

Abstract

In the cosmic history of the matter, at the end of the Epoch of recombination, tiny fluctuations in the matter densities were present. With the growth of these tiny fluctuations due to gravitational instabilities, the first luminous objects in the universe emerged. The era during which these first objects formed is termed as the Cosmic Dawn. The radiation from these luminous objects started ionizing the surrounding neutral hydrogen in the universe. As time progressed, the bubbles of ionized hydrogen increasingly overlapped, resulting in an entirely ionized universe. This process of phase transition of the universe from a neutral state to an almost completely ionized state is known as reionization. The Epoch of Re-ionization is one of the most important but less understood eras in the history of the universe's evolution. As neutral hydrogen was the dominant component of the universe during Cosmic Dawn, EoR, and post-EoR era, the radiation emerging from the forbidden spin-flip transition of neutral hydrogen provides excellent prob of these important stages of the universe. This radiation resulting from the line transition of the neutral hydrogen peaked at a frequency of 1420 MHz or equivalently at the 21-cm in wavelength units, hence getting its famous name "the 21- cm signal". Observing this 21- cm signal at redshifted frequencies can solve many mysteries, including our understanding of structure formation, the evolution, and the radiation mechanism of these first luminous objects. The direct imaging of the redshifted 21- cm signal is quite difficult due to the presence of very strong radiation coming from the foregrounds in similar observational frequency ranges, collectively known as foregrounds. Apart from the presence of orders of magnitude larger foregrounds in the observed frequency range, the instrumental effects of the interferometers, combined with the ionospheric effects, present a considerable challenge in the extraction of 21-cm signals from strong foregrounds. The combination of these instrumental and ionospheric effects, which is termed as "gain", is complex in nature and depends both on time and frequency.

The gains are estimated during the data preparation using various calibration procedures. Even after investing a good amount of effort, there are some residual errors in the gain. Although these residual gain errors are small in magnitude, these gain errors become important in high dynamic range observations with the presence of strong foregrounds. The systematic effects of these time and frequency-correlated residual gain errors originating from the measurement process introduce a bias and enhance the variance of the power spectrum measurements. Our work studies the effect of time and frequency-correlated residual gain errors in estimating the power spectrum of the sky brightness distribution in high dynamic range observations, such as EoR observations.

In the first project included in this thesis, we study the effect of residual gain errors in primary calibration. Modeling the residual gain errors as time-independent Gaussian random variables with zero mean, we analytically calculated the amplitude and variance of the visibility. Calculations showed that the visibility amplitude decreases with the phase gain error. Also, the variance of the visibility increases with both amplitude and phase gain error. For a single point source at the center of the field of view, we study the statistics of simulated visibilities and find that the simulated visibilities show similar behavior as predicted by analytical results.

In the next project, we discuss a methodology to estimate the bias in the power spectrum estimates of the redshifted 21-cm signal from neutral hydrogen in the presence of strong foregrounds. Considering time correlation in residual gain, we model the two-point correlation function of the residual gain errors using a power law and study its effect on the power spectrum. Through simulated observations using GMRT baseline configuration, we show that the presence of time-correlated residual gain errors introduces a bias in the power spectrum estimator that uses the correlation of visibilities in nearby baselines. We consider the contributions from various types of baseline pairs involved in the visibility measurement to be used in visibility correlation. We find that for the visibility correlation-

based power spectrum estimator, the bias in the power spectrum arises mainly from those types of baseline pairs that have at least one antenna in common. The bias also depends on the foreground model and the correlation properties of the gain errors. We define the ratio of the bias to the foreground power spectrum as the “Power spectral gain”. This quantity was later used to calculate the bias for the foreground model, including the bright point sources and the galactic diffuse synchrotron emission. Our results show that more than 10,000 hours of observation will be required using GMRT baseline configuration for the calibration accuracy of 1% for the full foreground. In this work, we do not consider the frequency dependence of the gain. Also, to check the effect of gain errors explicitly, we do not consider the thermal noise in our visibility measurements.

In the next project, we included this measurement noise in the estimation of bias and variance in the power spectrum with residual gain errors. We propagate errors in the presence of thermal noise and present a method to produce analytic estimates of the bias and variance in the power spectrum. We model the residual gain errors as time-correlated Gaussian random variables with zero mean. To incorporate the time correlation of the gain errors, we model the two-point correlation function of the gain errors using a Gaussian function with correlation time given as the half-width of the Gaussian. We study the effect of time-correlated residual gain errors in presence of strong foregrounds and thermal noise analytically and provide the mathematical expressions for bias and variance in the power spectrum measurements. We used simulated observations using GMRT baseline configuration in the presence of point source foregrounds. We observe that the simulation results are in quite good agreement with our analytical estimates. Later we use this analytical framework to understand various effects of the correlated gain errors. As the standard deviation in the residual gain errors increases, the bias in the estimation supersedes the variance. It is observed that an optimal choice of the time over which the gain solutions are estimated minimizes the combination of bias and variance, collectively known as the

risk. We also find that the interferometers with higher baseline densities are preferred instruments for these studies.

As the complex antenna gains are both time and frequency-dependent, we incorporate the frequency dependence of the gains in our analytical framework as a part of our final project. Similar to the time correlation of the gain errors, we model the frequency correlation of the gains also using a Gaussian function. Our analytical estimates show that the bias is more at low-frequency separations and increases with an increase in the correlation frequency. On the other hand, the variance is dominated by the contribution of thermal noise, and it shoots up at higher frequency separations. This is because, at higher frequency separations, the number of independent estimates of the visibility correlation will be less. We compare the performance of GMRT and SKA - 1 Low baseline configurations and find that SKA -1 Low baseline configuration will require ~ 50 times less observation time than the GMRT. This is primarily because of the baseline configuration and the larger collecting area of the SKA- 1 Low.

Our work sheds light on the importance of assessing the effect of time and frequency-correlated gain errors in high dynamic range observations, such as EoR experiments. Although the mathematical estimates of the bias and variance depend upon the array configuration, the methodology we present in this work is rather general and can be used to estimate the bias and variance of the power spectrum estimates for any telescope, given the antenna gain properties are known. We believe this would add and aid the quest of detecting the redshifted 21- cm signal from the Epoch of reionization.