Appendix

(A-1). Stehfest Method of Laplace Inversion

Let $\overline{f}(s)$ be the Laplace transform of a function, $f(t)$ which is piece-wise continuous in $[0, \infty)$ and is of exponential order. Then the Laplace inverse of the function, $f(t)$ is given by (see Stehfest (1970) and Kuhlman (2013)),

$$f(t) = \frac{ln(2)}{2} \sum_{k=1}^{N} V_k \overline{f}\left(k \frac{ln(2)}{t}\right) \tag{A1-1}$$

where N is a suitable positive integer and V_k is given by

$$V_k = (-1)^{(k+N/2)} \sum_{j=[(k+1)/2]}^{\min(k,N/2)} \frac{j^{\frac{N}{2}} (2j)!}{(\frac{N}{2} - j)! \, j! \; (j-1)! \, (k-j)! \, (2j-k)!} \tag{A1-2}$$

(A-2). Zakian Method of Laplace Inversion

The Laplace transform, $\bar{f}(s)$ of the function $f(t)$ is defined by

$$\overline{f}(s) = \int_0^{\infty} f(t) \mathrm{e}^{-\mathrm{st}} \mathrm{dt} \tag{A2-1}$$

where we assume, $Re(s) > \alpha$ and $f(t)$ is integrable and of exponential order α, then it may be inverted by using the economical and efficient formula (Zakian *et al.* (1969)).

i	K_i		α_i	
	Re(K_i)	Im(K_i)	Re(α_i)	Im(α_i)
1	-3.690208210 E+04	1.969904257 E+05	1.283767675 E+01	1.666063445 E+00
2	6.127702524 E+04	-9.540862551 E+04	21.222613209 E+0l	5.012718792 E+00
3	-2.891656288 E+04	1.816918531 E+04	31.093430308 E+01	8.409673116 E+00
4	4.655361138 E+03	-1.901528642 E+00	48.776434715 E+00	l.l92185389 E+0l
5	-l.l87414011 E+02	-1.413036911 E+02	55.225453361 E+00	1.572952905 E+01

Table-1. (Zakian *et al.*(1969)).

$$f_N(t) = \frac{1}{t} \sum_{i=-N}^{N} K_i \overline{f} \left(\frac{\alpha_i}{t} \right) \qquad \text{(A2-2)}$$

where $f_N(t)$ is the approximation to the function $f(t)$. The constants K_i and α_i are real or complex numbers. The approximation can be made with arbitrary accuracy since $f_N(t) \rightarrow f(t)$ as $N \rightarrow \infty$. For the most applications, sufficient accuracy can be obtained with $N = 5$ and the constants are listed in the Table-1. The arithmetic involved in computing the inversion formula given by (A2-2) with complex constants and for $N = 5$ can be modified as follows:

$$f_N(t) = \frac{2}{t} \sum_{i=1}^{5} Re \left[K_i \overline{f} \left(\frac{\alpha_i}{t} \right) \right] \qquad \text{(A2-3)}$$

Thus, the Eqn. (A2-3) can be used to compute the inverse of Laplace transform of the function $f(t)$.