Journal of Engineering Science and Technology Review 16 (2) (2023) 71 - 81

Research Article

JOURNAL OF
Engineering Science and Technology Review

www.jestr.org

Level Control in Conical Tank Using IMC-PID Controller

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Received 12 April 2022; Accepted 28 March 2023

Abstract

An IMC-PI/PID controller with a first or second order filter in series is designed for several types of processes with time delay. The tradeoff between performance and robustness is done by adjusting the tuning parameter of the controller. The present tuning method shows better performances in terms of overshoot (OS), settling time and ITAE error index. Simulated results of various processes are compared with recently published tuning methods and it provide better response in set point change and comparable in load change. A set point weighting parameter 'b' is used to remove the undesirable overshoot, which is easier to select in comparison to the set point filter. The controller's parameters are fixed in such a way that give same robustness level (maximum sensitivity Ms), by adjusting the tuning parameter. A 10% to 15% perturbation in all the parameters of process models is introduced to evaluate the robustness of the proposed tuning method. In the present study, a conical tank system is considered for the applicability of the proposed design method of PI/PID controller. The proposed tuning method is used for designing the PI controller and successfully applied to the transfer function model and process of conical tank. The simulated and experimental results are compared with two other recently published tuning techniques.

Keywords: IMC; PID; ITAE; Conical tank

1. Introduction

Numerous advancement has been done in the field of process control such as neural network, fuzzy logic, neuro-fuzzy logic, model predictive control (MPC), adaptive controller and sliding mode control to obtain desired quality and economical product. However, till now >95% of process industries such as chemical, biochemical, pharmaceutical industries uses proportional integral and derivative (PID), and conventional controller design techniques due to their simplicity and robustness [1]. These process industries exhibit various linear and nonlinear processes such as chemical reactions occurring in continuous stirred tank reactor, level control in variable area tank (conical or spherical tank) and pH control.

Control of nonlinear process is challenging task and therefore, to study or control the nonlinear process, the conical tank is selected and the various control technique is used for the level control. A nonlinear model based controller is designed by Arasu, Panda [2] and the parameters of the nonlinear model were determined using empirical approach. A neural network based IMC-PID control is designed by Srivignesh, Sowmya [3], a comparative study of different well known tuning methods is reported by Anusha, Karpagam [4] for the level control in conical tank. Ravi, Thyagarajan [5] designed a dynamic matrix controller (DMC) approach to control the level in two conical tanks connected in series. Therefore, a simple PI/PID controller based on a linear model is not effective and shows poor performance for controlling non-linear chemical processes (non-isothermal chemical reactors, pH processes). Thus, there is need of nonlinear control law [6]. Several modeling errors or random disturbances are arising in the operation of nonlinear processes and which are difficult to compensate. Hence, the control approaches are quite complex to small errors or disturbances, and the successive implementation of actual processes are often unsuccessful [7]. Various other advanced techniques have been developed to control the nonlinear process. In recent years, different advanced control techniques have been established. Artificial Neural Networks (ANNs) [3, 8], Fuzzy PID controller [9, 10], and others have been used for the nonlinear system control.

Model-based control has also been used for the nonlinear process using global linearization around the operating point. A great work on model-based control and PID design rules are existing in the literature and processes are asummed to first order with time delay (FOPDT) for the designing of the controller. Åström, Hägglund [11] and O'Dwyer [12] have done great work to design model-based PI or PID controllers. Tuning rule based on process reaction or step response curve are widely used and conventional PI/PID controller tuning approaches have been suggested by the various authors [13-17]. Ziegler-Nichols method shows good responses for the disturbance rejection but give high overshoot in case of servo problems. Set point weighting parameters are used by various authors for reducing the excessive overshoots [16, 18]. Skogestad [19] proposed a simple analytical tuning rule (SIMC) for the PI/PID controller based on IMC-PID techniques. The authors modified the integral terms to achieve the good disturbance rejection for the integrating process and also proposed a single tuning rule for the various forms of transfer functions. An IMC-PID tuning rule [20-24] has been designed for stable or unstable processes which show very good results for the servo as well as regulatory problems.

Recent developments have been done by the researchers to improve the IMC-PID controller performances and

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ISSN: 1791-2377
doi:10.25103/jestr.162.10

robustness [25-27]. The tuning parameter in IMC-PID controller can be calculated by time constant and time delay. The maximum sensitivity (*Ms)* play important role in the case of model uncertainty or model mismatch. The model uncertainty occurs mainly due to process's parameters variations and nonlinearity of the processes. Therefore, it is required to design robust controller which can perform well in case of model uncertainty. Several researchers have proposed maximum sensitivity (*Ms*)-based IMC-PID controller for the various processes with time delay [28-31]. Shamsuzzoha and Lee [27] developed an advanced PID controller which cascaded with a lead-lag compensator, based on IMC strategy for second-order processes plus dead time. The obtained controller shows very good results for the reference tracking along with load change. Direct synthesis is used to design of PID controller in series with a lead-lag compensator for all class of processes having at least one pole at the origin plus time delay and the controller's parameters are selected by adjusting tuning parameter λ for different robustness levels by evaluating *Ms* value [32].

In the present study, an IMC based PID controller is designed for second order time delay process model and applied on different stable and unstable models. The proposed controller is also applied on conical tank for level control. The modelling of conical tank and its transfer function is calculated in the last section. The MATLAB/SIMULINK 2013b is used for simulation works and the simulated results of different examples taken from literature shows better results for set point as well as regulatory problems in terms of ITAE. The proposed designed controller successfully tested on conical tank for level control.

2. IMC PID controller design

Internal model control and simple feedback control structure are given in Figs. 1(a) and (b). Where G_p is the plant transfer function, G_m the plant model, Q IMC controller and C is the equivalent classical feedback controller. It is considered that the process model may be first or second order with time delay (FOPDT or SOPDT) with or without a zero and the process may be unstable i.e. system has at least one right hand pole (RHP).

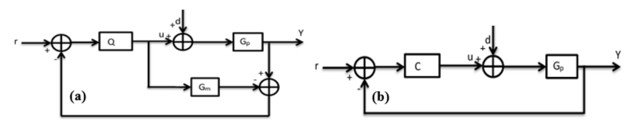


Fig. 1. Block diagram of (a) IMC (b) simple feedback control.

$$G_p = \frac{-fs+1}{as^2+bs+1} e^{-ds} \tag{1}$$

Where a, b, c, d, and f are constant coefficients. An IMC based PID controller with a filter in series, is proposed for the process transfer function given in Eq. 1. The IMC controller q is augmented with low pass filter *f* (Q = q*f*) which is used for altering the robustness of the controller. For nominal case i.e. perfect model $G_p = G_m$ and IMC parameterization [24] split the process model into minimum and non-minimum phase as:

$$G_m = G_{m-}G_{m+} \tag{2}$$

Where:

$$G_{m-} = \frac{fs+1}{as^2+bs+1} \text{ and } G_{m+} = \frac{-fs+1}{fs+1} e^{-ds} \tag{3}$$

To calculate the IMC controller Q, a filter of following form is selected:

$$f = \frac{\gamma s+1}{(\lambda s+1)^3} \tag{4}$$

Therefore, the IMC controller *Q =qf* is obtained.

Or,

$$Q = (G_{m-})^{-1} f$$

Or,

$$Q = \frac{(as^2+bs+c)}{(fs+1)} \times \frac{\gamma s+1}{(\lambda s+1)^3} \tag{5}$$

here λ is the closed loop tuning parameter and γ can be calculated from the conditions of internal stability for IMC structure [24, 29]. The following conditions must be satisfied for internal stability, which are given below:

Condition 1: Q should cancel the right half plane poles of G_m and must be stable
Condition 2: QG_m must be stable.
Condition 3:1- QG_m at the RHP poles of the process should be zero.

The conditions 1st and 2nd are satisfied from above design procedure and third condition can be fulfilled by:

$$(1 - QG_m)|_{s=\frac{1}{\tau 1},\frac{1}{\tau 2}} = 0 \tag{6}$$

The above conditions must be followed in designing the controller for unstable process. These criterions are also beneficial in performance improvement of the control system. The first condition is satisfied automatically by IMC controller Q is designed as Q = $(G_{m-})^{-1}f$ and 2nd condition is fulfilled by designing the IMC filter (*f*).

The IMC control loop shown in Fig.1 can be converted to conventional feedback control loop by the equation $C(S) = \frac{Q(S)}{1-G_m Q(S)}$ and from Eq. (1) and (5), '*C(s)*' can be written as:

$$C(s) = \frac{\frac{(as^2+bs+c)}{(fs+1)} \times \frac{\gamma s+1}{(\lambda s+1)^3}}{1 - \frac{(as^2+bs+c)}{(fs+1)} \times \frac{\gamma s+1}{(\lambda s+1)^3} \times \frac{-fs+1}{as^2+bs+1} e^{-ds}} \tag{7}$$

The controller *C(S)* is not in standard form of PID controller due to time delay in denominator. Hence, the time delay term is approximated by Taylor's series expansion i.e. e^{-ds}= *1-ds* and the Eq. (7) is obtained as:

$$C(s) = \frac{(as^2+bs+c)(\gamma s+1)}{(fs+1)(\lambda s+1)^3 - (-fs+1)(\gamma s+1)(1-ds)} \tag{8}$$

Further, Eq. (8) can be written as:

$$C(s) = \frac{a\gamma s^3 + (b\gamma + a)s^2 + (c\gamma + b)s + c}{f\lambda^3 s^4 + (3f\lambda + \lambda^3 - fd\gamma)s^3 + (3\lambda + 3f\lambda + f\gamma + \gamma d - fd)s^2 + (3\lambda^2 + 2f + d - \gamma)s}$$

to convert *C(S)* into standard form of PID controller, the constant term c does not affect the controller performance, is neglected in numerator and by solving the above equation we find the *C(S)*,

$$C(s) = \frac{a\gamma s^2+(b\gamma+a)s+(c\gamma+b)}{f\lambda^3 s^3+(3f\lambda+\lambda^3-fd\gamma)s^2+(3\lambda+3f\lambda+f\gamma+\gamma d-fd)s+(3\lambda^2+2f+d-\gamma)}$$

now the constant term in denominator equating to zero, we find the value of $'\gamma'$.

Or,

$$C(s) = \frac{a\gamma s^2+(b\gamma+a)s+(c\gamma+b)}{s(f\lambda^3 s^2+(3f\lambda+\lambda^3-fd\gamma)s+(3\lambda+3f\lambda+f\gamma+\gamma d-fd))}$$

And

$$(3\lambda^2+2f+d-\gamma)=0$$

or,

$$\gamma = 3\lambda^2+2f+d \tag{9}$$

And

$$C(s) = \frac{(b\gamma+a)}{(3\lambda+3f\lambda+f\gamma+\gamma d-fd)}\left(1+\frac{(c\gamma+b)}{(b\gamma+a)s}+\frac{a\gamma}{(b\gamma+a)}s\right)\times\frac{1}{\alpha s^2+\beta s+1} \tag{10}$$

Where,

$$\alpha = \frac{f\lambda^3}{(3\lambda+3f\lambda+f\gamma+\gamma d-fd)},\ \beta = \frac{(3f\lambda+\lambda^3-fd\gamma)}{(3\lambda+3f\lambda+f\gamma+\gamma d-fd)} \tag{11}$$

Therefore,

$$k_c = \frac{(b\gamma+a)}{(3\lambda+3f\lambda+f\gamma+\gamma d-fd)},\ \tau_i = \frac{(c\gamma+b)}{(b\gamma+a)} \text{ and } \tau_D = \frac{a\gamma}{(b\gamma+a)} \tag{12}$$

3. Simulation Studies

Various forms of processes (stable, unstable and integrating) are taken from the literature to show the usefulness of the proposed method. The controller's performances of the proposed method are compared with recently developed methods by several authors [19, 21, 27, 33-37]. The time-weighted absolute error (ITAE) minimises by adjusting tuning parameter λ. The controller performance is calculated in terms of ITAE, overshoot (OS) and compared to other design methods for both servo and regulatory problems. The proposed method is also tested on the transfer function model of the conical tank shown in Fig.9 for level control. The set-point weighting method is applied in the present method to minimise the peak overshoot and the parameter is selected so that it minimizes the overshoot and give fast response [38, 39]. The proposed method shows better performance in terms of ITAE in example (4) and (5) and also shows comparable results in other examples.

3.1. Example 1: Stable SOPDT process

Consider the following SOPDT process [25, 33]:

$$G_p = \frac{2e^{-1s}}{(10s+1)(5s+1)}$$

The simulated results of the proposed tuning method of PID controller are compared with the other two methods [27, 33]. The closed loop output results,controller parameters, including the performance and robustness parameters are recorded in Table 1. Robustness lavel (*Ms*=1.87) is selected same as the other two methods by adjusting the tuning parameter 'λ' to confirm the fair comparison of the various methods.

Figure 2(a) and (b) show the performances of the three different controllers for servo and regulatory problems of both respectively. A set-point weighting parameter 'b' is used to minimize the peak overshoot in this method and a set point filter is used for the same purpose in the other's two methods. The proposed controller shows faster response for load change than those by Chen and Seborg [33] and Shamsuzzoha & Lee (2008) methods and almost similar overshoot is obtained for the other two methods. The settling time is almost equal in all the cases but the proposed methods shows less rise time. Table 1 confirms similar results are obtained in case of servo problem.

Shamsuzzoha and Lee [27] inserted a 10% perturbation uncertainty in all three parameters and obtained the worst direction of the actual process to get the robust performance as, $G_p = 2.2e^{-1.1s}/(9s+1)(4.5s+1)$. Table 1(a) and 1(b) show the closed loop results in case of model mismatch for the three methods. The various results listed in Table1 confirms the superiority of the proposed controller.

Table 1(a). Various controller and performance parameters of unit step change in set-point.

PID parameters				Nominal Case			10 % Mismatch	
Method	**Kc**	τ_i	τ_D	**ITAE**	**Overshoot**	**TV**	**ITAE**	**Overshoot**
Proposed,λ=1.3	7.86	5.79	1.82	13.31	1.0270	10.92	14.57	1.033
[27], λ=1.182	9.81	5.45	1.69	10.89	1.009	13.58	11.28	1.004
[33], λ=2.4	6.384	7.6045	2.1	12.68	1.009	5.46	13.02	1.004

Proposed method, set point weighting parameter b=0.45 [27]:

$f_R = (1.6351s+1)/(9.21s^2+5.45s+1)$,

$a = 0.5$,

$b = 0.0341[33]$,

$f_R = (3.8022s+1)/(15.9520s^2+7.6045s+1)$.

Table 1(b). Various performance parameters of unit step change in load.

Nominal Case				10 % Mismatch	
Method	**ITAE**	**Overshoot**	**TV**	**ITAE**	**Overshoot**
Proposed, λ=1.3	6.82	0.145	2.41	7.095	0.16
[27] λ=1.182	3.5	0.089	1.70	3.49	0.091
[33], λ=2.4	9.77	0.148	1.71	9.7	0.156

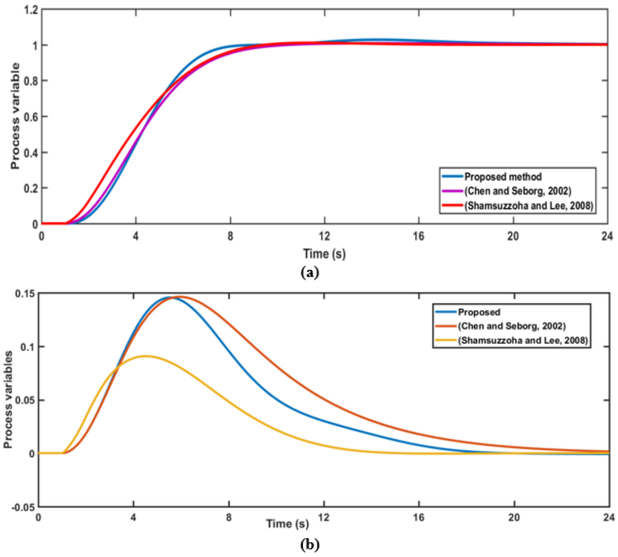


Fig.2 (a) Closed-loop response of the unit step change in set-point for the nominal case. **(b)** Closed-loop response of the unit step change in load for the nominal case.

3.2. Example 2: SOPDT with Inverse Response

Various process unit in the chemical industries such as the boiler drum level control by the heating medium flow rate, the tray composition control of a distillation column by the vapor flow rate, and the exit temperature of a tubular exothermic reactor show inverse response. The main characteristics of the process that shows the inverse response is that the transfer function must have an odd number of zeros in the open right half plane. In this study an example is taken from the literature that shows same nature has also been studied by Luyben [40] and Chien, Chung [34] is selected for the simulation study:

$$G_p = \frac{(-0.2s+1)e^{-0.2s}}{(1s+1)(1s+1)}$$

The closed-loop response of the present study and the tuning methods given by Chien, Chung [34] and Shamsuzzoha and Lee [27] for a unit step change in both the set-point and load disturbance are shown in Figure 3(a) and 3(b) respectively. The tuning parameter is adjusted to attain same robustness (*Ms* = 1.88) as used by other two methods. The PID parameters of each method with the performance indices for set-point change are listed in Table 2(a) and for the disturbance change the performance parameters are shown in Table 2(b). The Figure 3(a) and (b) show the responses of the unit step change set-point as well as load change. These figures show the smaller overshoot than Chien, Chung [34] and faster settling time in this method is obtained. The similar results for closed-loop are obtained of the proposed controller and other two controllers. A set point weighting parameter is used in the proposed method to decrease the overshoot while the other method used a set point filter for the same purpose. $G_p = (-0.23s+1)1.15e^{-0.23s}/(0.85s+1)(0.85s+1)$ is obtained worst case under a 15% uncertainty in all four parameters to evaluate the robustness of the controller. The simulation results for all design methods are given in Table 2(a) for the set point change and Table 2 (b) shows the load change performance parameters. The time integral error has similar values for the proposed method.

Proposed method, set point weighting parameter b=0.2 [27]:

$f_R = 1/(0.7044s^2 + 1.6399.45s + 1)$,

$a = 0.2$,

$b = 0.1715$

Chien et al., no set point filter is used.

Ms = 1.88

Table 2(a). Various performance parameters of unit step change in set-point.

PID parameters				Nominal Case			15% Mismatch	
Methods	**Kc**	τ_i	τ_D	**ITAE**	**Overshoot**	**TV**	**ITAE**	**Overshoot**
Proposed, λ=0.9	1.6002	1.404	0.4292	4.6	1.16	2.41	2.95	1.13
[27], λ=0.443	3.0819	1.639	0.4295	1.88	1.027	6.0	2.12	1.499
Chien, Chung, Chen and Chuang, 2003),λ=0.091	1.7182	1	1	7.805	1.35	3.65	5.689	1.351

Table 2(b). Various performance parameters of unit step change in load.

Nominal Case				15% Mismatch	
Method	**ITAE**	**Overshoot**	**TV**	**ITAE**	**Overshoot**
Proposed, λ=1.3	3.74	0.44	1.78	2.83	0.55
[27] λ=1.182	1.17	0.26	2.053	1.20	0.367
(Chien, Chung, Chen and Chuang, 2003), λ=0.091	3.75	0.277	2.177	3.034	0.343

3.3. Example 3: First order delay integrating process

An integrating process having at least a pole at origin in transfer function model is observed in many chemical and biochemical industries. Due to presence of a pole at the origin the step response of these processes becomes unstable and it is difficult to control. The process transfer used by Shamsuzzoha and Lee [27] and Sree and Chidambaram [35] is chosen to study the applicability of the present design for the integrating processes.

$$G_p = \frac{1e^{-4s}}{s(4s+1)}$$

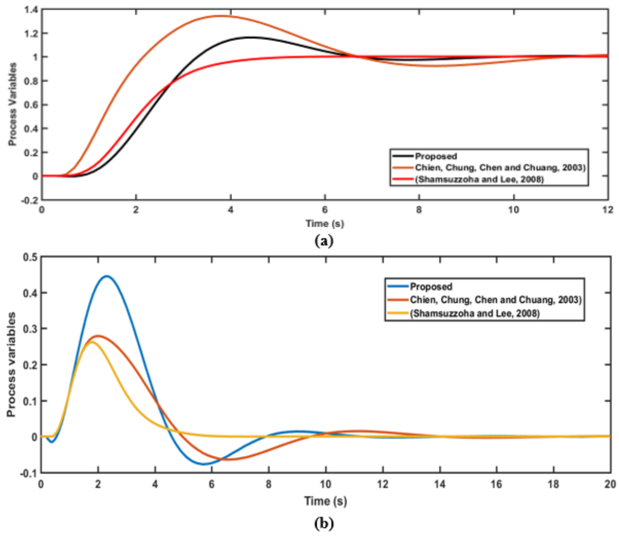


Fig. 3. (a) Closed-loop response of the unit step change in set-point for the nominal case. **(b)** Closed-loop response of the unit step change in load for the nominal case.

The performance of the proposed study is compared with the another two well-known PID controller design Shamsuzzoha and Lee [27] and Sree and Chidambaram [35]. In the simulation study, the *Ms* = 3.28 is obtained by adjusting the controller tuning parameter to get the same robustness for the proposed method and other two methods.

Above process is converted to $G_p = 100e^{-4s}/(100s+1)(4s+1)$ and then after controller is designed [27]. Output response for a unit step change in both the set-point and load disturbance are shown in Figure 4(a) and 4(b) respectively. Present study shows lower overshoot and settling time as compared to Sree and Chidambaram [35] and similar to Shamsuzzoha and Lee [27]. But in case of disturbance rejection, [27] provide best results. PID parameters and performance indices for the servo problem are listed in Table 3(a) and for the load are listed in Table 3(b). A set-point weighting parameter of b=0.3 is used to minimize the maximum overshoot in the set-point response however, the other two methods a set-point filter is used for the same purpose. 10% model uncertainty is introduced in to all the three parameters and the process transfer obtained as $G_p = 1.1e^{-4.4s}/s\,(3.6s+1)$ to evaluate the robustness of the controller.

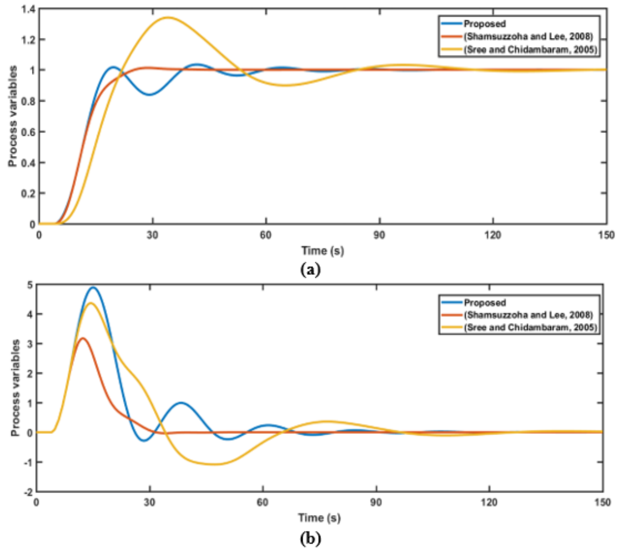


Fig. 4. (a) Closed-loop response of the unit step change in set-point for the nominal case. **(b)** Closed-loop response of the unit step change in load for the nominal case.

Table 3(a). Various performance parameters of unit step change in set-point.

PID parameters				Nominal Case			10% Mismatch	
Methods	Kc	τ_i	τ_D	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed, λ=1.9	0.2896	18.83	3.1503	161.4	1.0157	0.5696	644.7	1.15
[27], λ=2.117	0.3593	12.13	2.704	87.31	1.012	0.393	128.9	1.050
[35]	0.199	10.0	7.02	560.0	1.339	0.292	478.6	1.335

Table 3(b). Various performance parameters of unit step change in load.

Nominal Case				10% Mismatch	
Method	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed	4.613	4.912	4.613	6440	5.50
[27], λ=2.117	499.4	3.179	3.36	953.1	3.741
[35]	3092.0	4.363	3.17	2835	4.960

Proposed method, set point weighting parameter b=0.3

[27]:

$f_R = 1/(32.8106s^2 + 12.1304s + 1)$,

$a = 2$,

$b = 0.049$ [35],

$f_R = 1/(70.2s^2 + 10s + 1)$,

Ms = 3.28

3.4. Example 4: Unstable second order delay process (USOPDT)

We consider the unstable second order process used by Rao and Chidambaram [36] and Shamsuzzoha and Lee [27].

$$G_p = \frac{e^{-0.939s}}{(5s-1)(2.07s+1)}$$

The proposed method is applied to the above process model performances of the controller are compared with [27] and [36]. *Ms* = 2.34 is chosen same as the other two methods for fair comparison in terms of the robustness. Evaluate its relative performance. Figure 5(a) and 5(b) shows the unit step response in set point as well as in load change. The responses show that the proposed method has higher overshoot and settling time but the ITAE is the comparable to other two methods and the total variation is lower than the other methods. Therefore, the present method can be useful to control the unstable system. The robustness of the controller is evaluated by giving of 10% perturbation in all the three parameters and the process transfer function is obtained as:

$$G_p = 1.1e^{-1.0329}/(4s-1)(1.863+1)$$

and the results are compared in Table 4(a) and 4(b).

Table 4(a). Various performance parameters of unit step change in set-point.

PID parameters				Nominal Case			10% Mismatch	
Methods	Kc	τ_i	τ_D	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed, λ=0.68	4.0636	28.4296	1.4026	11.88	1.170	10.234	31.82	1.193
[27], λ=0.9296	6.7051	5.4738	1.3330	7.929	1.030	114.48	12.40	1.052
[36], λ=01.5	6.4285	6.4409	1.4135	6.762	1.032	153.66	12.70	1.055

Table 4(b). Various performance parameters of unit step change in load.

Nominal Case				10% Mismatch	
Method	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed, λ=0.68	145.3	0.383	0.7634	147	0.422
[27] λ=0.9296	4.365	0.163	2.54	5.94	0.192
[36] λ=1.50	5.921	0.166	3.41	7.01	0.190

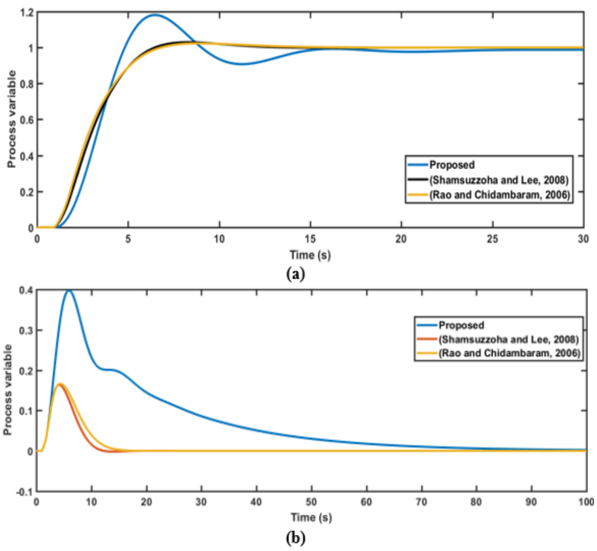


Fig. 5. (a) Closed-loop response of the unit step change in set-point for the nominal case. **(b)** Closed-loop response of the unit step change in load for the nominal case.

3.5. Example 5: First order plus time delay process (FOPDT)

FOPDT process is extensively obtained in the process industries such as for level control, temperature control etc. we consider the FOPDT model which was used by Shamsuzzoha [21] and Skogestad [19] to check controller's performances.

$$G_p = \frac{1}{10s+1}e^{-0.5s}$$

Figure 5(a) and (b) show the closed-loop response of the unit step change in the refernce as well as in the load. The performance of the proposed controller are compared to the other two methods given by Shamsuzzoha [21] and Skogestad [19]. The overshoot for the servo and regulatory problems is slightly greater than other two methods in the nominal case but in the case of model uncertainty, it is lower than the other methods. the maximum sensitivity (*Ms*) is taken 1.65 which is same as the other methods for the fair comparison of robustness. A 10% of perturbation for the model uncertainty is introduced into all the three parameters of the process and the perturbed process transfer obtained as:

$$G_p = \frac{1.1}{9s+1}e^{-0.55s}.$$

Performance matrix of the various controller is shown in Table 5(a) and 5(b) for both servo as well as regulatory problems.

Table 5(a). Various performance parameters of unit step change in set-point.

				Nominal Case			10% Mismatch	
Methods	Kc	τ_i	τ_D	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed, λ = 0.715	6.432	1.69	0	5.55	1.20	8.60	3.721	1.1933

[19], λ = 0.5	10.0	4	0	3.50	1.17	13.3	2.806	1.2994
[21], λ =1.286	9.41	2.78	0	4.11	1.33	12.5	2.738	1.3521

Table 5(b). Various performance parameters of unit step change in load.

Nominal Case				10% Mismatch	
Method	**ITAE**	**Overshoot**	**TV**	**ITAE**	**Overshoot**
(Proposed), λ =0.715	1.298	0.105	1.915	1.05	0.119
[19], λ = 0.5	1.730	0.090	1.34	1.71	0.102
[21], λ =1.286	0.908	0.091	1.50	0.90	0.104

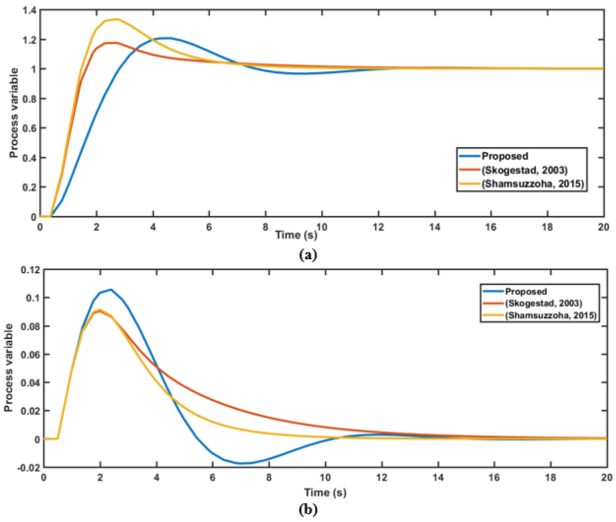


Fig. 6. (a) Closed-loop response of the unit step change in set-point for the nominal case. **(b)** Closed-loop response of the unit step change in load for the nominal case.

3.6. Example 6: Unstable first order plus time delay process (UFOPDT)

We consider the following unstable first order plus time delay process (UFOPDT) which was also used by Shamsuzzoha [21] and Shamsuzzoha and Skogestad [37]:

$$G_p = \frac{1}{5s-1} e^{-1s}$$

The maximum sensitivity value Ms = 2.33 is selected equal to other two methods for the fair evaluation of the proposed PID with other tuning techniques mentioned in Table 6(a) and 6(b). Figure 7(a) and 7(b) show the output closed-loop response of the unit step change in set point change as well as in load change. The proposed method shows faster response and lower overshoot compared to other methods in case of the servo problem. However, in case of load change, overshoot is more than Shamsuzzoha [21] and lower than Shamsuzzoha and Skogestad [37]. 10% of perturbation is introduced in all the three parameters for model uncertainty and the transfer function is obtained as:

$$G_p = 1.1e^{-1.1s}/(4.5s-1)$$

to estimate the robustness and performance of the proposed controller.

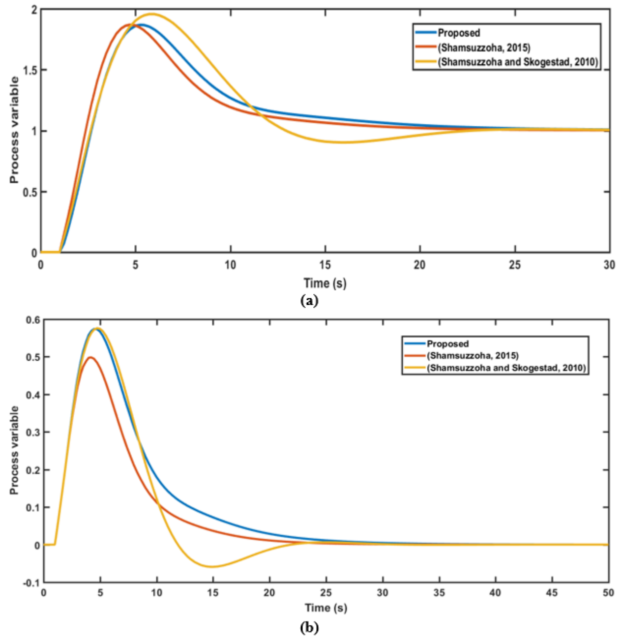


Fig. 7. (a) Closed-loop response of the unit step change in set-point for the nominal case. **(b)** Closed-loop response of the unit step change in load for the nominal case.

3.7. Example 7: First order plus time delay process (FOPDT)

To evaluate the performance of proposed PID controller design technique, an experiment was performed on the conical tank for the level control. The mathematical model of the conical tank is obtained as:

$$(35.53h - 0.052h^2)\frac{dh}{dt} = F_i - 2835.4h^{0.422}$$

Here, 'h' is the liquid level in the tank at any time 't' and F_i is the inlet flowrate to the tank.

PI/PID controller cannot be design based on nonlinear process model, therefore, it is required to linearize the model and convert into the transfer function model for the designing of the PI/PID controller. A first order with time delay transfer function model is obtained and which is given by:

$$G_p = \frac{0.01307}{48.8S+1} e^{-2S} .$$

The time delay of 2 sec is obtained by the open-loop experiment.

Table 6(a). Various performance parameters of unit step change in set-point.

PID parameters				Nominal Case			10% Mismatch	
Methods	Kc	τ_i	τ_D	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed, λ = 0.92	2.80	12.12	0	48.32	1.867	7.77	38.51	2.144
[21], λ =2.77	3.04	9.76	0	40.12	1.868	7.84	38.06	2.156
[37],	2.48	7.85	0	50.10	1.955	7.66	36.69	2.147

Table 6(b). Various performance parameters of unit step change in load.

Nominal Case				10% Mismatch	
Method	ITAE	Overshoot	TV	ITAE	Overshoot
(Proposed),	30.27	0.574	2.80	29.22	0.682
[21]	23.88	0.498	2.73	26.86	0.595
[37]	28.51	0.576	3.12	21.88	0.670

Figure 8(a) and 8 (b) show that the output response of the unit step change in set point as well as load change respectively. The proposed method shows better response and lower overshoot in both the cases set point change as well as in load change compared to Shamsuzzoha [21] and Skogestad [19]. The performance of the proposed controller listed in Table 7(a) for the set point change and Table 7(b) shows for load change. Robustness of the proposed controller is calculated by introducing a 10 % perturbation in all the three process parameters and the transfer function of perturbed model is obtained as:

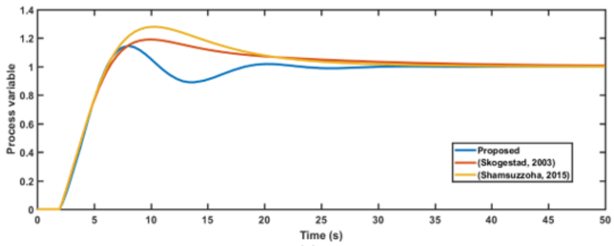
$$G_p = \frac{0.0144}{43.92S+1} e^{-2.2S}.$$

Table 7(a). Various performance parameters of unit step change in set-point.

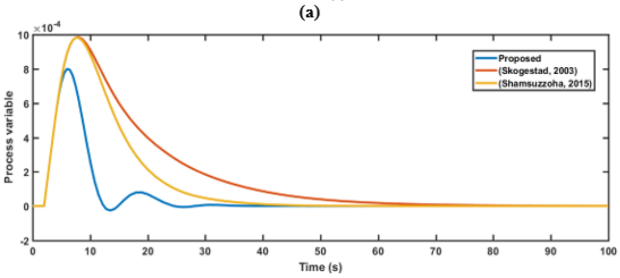
PID parameters				Nominal Case			10% Mismatch	
Methods	Kc	τ_i	τ_D	ITAE	Overshoot	TV	ITAE	Overshoot
Proposed, λ = 1.5	1484	7.42	0	22.09	1.145	2339.48	78.49	1.178
[21], λ = 5	906	11	0	53.02	1.279	1315.5	49.49	1.40
[19], λ = 2	933.43	16	0	56.65	1.190	1176.202	44.23	1.318

Table 7(b). Various performance parameters of unit step change in load.

Nominal Case				10% Mismatch	
Method	ITAE	Overshoot	TV	ITAE	Overshoot
(Proposed),	0.0661	7.611×10^{-4}	2.777	0.111	9.781×10^{-4}
[21]	0.1451	9.853×10^{-4}	1.557	0.144	12.10×10^{-4}
[19]	0.2971	9.822×10^{-4}	1.380	0.293	12.10×10^{-4}



(a)



(b)

Fig. 8. (a) Closed-loop response of the unit step change in set-point for the nominal case. **(b)** Closed-loop response of the unit step change in load for the nominal case.

4. Mathematical Modeling of a Conical Tank

A nonlinear level control process is taken as the process to provide the applicability of the proposed IMC-PID tuning method. In experimental setup as shown in Fig.9, a conical tank is inserted in the middle of the cylinder and whose base diameter is nearly equal to the diameter of the cylinder. The bottom part of the cone has a greater diameter as shown in below Fig. 9 and therefore the liquid flows into the annular part of the cone and cylinder. The parameters of the cone and cylinder are given Table 8.

The mathematical model for the material balance can be written as:

$$A\frac{dh}{dt} = Fi - Fo \tag{13}$$

Where, Fi and Fo are the inlet and outlet flow rate respectively and expressed in mm^3/sec. h is the liquid level in the tank expressed in mm. since, a cone is inserted in the cylinder, the annular area of the tank is varying with the tank height.

Table 8. Process Parameters and their values.

Parameter	Values
The inner radius of cylinder, R	46 mm
The smaller or top outer radius of cone, R1	12.5 mm

The larger or bottom outer radius of cone, R2	44 mm
Maximum Height of the cone, H	245 mm

Assuming that the radius of the cone at any height of 'h' is = r. Therefore, from the similar triangle relation we can write the relation between height and radius at any instant:

$$r = R2 - (R2 - R1)\frac{h}{H} \tag{14}$$

(from the similar triangle principle)

And the area of the annular part of cone and cylinder at height h is:

$$A = \pi\,(R^2 - r^2) \tag{15}$$

Or,

$$A = \pi\left(R^2 - \left(R2 - (R2 - R1)\frac{h}{H}\right)^2\right) \tag{16}$$

Therefore, from Eq. (13) and (16) we can write mathematical model for the material balance.

Or,

$$\pi\left(R^2 - (R2 - (R2 - R1)\frac{h}{H})^2\right)\frac{dh}{dt} = Fi - F0 \tag{17}$$

Now, we assume that the outer radius of the bottom part of the cone and the inner radius of the cylinder is nearly equal i.e. R=R2.

After simplification and calculation of the eq. (17) we get:

$$\pi(R2 - R1)\left(2 \times R2 \times \left(\frac{h}{H}\right) - (R2 - R1)\left(\frac{h}{H}\right)^2\right)\frac{dh}{dt} = Fi - Fo \tag{18}$$

Now, Fo is required to solve the Eq. (18) in terms of the discharge coefficient and characteristics coefficient of the discharge valve. Several open-loop experiments have been performed to get the relationship between outlet flow rate and Fo and the discharge valve's coefficients. At steady state:

$$Fin = Fo \tag{19}$$

$$Fo = \beta\,(h)^n \tag{20}$$

The value of parameter 'β' and 'n' are determined by a plot of steady state outlet flow rate *'Fo'* and corresponding steady state height '*h*' as shown in Fig. 10.

The discharge coefficient 'β'' and characteristic coefficient 'n' are determined to be 2835.4 and 0.422 respectively.

Hence the final equation of the mathematical model of the process can be written as:

$$\frac{dh}{dt} = \frac{Fi}{\pi(R2-R1)\left(2\times R2\times\left(\frac{h}{H}\right)-(R2-R1)\left(\frac{h}{H}\right)^2\right)} - \frac{2835.4h^{0.422}}{\pi(R2-R1)\left(2\times R2\times\left(\frac{h}{H}\right)-(R2-R1)\left(\frac{h}{H}\right)^2\right)} \tag{21}$$

The above model equation (21) is solved by state space method, and we can get the various process transfer function at different steady-state height. Now,

$$F(h, Fi) = \frac{dh}{dt} = \frac{Fi}{\eta(h)} - \frac{\varphi(h)}{\eta(h)} \tag{22}$$

Where,

$$\eta(h) = \pi(R2 - R1)\left(2 \times R2 \times \left(\frac{h}{H}\right) - (R2 - R1)\left(\frac{h}{H}\right)^2\right)$$

And

$$\varphi(h) = 2835.4h^{0.422}$$

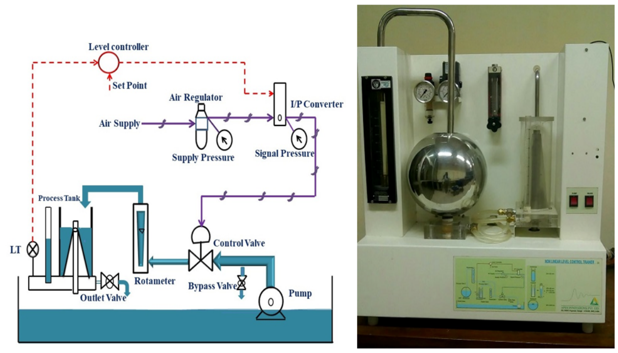


Fig. 9. Schematic diagram of experimental setup.

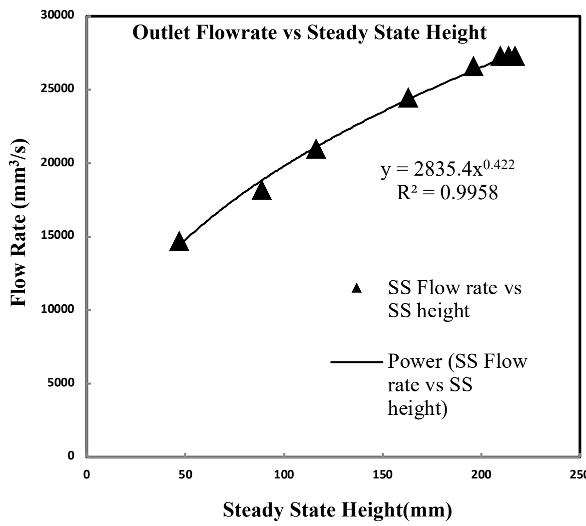


Fig. 10. Data fit between steady height and steady state outlet flow rate.

we can write the equation (22) in state space form as:

$$\dot{X} = AX + BU \tag{23}$$

$$Y = CX + DU \tag{24}$$

Since in the present study, the process is described as single input single output (SISO). Therefore, X is state variable, U is the input variable and Y is the output variable respectively. The equation (10) is linearized at steady state of state variable i.e. at h = hs and we can determine the state matrix 'A' and the input matrix 'B' as written bellow equation.

$$A = \left.\frac{\partial F(h,Fi)}{\partial h}\right|_{h=hs} \tag{25}$$

$$B = \left.\frac{\partial F(h,Fi)}{\partial Fi}\right|_{h=hs} \tag{26}$$

For SISO system C the output matrix is the identity matrix, therefore, C=*I* and D=0 for the present case study.

Now, the output-input transfer function of the process is written as:

$$\frac{Y(S)}{U(S)} = \frac{h(S)}{Fi(S)} = C(SI - A)^{-1}B \tag{27}$$

From the above eq. (15) we determine the transfer function of the process at steady state height i.e. h=hs= 116 mm is:

$$\frac{Y(S)}{U(S)} = \frac{h(S)}{Fi(S)} = \frac{0.01307}{48.8S+1}e^{-2S} \tag{28}$$

The time delay term in the eq. (28) comes from the various open loop test of the experiment and it is found to be of 2 sec.

The proposed method of design of IMC-PID controller is used to design the PI controller for the nonlinear system of conical tank and the simulation studies of the closed loop is shown in Fig. 8. The proposed method shows the better results than the other two methods [19, 21]. The ITAE value of the present method is much lower than the other both methods, while the settling time, overshoot is much lower than the other two methods as mentioned in Table 7.

The experimental results shown in Figs. (11), (12) and (13) are obtained by using the proposed tuning method and other two tuning methods mentioned above. All the three tuning methods used for the level control, the initial responses of all the methods shows oscillatory response due to nonlinearity in the process and get stable after some time. The settling time for the proposed method is greater than the Skogestad [19] but lower than Shamsuzzoha [21]. The proposed tuning technique shows greater overshoot than the Skogestad [19] but comparable to Shamsuzzoha [21]. The performance of the design techniques done in the terms of error index IAE and found to be 1409.485, 1746.85 and 2259 respectively for the Skogestad [19], Shamsuzzoha [21] and the proposed method.

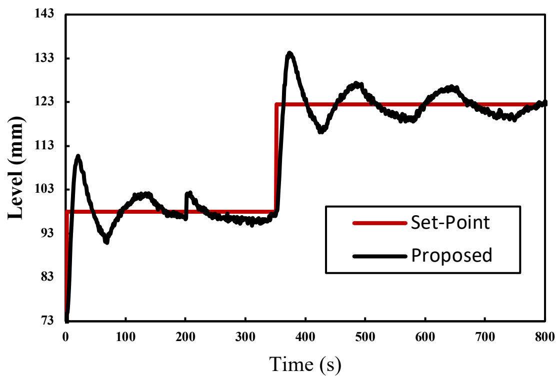


Fig. 11. Closed loop response of method by proposed method on conical tank setup.

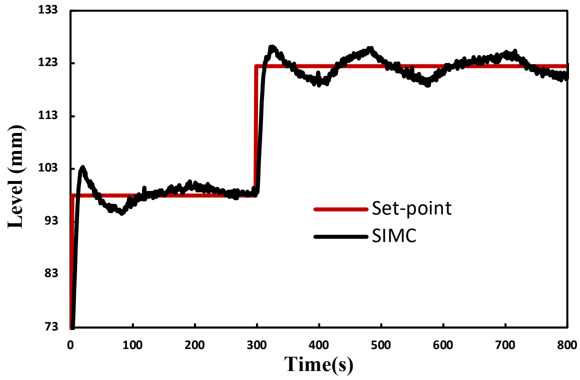


Fig. 12. Closed loop response of method by Skogestad (2003) on conical tank setup.

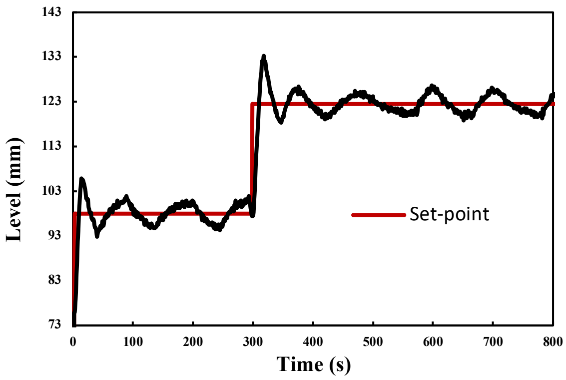


Fig. 13. Closed loop response of method by Shamsuzzoha (2015) on conical tank setup.

5. Conclusions

The IMC based PI/PID controller is designed and applied to various types processes. The tuning parameter λ is selected such that the maximum sensitivity *Ms* values matches for the particular process given in the literature and also which gives minimum ITAE values. A set point filter is used by different researchers to remove the excessive overshoot. However, in the present studied a set point weighting parameter is used for the same minimize the undesirable overshoot where ever it is required. Since, the setpoint weighting parameter design procedure is very easier than set point filter design. The robustness analysis is done by inserting a 10% and 15 % perturbation in each of the process parameters in the worst direction, and robustness of the proposed method is evaluated against the parameters uncertainty. The proposed method is successfully applied to the conical tank transfer function model and also on real system for level control.

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