 0 \tag{6.26}$$

where $b_1(s) = \frac{2s^2}{b_0(s)} + \frac{2\epsilon s^2}{b_0(s)} + s^2$ and $b_2(s) = \frac{2s^4}{b_0(s)}$.

Applying $\left(\nabla^2 - s^2 \right)$ operator on equation (6.20) and using equation (6.23), we get

$$\left[\nabla^4 - b_1(s) \nabla^2 + b_2(s) \right] \bar{\Phi} = 0 \tag{6.27}$$

Now, equations (6.26) and (6.27) can be rewritten as

$$\left(\nabla^2 - n_1^2 \right) \left(\nabla^2 - n_2^2 \right) \left(\bar{\Phi}, \bar{e} \right) = 0 \tag{6.28}$$

where n_1^2 and n_2^2 satisfy the equation

$$n^4 - b_1(s) n^2 + b_2(s) = 0 \tag{6.29}$$

Since the equations (6.27)-(6.28) represent the modified spherical Bessel differential equations, hence its general solution can be obtained as

$$\bar{\Phi} = \frac{1}{\sqrt{r}} \sum_{i=1}^{2} \left[A_i I_{1/2} \left(n_i r \right) + B_i K_{1/2} \left(n_i r \right) \right] \tag{6.30}$$

$$\bar{e} = \frac{1}{\sqrt{r}} \sum_{i=1}^{2} \left[C_i I_{1/2} \left(n_i r \right) + D_i K_{1/2} \left(n_i r \right) \right] \tag{6.31}$$

where, A_i, B_i, C_i and D_i are arbitrary constants and $I_\alpha(r)$, $K_\alpha(r)$ are the representation of modified Bessel functions of order α of first and second kinds, respectively.

Using equations (6.23), (6.30) and (6.31) , we get

$$A_i = f_i C_i \text{ and } B_i = f_i D_i. \tag{6.32}$$

where $f_i = \frac{n_i^2 - s^2}{a_1 n_i^2},\ i = 1, 2.$

With the help of equations (6.19),(6.21),(6.22),(6.30), and (6.31), we get the solution of other physical variables in Laplace transform domain as follows:

$$\bar{u} = \frac{1}{\sqrt{r}} \sum_{i=1}^{2} \left[\frac{C_i}{n_i} I_{3/2}\left(n_i r\right) - \frac{D_i}{n_i} K_{3/2}\left(n_i r\right) \right] \tag{6.33}$$

$$\bar{\sigma}_{rr} = \frac{1}{\sqrt{r}} \sum_{i=1}^{2} \left[C_i \left\{ \frac{s^2}{n_i^2} I_{1/2}\left(n_i r\right) - \frac{2\left(1-\lambda_1\right)}{\left(n_i r\right)} I_{3/2}\left(n_i r\right) \right\} + D_i \left\{ \frac{s^2}{n_i^2} K_{1/2}\left(n_i r\right) - \frac{2\left(1-\lambda_1\right)}{\left(n_i r\right)} K_{3/2}\left(n_i r\right) \right\} \right] \tag{6.34}$$

$$\begin{aligned} \bar{\sigma}_{\varphi\varphi} &= \bar{\sigma}_{\vartheta\vartheta} = \frac{1}{\sqrt{r}} \sum_{i=1}^{2} \left[C_i \left\{ \left(\lambda_1 - \frac{n_i^2 - s^2}{n_i^2} \right) I_{1/2}\left(n_i r\right) + \frac{\left(1-\lambda_1\right)}{\left(n_i r\right)} I_{3/2}\left(n_i r\right) \right\} \right] \\ &+ \frac{1}{\sqrt{r}} \sum_{i=1}^{2} \left[D_i \left\{ \left(\lambda_1 - \frac{n_i^2 - s^2}{n_i^2} \right) K_{1/2}\left(n_i r\right) - \frac{\left(1-\lambda_1\right)}{\left(n_i r\right)} K_{3/2}\left(n_i r\right) \right\} \right] \end{aligned} \tag{6.35}$$

6.4 Applications of the Problem

6.4.1 Case I: Unit Step Increase in Temperature and Zero Stress of the Boundary of the Spherical Shell

We consider the thermoelastic spherical shell with initial conditions as homogeneous and it is assumed that the inner and outer boundaries $r = a$ and $r = b$ of the spherical shell are traction free and are subjected to a unit step increase in temperature. Therefore, the boundary conditions in the dimension-less forms can be written as:

$$\theta(r,t) = \theta_1^* H(t) \text{ and } \sigma(r,t) = 0 \text{ at } r = a \tag{6.36}$$

$$\theta(r,t) = \theta_2^* H(t) \text{ and } \sigma(r,t) = 0 \text{ at } r = b \tag{6.37}$$

where θ_1^* and θ_2^* are constant temperatures and $H(t)$ is the Heaviside unit-step function.

Therefore, using equations (6.10) and applying Laplace transform to the boundary conditions given by(6.36) and (6.37), we find that

$$\bar{\Phi}(a,s) = \frac{\theta_1^*}{s}\left(1 + \frac{1}{2}K_1\theta_1^*\right), \bar{\Phi}(b,s) = \frac{\theta_2^*}{s}\left(1 + \frac{1}{2}K_1\theta_2^*\right), \bar{\sigma}(a,s) = 0 = \bar{\sigma}(b,s). \tag{6.38}$$

6.4.2 Case II: Exponential Variation in Temperature and Zero Stress of the Boundary of the Shell

It is assumed that both the inner boundary $r = a$ and outer boundary $r = b$ of the spherical shell are traction free. The inner boundary is subjected to an exponential variation in temperature and the outer boundary is maintained to be insulated as follows:

$$\theta(r,t) = \frac{t^2}{t_s}\mathrm{e}^{-t/t_s},\ t > 0 \text{ and } \sigma(r,t) = 0 \text{ at } r = a \tag{6.39}$$

$$\frac{\partial\theta(r,t)}{\partial r} = 0 \text{ and } \sigma(r,t) = 0 \text{ at } r = b \tag{6.40}$$

where t_s is a constant parameter to control the nature of temperature given on the inner boundary.

Hence, using equations (6.10), (6.39) and (6.40), we get

$$\bar{\Phi}(a,s) = \frac{2t_s}{(1+st_s)^3} + \frac{3K_1 t_s}{4\,(2+st_s)^5}, \bar{\Phi}(b,s) = 0, \bar{\sigma}(a,s) = 0 = \bar{\sigma}(b,s). \tag{6.41}$$

6.4.3 Case III: Sinusoidal Varying Temperature and Zero Displacement at the Boundary of the Spherical Shell

Now, we assume that the inner boundary $r = a$ of the spherical shell is rigidly fixed and is subjected to a sinusoidal variation in temperature as follows:

$$\theta(r,t) = \begin{cases} \theta_0 \sin\left(\frac{\pi t}{t_0}\right) & 0 < t < t_0 \\ 0 & \text{otherwise} \end{cases}, \; t > 0 \text{ and } u(r,t) = 0 \text{ at } r = a \tag{6.42}$$

The outer boundary is also kept rigidly fixed and insulated, i.e.,

$$\frac{\partial\theta(r,t)}{\partial r} = 0 \text{ and } u(r,t) = 0 \text{ at } r = b \tag{6.43}$$

where t_0 is a constant that controls the range of temperature to be positive on the inner boundary.

Therefore, we get in this case

$$\bar{\Phi}(a,s) = \frac{\theta_0 \pi t_0}{(\pi^2 + s^2 t_0^2)}\left(1 + \mathrm{e}^{-st_0}\right) + \frac{K_1 \theta_0^2 \pi^2}{s\left(4\pi^2 + s^2 t_0^2\right)}\left(1 - \mathrm{e}^{-st_0}\right), \bar{\Phi}(b,s) = 0, \tag{6.44}$$

$$\bar{u}(a,s) = 0 = \bar{u}(b,s). \tag{6.45}$$

Now, for the Case-I, from equations (6.30), (6.34)and (6.38) we obtain a linear system of four equations in four unknowns as given by

$$\sum_{i=1}^{2}\left[f_i I_{1/2}\left(n_i a\right) C_i + f_i K_{1/2}\left(n_i a\right) D_i\right] = \frac{\sqrt{a}\theta_1^*}{s}\left(1 + \frac{1}{2}K_1\theta_1^*\right) \tag{6.46}$$

$$\sum_{i=1}^{2}\left[f_i I_{1/2}\left(n_i b\right) C_i + f_i K_{1/2}\left(n_i b\right) D_i\right] = \frac{\sqrt{b}\theta_2^*}{s}\left(1 + \frac{1}{2}K_1\theta_2^*\right) \tag{6.47}$$

$$\sum_{i=1}^{2}\left\{\left[\frac{s^2}{n_i^2}aI_{1/2}\left(n_ia\right)-\frac{2\left(1-\lambda_1\right)}{n_i}I_{3/2}\left(n_ia\right)\right]C_i+\left[\frac{s^2}{n_i^2}aK_{1/2}\left(n_ia\right)+\frac{2\left(1-\lambda_1\right)}{n_i}K_{3/2}\left(n_ia\right)\right]D_i\right\}=0 \tag{6.48}$$

$$\sum_{i=1}^{2}\left\{\left[\frac{s^2}{n_i^2}bI_{1/2}\left(n_ib\right)-\frac{2\left(1-\lambda_1\right)}{n_i}I_{3/2}\left(n_ib\right)\right]C_i+\left[\frac{s^2}{n_i^2}bK_{1/2}\left(n_ib\right)+\frac{2\left(1-\lambda_1\right)}{n_i}K_{3/2}\left(n_ib\right)\right]D_i\right\}=0 \tag{6.49}$$

After solving the equations (6.46)-(6.49), we can find the unknowns C_i and D_i, hence the A_i and $B_i,\ i=1,2$ from equation (6.32) and this completes the solution of the present problem in case-I in Laplace transform domain. We can obtain the solution for temperature θ by using equations (6.10) and (6.30) in the Laplace transform domain. The solutions in Laplace transform domain for the cases II and III can also be obtained in the similar way.

6.5 Numerical Results and Discussion

The solution in the physical domain can be derived by inverting the solutions obtained in the Laplace transform as found out in the previous section. Now, we find the Laplace inversion for the physical variables temperature, displacement, radial stress and shear stress with the help of Mat lab software and by employing a suitable numerical method of Laplace inversion. We employ here the method proposed by Stehfest (1970) (see the Appendix section (A-1)).

We assume that the spherical shell is made of copper material and the physical data points for which are taken as below (Sherief and Salah, 2005).

$\lambda = 7.76\times10^{10}\ Nm^{-2}$, $\mu = 3.86\times10^{10}\ Nm^{-2}$, $\alpha_t = 1.78\times10^{-5}\,K^{-1}$, $\eta = 8886.73\ sm^{-2}$, $c_E = 383.1\ JKg^{-1}K^{-1}$, $\rho = 8954\ Kg\ m^{-3}$, $T_0 = 293K$.

We assume the following dimension-less values of the constants:

$\tau = 0.01$, $t_s = 0.2$, $t_0 = 1.0$, $\theta_0 = 1$, $\theta_1^* = 1$, $\theta_2^* = 1$.

We compute the numerical solutions for non-dimensional temperature, displacement, radial stress and tangential stress in space-time domain inside the spherical shell. The results under three different cases are displayed in Figs. 6.1(a,b,c,d)-3(a,b,c,d). In each figure, we plotted the graphs for the fields at three different times, $t = 0.30$, $t = 0.35$, $t = 0.40$ and for three different values of the coefficient of temperature dependent effect, $K_1(0.0, -0.3, -0.5)$. Specially, we aim to understand the effect of temperature dependency on the solutions at various time of interaction. The specific features related to the effect of temperature dependency under various cases of prescribed boundary conditions arising out from our investigation are highlighted as follows:

Case-I:

In this case, we find the effect of temperature dependent material properties on the distributions of various fields inside the spherical shell when the inner and outer surfaces of the shell are subjected to thermal shock. The variation of displacement, temperature, radial stress and circumferential stress is shown in Figs. 6.1(a-d), respectively. Fig. 6.1(a) shows that the variation in displacement is prominently affected only near the boundaries and through the middle region of the shell the effect of time and temperature dependent property on displacement is negligible. The amplitude of displacement u increases with time t. It is further evident that at higher time, the absolute value of displacement decreases with larger numerical

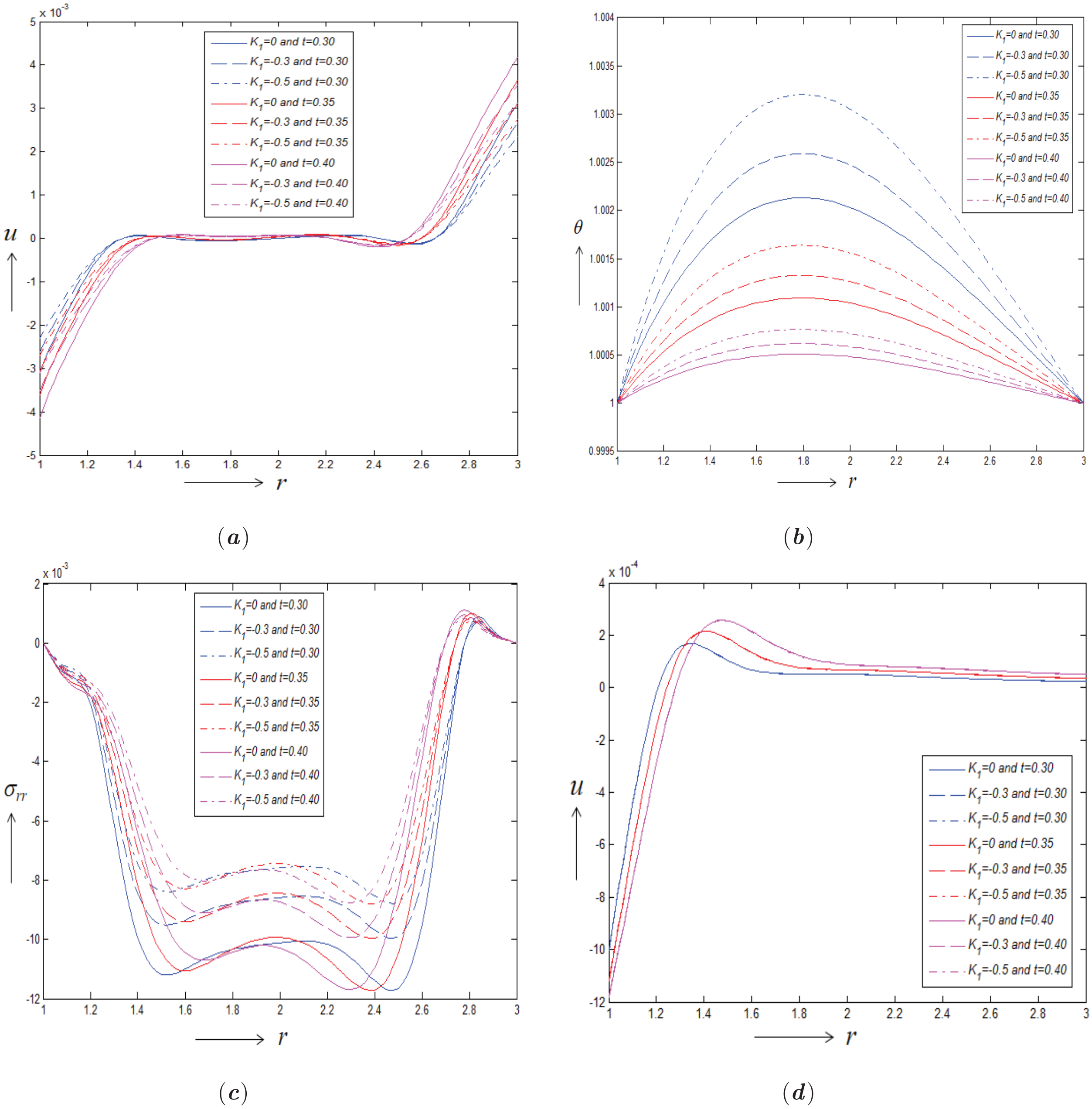

Figure 6.1: ***(a).*** Variation of displacement, u versus r for the Case-I, ***(b).*** Variation of temperature, θ versus r for the Case-I, ***(c).*** Variation of radial stress, σ_{rr} versus r for the Case-I and ***(d).*** Variation of tangential stress, $\sigma_{\varphi\varphi}$ versus r for the Case-I

values of parameter, K_1 and for smaller time, the dependency of thermal parameters on temperature distributions is negligible for displacement.

Fig. 6.1(b) shows the variation of temperature, θ in spherical shell for Case-I. It is clear from Fig. 6.1(b) that for a fixed time, the temperature starts increasing from inner boundary of the shell and after getting maximum value starts decreasing till the outer boundary of the shell. In this case, the temperature field is more sensitive to the temperature dependent material properties. The temperature increases with the increase of the decreasing coefficient K_1. It can also be seen from Fig.6.1(b) that temperature distribution is in agreement with the boundary conditions given for the problem.

Radial stress σ_{rr} and shear stress $\sigma_{\varphi\varphi}$ are shown in the Figs. 6.1(c) and 6.1(d), respectively and it is observed that both the stress components show significant variation near the boundaries of the shell and in the middle region of the shell. The stresses increase with higher negative value of K_1 for all time. One important fact is observed that the effect of temperature dependency of stresses is independent of time, implying that at any time, a similar effect of temperature dependency is observed for the stress components in case-I.

Case-II:

Figs. 6.2(a,b,c,d) display the variation of field variables for the exponentially varying temperature on the inner boundary. It is observed that there is no adequate effect of temperature dependent material properties on any physical quantities except the field temperature. At any time, the same

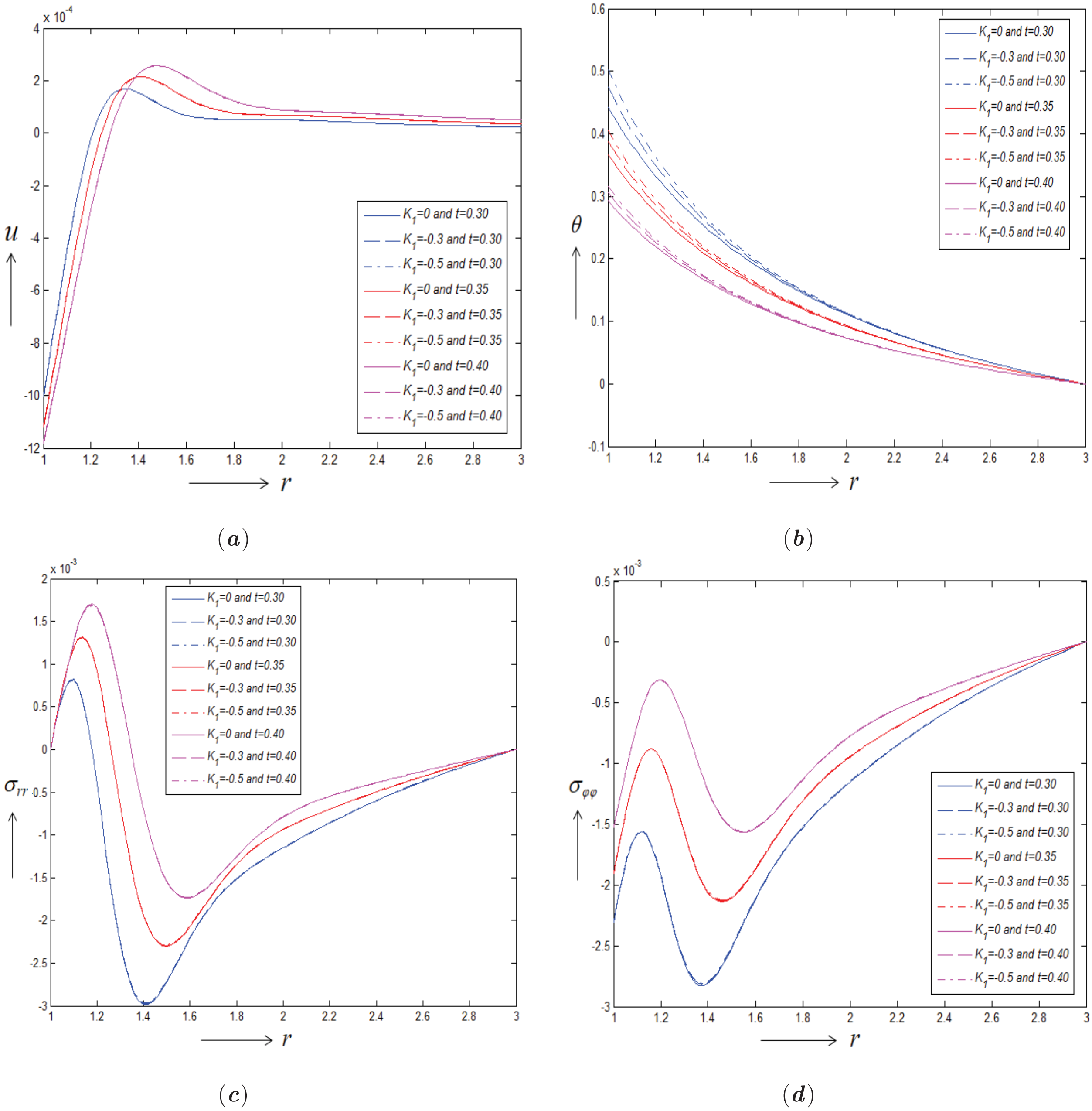

Figure 6.2: $(\boldsymbol{a})$. Variation of displacement, u versus r for the Case-II, $(\boldsymbol{b})$. Variation of temperature, θ versus r for the Case-II, $(\boldsymbol{c})$. Variation of radial stress, σ_{rr} versus r for the Case-II and $(\boldsymbol{d})$. Variation of tangential stress, $\sigma_{\varphi\varphi}$ versus r for the Case-II

effect of temperature dependency is noted for all the field variables. Fig. 6.2(a) shows the distribution of displacement u. It can be seen that the effective value of displacement u increases with increase of time. Displacement tends to zero through the radial distance r for all times. Fig. 6.2(b) shows the variation of temperature with radial distance and it can be seen that values of temperature increases with respect to the time and also w.r.t. K_1. Figs. 6.2(c,d) measure the variation of stress components and it can be seen that radial stress σ_{rr} satisfies the boundary condition. It could be observed that the effective regions of stress components increases with increase of time t.

Case-III:

Distributions of displacement, temperature and stress components for the current generalized thermoelasticity in case-III are displayed in Figs. 6.3(a,b,c,d), respectively. It is observed that like case-I, the influence of temperature dependent properties is prominent in this case. From Fig. 6.3(a), it is clear that the displacement is in agreement with the boundary conditions. The displacement, u is positive throughout the distance r and gets a local maximum value within middle region of the shell. The effect of temperature dependent property is more prominent at higher times. The displacement increases with time implying that the region of influence for u increases with the time. However, it decreases with larger numerical value of K_1.

Fig. 6.3(b) shows the variation for temperature θ. It is depicted in the graph that the temperature is influenced significantly by the temperature dependent property of the material. The temperature field has an increas-

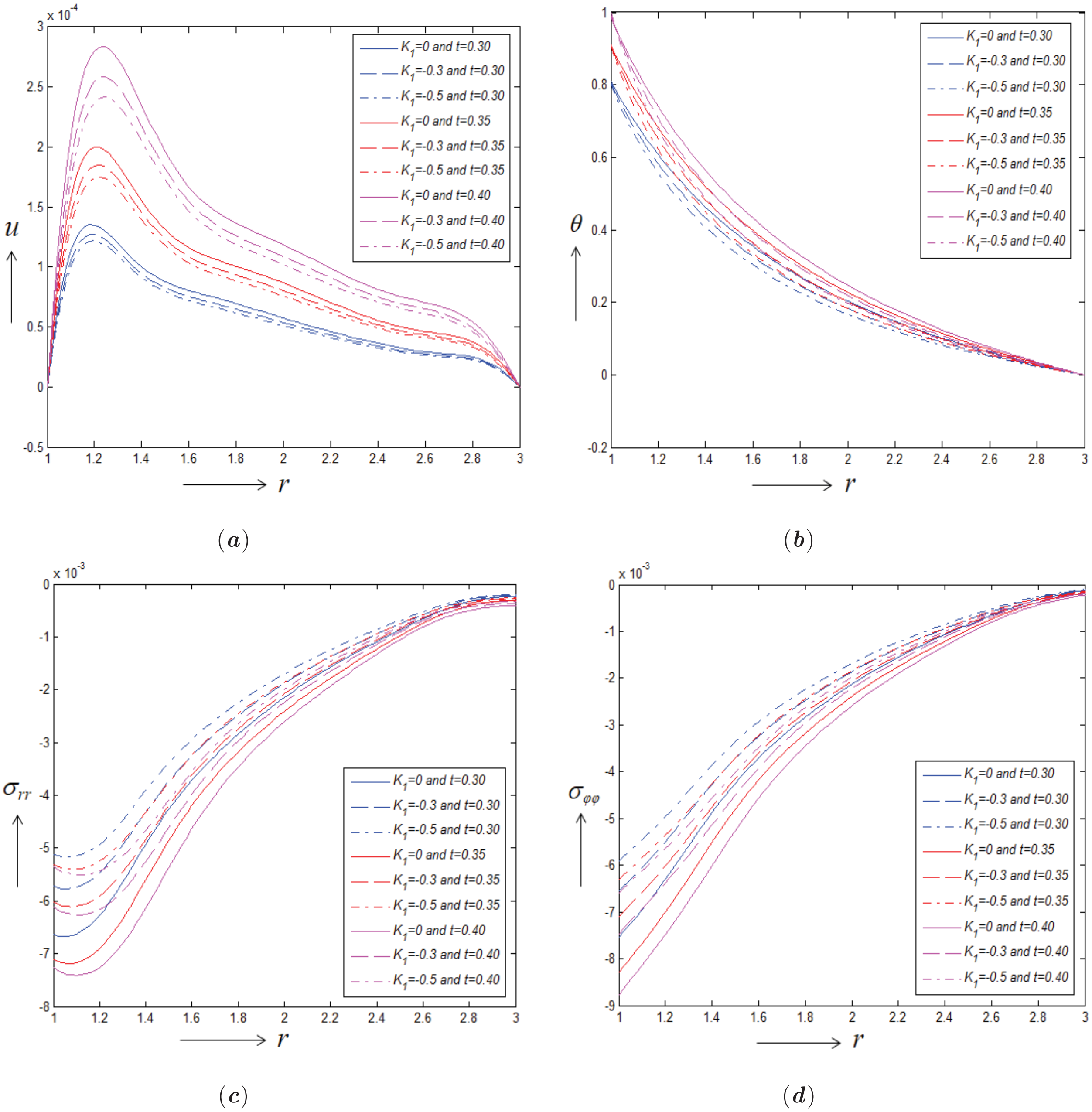

Figure 6.3: $(\boldsymbol{a})$. Variation of displacement, u versus r for the Case-III, $(\boldsymbol{b})$. Variation of temperature, θ versus r for the Case-III, $(\boldsymbol{c})$. Variation of radial stress, σ_{rr} versus r for the Case-III and $(\boldsymbol{d})$. Variation of tangential stress, $\sigma_{\varphi\varphi}$ versus r for the Case-III

ing trend with time and also with coefficient K_1. The variation of stress components in case-III are shown in the Figs. 6.3(c,d). The stresses are compressive in nature throughout region of the medium and the influence of temperature dependent properties on stress distributions is much more prominent in this case. This effect on stresses is prominent near the inner boundary of the shell and it approaches to zero as $r \to b$, where b is the outer radius of the spherical shell.