MULTI-FUNCTIONAL ANTENNA (MFA) ARRAYS USING SPARSE CONCEPT

4.1 Context and Background

In chapters 2 and 3, thinned and non-uniformly spaced configurations of sparse antenna arrays dealing with antenna elements operating at single frequency were examined. Chapter 4 discusses the design of randomly spaced configuration of sparse antenna arrays, consisting of identical sized radiating elements operating at one frequency and differently sized radiating elements operating at different frequency forming a multi-functional array. This approach is considered to meet multi-functional requirements of today's military radar system by combining functionalities of several antenna arrays into a single aperture. As explained in chapter 1, the necessary space on array aperture for populating differently sized elements can be created by using sparse configurations (configurations with non-regular distribution of elements and inter-element spacings generally greater than half-wavelength). The gaps between radiating elements of one sub-aperture can be utilized to design and fill with another sub-aperture for different functionality [Coman (2006)]. During synthesis of such configurations, care is to be taken to prevent overlapping of the array elements concerning distinct sparse configurations.

In this chapter, the synthesis approaches to design randomly spaced and multifunctional planar antenna (RSPA and MFA) arrays are described. PSO optimization technique based synthesis approach developed for non-uniformly spaced planar antenna (NUSPA) arrays in chapter 3 is modified to design randomly spaced planar antenna and multi-functional antenna arrays in this chapter. These synthesis approaches are based on MBC-GA optimizer take into consideration the mutual coupling effect and physical size of the antenna elements. Research work in this chapter includes studies on 8×16 -element RSPA, and 4×4 - and 8×8 -elements MFA arrays.

4.2 Randomly Spaced Planar Antenna (RSPA) Arrays

4.2.1 Introduction

The research work on RSPA arrays deals with PSLL Optimization in 8×16 -element RSPA array. Two constraints are assumed during the synthesis process. First constraint is to maintain constant amplitude excitation across the entire array elements and the second one is to vary the inter-element spacing randomly within 0.5λ - 1.5λ . Present study employs MBC-GA optimization technique in order to find the most optimum arrangement of inter-element spacings corresponding to the radiation pattern with low peak side lobe level (PSLL). To authenticate the efficacy of the proposed technique, an 8×16 -element RSPA array is numerically analysed for its performance.

In the field of array synthesis, a method of randomly varying the inter-element spacing between elements across the aperture has gained attention of many engineers [Coman (2006)]. Such arrays are expected to be more efficient than the uniformly spaced arrays in terms of producing a radiation pattern with reduced peak side lobe level, minimizing the occurrence of grating lobes, and reduction in the effect of mutual coupling as well as ease in the design of radiating elements. In order to determine the most optimum inter-element spacings, various optimization techniques have been devised. In the recent past, global optimization techniques such as genetic algorithm (GA) [Haupt (1995)],[Chen *et al.*(2006)], [Donelli *et al.* (2009)], [Cen *et al.* (2012)],

and [Wang *et al.* (2012)], particle swarm optimization (PSO) [Khodier and Christodoulou (2005)], [Goudos *et al.* (2010)], differential evaluation algorithms (DE) [Kurup *et al.* (2003)], iterative algorithms (IA) [Fuchs *et al.* (2012)] and firefly algorithm (FA) [Zaman and Matin (2012)] have been developed to design randomly spaced antenna (RSA) arrays for a variety of applications.

The objective of the present study is to synthesize RSPA array with low peak side lobe level and also to reduce the number of array elements by randomizing the spacing between the elements. Modified genetic algorithm (MBC-GA) is devised to compute the optimum arrangement of randomly spaced elements related to the lowest achievable peak side lobe level (PSLL). To exhibit the performance of the proposed method, 8×16-element RSPA array with equal amplitude excitation and random inter-element spacings is studied and its results obtained through proposed synthesis technique are reported.

4.2.2 Mathematical Formulation for 8×16-element RSPA Arrays

Consider the geometrical configuration of the randomly spaced planar antenna (RSPA) array as shown in Fig. 4-1 with 2N×2M sources placed symmetrically in X-Y plane and centred at the origin. The total field radiated by this array in X-Z plane is expressed as

$$E(\theta, \Phi) = \left[E(\text{element at ref}(\theta, \Phi))\right] \times [AF(\theta, \Phi)]$$
(4.1)

where

$$AF(\theta, \Phi) = \sum_{n=1}^{N} \sum_{m=1}^{M} a_{nm} \exp\left[j\left(k\sum_{i=1}^{N} d_{i}\cos\left(\theta\right)\right)\right] \times \exp\left[j\left(k\sum_{i=j}^{M} d_{j}\cos\left(\theta\right)\right)\right]$$
(4.2)

is the Array Factor, $E_{ele}(\theta)$ is the antenna element radiation pattern, $k = \frac{2\pi}{\lambda}$, λ is the free space wavelength, θ is the angle of arrival of the incident wave measured with respect to Z-axis, d_i is the inter-element spacing between the (i-1)th and ith elements along X-axis, d_j is the inter-element spacing between the (j-1)th and jth elements along Y-axis, and A_{nm} is the amplitude of the (n,m)th element which is assumed to be 1 for all elements in this study.

$$x_n = \begin{cases} \sum_{i=1}^{N} d_i, \text{ for } N \ge 1\\ 0, \text{ for } N = 0 \end{cases}$$
(4.3)

$$y_m = \begin{cases} \sum_{i=1}^{M} d_j, \text{ for } M \ge 1\\ 0, \text{ for } M = 0 \end{cases}$$
(4.4)

where x_n is the distance between 1st and nth antenna elements of the RSPA array along X-axis, and y_m is the distance between 1st and mth antenna elements of the RSPA array along Y-axis. The element positions in the proposed array shall satisfy the following constraints.

where d_{min} and d_{max} are the minimum and maximum inter-element spacings along both X- and Y-axes.



Fig. 4-1: Geometrical configuration of the RSPA array (X-positive Y-positive plane) with N \times M-elements.

The fitness function is the peak side lobe level (PSLL) in principal planes $(\Phi = 0^{\circ} \text{ and } \Phi = 90^{\circ})$ of the radiation pattern. The objective of the present study is to minimize the PSLL of RSPA array by computing the most optimum inter-element spacing combination vectors $D = [x_n, y_m]$. The fitness function for calculating PSLL can be formulated as

$$F_{PSLL}(D) = \max_{\forall \theta \in R} \left[20 \log \left| \frac{AF(D, \theta, \Phi)}{AF_{max}} \right| \right]$$
(4.7)

where 'R' denotes the angular sector excluding the main beam, and AF_{max} is the maximum value of array factor (AF).

4.2.3 Optimization Technique for the Synthesis of 8×16-element RSPA Array

In the present study, MBC-GA optimization technique as described in section A1.2 (Appendix-A) is used to determine the best configuration of RSPA array that would

result in lowest possible value of PSLL. During search this technique optimizes interelement spacing vector $D \in \{d_i, d_j\}$, which reduces $F_{PSLL}(D)$ in Eq. (4.7).

4.2.4 Numerical Analysis, Results and Discussion of 8×16-element RSPA Array using MBC-GA

The effectiveness of the proposed optimization algorithm is validated by numerically examining 8×16-element RSPA array. Uniform amplitude excitation is considered across array elements and the spacing between elements was randomly perturbed over a range 0.5λ -1.5 λ . The algorithm with a population of 1000 chromosomes was iterated over 300 times. The convergence plot is depicted in Fig. 4-2, which shows the variation of the fitness value with the number of iterations. It is evident from the convergence plot that the proposed algorithm converges to an optimum solution within less than 100 numbers of iterations. The values of inter-element spacing in terms of λ obtained using proposed MBC-GA based method are listed in Table 4-1 and 4-2 along X-and Y-axes respectively. The distribution of antenna elements on the aperture of RSPA array is shown in Fig. 4-3. The radiation patterns of the proposed 8×16-element RSPA array in the azimuth and elevation planes are shown in Figs. 4-4 and 4-5 respectively. The 3D radiation pattern and intensity plot are shown in Figs. 4-6 and 4-7 respectively. The radiation characteristics extracted from Figs. 4-4 - 4-7 are tabulated in Table 4-3. It can be observed from Figs. 4-4 -4-7 and Table 4-3 that PSLLs better than -19.08 dB and -18.04 dB have been obtained through the proposed method in azimuth and elevation planes respectively.



Fig. 4-2: Variation in fitness value i.e. PSLL with number of iterations for synthesis of the proposed 8×16-element RSPA array.

Table 4-1: Variation of inter-element spacing obtained by the proposed MBC-GA

 based method along X-axis normalized to wavelength for 8×16-element RSPA array.

0	0.91	0.88	0.69	0.64	0.63	0.69	0.80	0.69	0.80	0.69	0.63	0.64	0.69	0.88 0.91
0	0.77	0.80	0.97	0.73	0.62	0.62	0.63	0.57	0.63	0.62	0.62	0.73	0.97	0.80 0.77
0	0.83	0.86	0.87	0.76	0.75	0.59	0.55	0.53	0.55	0.59	0.75	0.76	0.87	0.86 0.83
0	0.99	0.97	0.95	0.92	0.75	0.64	0.57	0.51	0.57	0.64	0.75	0.92	0.95	0.97 0.99
0	0.99	0.97	0.95	0.92	0.75	0.64	0.57	0.51	0.57	0.64	0.75	0.92	0.95	0.97 0.99
0	0.83	0.86	0.87	0.76	0.75	0.59	0.55	0.53	0.55	0.59	0.75	0.76	0.87	0.86 0.83
0	0.77	0.80	0.97	0.73	0.62	0.62	0.63	0.57	0.63	0.62	0.62	0.73	0.97	0.80 0.77
0	0.91	0.88	0.69	0.64	0.63	0.69	0.80	0.69	0.80	0.69	0.63	0.64	0.69	0.88 0.91

 Table 4-2:
 Variation of inter-element spacing obtained by the proposed MBC-GA

 based method along Y-axis normalized to wavelength for 8×16-element RSPA array.

0	0.51	0.88	0.92	0.91	0.89	0.81	0.72	0.72	0.81	0.89	0.91	0.92	0.88	0.86	0.51
0.74	0.78	0.80	0.85	0.81	0.67	0.59	0.64	0.64	0.59	0.67	0.81	0.85	0.80	0.78	0.74
0.63	0.64	0.65	0.65	0.74	0.63	0.57	0.59	0.59	0.57	0.63	0.74	0.65	0.65	0.64	0.63
0.77	0.69	0.57	0.55	0.53	0.55	0.51	0.51	0.51	0.51	0.55	0.53	0.55	0.57	0.69	0.77
0.77	0.69	0.57	0.55	0.53	0.55	0.51	0.51	0.51	0.51	0.55	0.53	0.55	0.57	0.69	0.77
0.63	0.64	0.65	0.65	0.74	0.63	0.57	0.59	0.59	0.57	0.63	0.74	0.65	0.65	0.64	0.63
0.74	0.78	0.80	0.85	0.81	0.67	0.59	0.64	0.64	0.59	0.67	0.81	0.85	0.80	0.78	0.74
0.85	0.51	0.88	0.92	0.91	0.89	0.81	0.72	0.72	0.81	0.89	0.91	0.92	0.88	0.86	0.51



Fig. 4-3: Antenna element distribution at the aperture of the proposed 8×16- element RSPA array.



Fig. 4-4: Radiation pattern of the proposed 8×16 -element RSPA array in azimuth ($\phi = 0^0$) plane.



Fig. 4-5: Radiation pattern of the proposed 8×16 -element RSPA array in elevation ($\phi = 90^{\circ}$) plane.



Fig. 4-6: 3D radiation pattern of the proposed 8×16-element RSPA array.



Fig. 4-7: Intensity plot of the proposed 8×16-element RSPA array.

Table 4-3: Numerically evaluated PSLL for the proposed 8×16-element RSPA a	irray
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Parameters	RSPA array using MBC-GA
PSLL (dB) in Azimuth ($\phi = 0^0$) plane	-19.08
PSLL (dB) in Elevation ($\phi = 90^{\circ}$) plane	-18.04

4.3 Multi-functional Antenna (MFA) Arrays

The research work in this study deals with PSLL optimization in 4×4- and 8×8-elements multi-functional array sharing the same physical aperture. The MBC-GA optimization technique, which was utilized for the synthesis of randomly spaced planar antenna (RSPA) array is extended to design MFA arrays.

4.3.1 Introduction

The antenna array functional diversity can be increased by making optimum use of its aperture. Antenna systems that consist of two or more arrays operating at different frequencies can be realized by utilizing sparse array concept. Such arrays offer various advantages, including increased aperture efficiency and cost effectiveness. This study presents a synthesis method to determine the best possible combinations for interelement spacings between elements of same frequency and elements of different frequency to avoid overlapping of elements and result in a desirable radiation pattern. Synthesis of such antenna arrays is a challenging task due to the involvement of risk in terms of overlapping of elements. This needs to be carefully addressed during optimization stage. Steps involved in the synthesis of multi-functional antenna array are as follows:

- The inter-element spacings corresponding to the array operating at higher frequency are determined initially and are mapped into physical area of the array.
- After this, the process for determining inter-element spacings for the lower frequency begins. After the values are generated, they are also mapped into the same physical area of the higher frequency elements. If the lower frequency elements cross the limit of minimum spacing between the elements of higher frequency (limit is set according to user design), then the spacing values for the

higher frequency elements are regenerated until it satisfies the minimum interfrequency spacing criterion.

- After the aforesaid criterion is met, each set of spacing combinations is considered as a chromosome and each chromosome is evaluated for PSLL performance. Fitness value for respective chromosome is calculated by using Eq. (4.7). The process of crossover and mutation is applied. Once the crossover and mutation process is completed, both arrays are again mapped and the condition for minimum spacing between inter-frequency elements is checked. If it satisfies the condition, the algorithm proceeds to the next iteration.
- This process is repeated until the termination criterion is met.

It is well known that, by increasing the distances between elements, the mutual coupling effect in array architectures can be reduced. In the aforesaid synthesis approach since inter-element spacings between the elements of same and different frequencies are greater than or equal to the intended limit, the effect of mutual coupling is expected to be negligible when such arrays would be realized.

4.3.2 Mathematical Formulation for MFA Arrays

In this study, the geometrical configuration of the randomly spaced planar antenna (RSPA) array as shown in Fig. 4-1 with $2N \times 2M$ sources placed symmetrically in X-Y plane and centred at the origin is considered. The total field radiated by this array in X-Z plane can be expressed by Eq. (4.1).

The fitness function is the peak side lobe level (PSLL) in principal planes $(\Phi = 0^{0} \text{ and } \Phi = 90^{0})$ of the radiation pattern. The objective of the present study is to minimize the PSLL of MFA array by computing the most optimum inter-element spacing combination vectors $D = [x_n, y_m]$. The fitness function for calculating PSLL as given in Eq. (4.7) is used, which reduces during the optimization process.

4.3.3 Optimization Technique for the Synthesis of 4×4- and 8×8elements MFA Array

Synthesis approach based on MBC-GA technique as described in section A1.2 (Appendix-A) is used to determine the best configuration of MFA array that would result in lowest possible value of PSLL. The optimum inter-element spacing vector $D \in \{d_i, d_j\}$ associated with lowest possible value of $F_{PSLL}(D)$ in Eq. (4.7) is determined by using MBC-GA method.

4.3.4 Numerical Analysis, Results and Discussion of 4×4- and 8×8elements Multi-functional Antenna Array Sharing the Same Physical Aperture Using MBC-GA

In order to verify the performance of proposed method for the synthesis of multifunctional array, two RSPA arrays, consisting of a 4×4-element array operating at 1.5 GHz and a 8×8-element array operating at 3.3 GHz have been interleaved into common aperture and examined numerically. Initially 4×4-element RSPA array is analysed. The variation of best fitness value i.e. PSLL for this array with number of iterations is shown in Fig. 4-8. The best inter-element spacings obtained for this array along X- and Y-axes are listed in Table 4-4 and 4-5 respectively. The corresponding array architecture is shown in Fig. 4-9. The radiation patterns of the array obtained in $\Phi = 0^0$ and $\Phi = 90^0$ planes at the frequency of 1.5 GHz are demonstrated in Figs. 4-10 and 4-11 respectively. The 3D radiation pattern and intensity plot of the array are shown in Figs. 4-12 and 4-13 respectively. The radiation parameters extracted from Figs. 4-10 - 4-13 are tabulated in Table 4-8. It can be observed from Figs. 4-10 - 4-13 and Table 4-8 that values of PSLL \leq - 12.4 dB in $\Phi = 0^0$ plane and \leq - 13.3 dB in $\Phi = 90^0$ plane have been obtained for this array.



Fig. 4-8: Variations in best fitness value i.e. PSLL with number of iterations for synthesis of the proposed 4×4-element RSPA array.



Fig. 4-9: Antenna element distribution at the aperture of 4×4-element array operating at 1.5 GHz.

Table 4-4: Variation of inter-element spacing along X-axis normalized to wavelength for 4×4-element RSPA array obtained by proposed method.

0.76	0.785	0.65	0
0.855	0.8	0.845	0.16
0.8	0.79	0.845	0.15
0.805	0.735	0.65	0.16

Table 4-5: Variation of inter-element spacing along X-axis normalized to wavelengthfor 4×4-element RSPA array obtained by proposed method.

0.19	0.25	0.19	0
0.94	1.05	0.9	0.94
0.835	0.89	0.875	0.835
0.94	0.985	0.985	0.94



Fig. 4-10: Radiation pattern of the proposed 4×4-element RSPA array at 1.5 GHz in azimuth ($\phi = 0^0$) plane.



Fig. 4-11: Radiation pattern of the proposed 4×4-element RSPA array at 1.5 GHz in elevation ($\phi = 90^{\circ}$) plane.



Fig. 4-12: 3D radiation pattern of the proposed 4×4-element RSPA array operating at 1.5 GHz.



Fig. 4-13: Intensity plot of the proposed 4×4-element RSPA array operating at 1.5 GHz.

The numerical analysis of the proposed 8×8-element RSPA array operating at 3.3 GHz is now described. Fitness function illustrated in Eq. (4.7) is applied to suppress PSLL in $\Phi = 0^{0}$ and $\Phi = 90^{0}$ planes by utilizing the proposed MCB-GA optimization technique. The variation of best fitness value measured by PSLL as a function of iteration number is depicted in Fig. 4-14. It can be noticed from Fig. 4-14 that the optimal solution (Best PSLL) occurs after around 75 iterations. The best combination of

inter-element spacings along X-and Y-axes obtained through the proposed algorithm are depicted in Table 4-6 and 4-7 respectively and optimum distribution of antenna elements at the aperture of the proposed RSPA array is shown in Fig. 4-15. The numerically computed array patterns obtained through the proposed technique are depicted in Figs. 4-16 and 4-17 in $\Phi = 0^{0}$ and $\Phi = 90^{0}$ planes respectively. The 3D radiation pattern and Intensity plot are depicted in Figs. 4-18 and 4-19 respectively. Radiation parameters evoked from Figs. 4-16 - 4-19 are indexed in Table 4-8. It can be noticed clearly form Figs. 4-16 - 4-19 and Table 4-8 that the lowest possible values of PSLL achieved by the proposed MBC-GA technique are -18.27 dB and -19.3 dB in $\Phi = 0^{0}$ and $\Phi = 90^{0}$ planes respectively. This demonstrates that the proposed RSPA array is suitable for multi-functionality in shared aperture array environment.

The distribution of antenna elements at the aperture of 4×4- and 8×8-element multi-functional arrays operating at 1.5 GHz and 3.3 GHz is depicted in Fig. 4-20 and radiation patterns are shown in Fig. 4-21 and 4-23 in $\Phi = 0^{\circ}$ and $\Phi = 90^{\circ}$ planes respectively.



Fig. 4-14: Variation in best fitness value i.e. PSLL with number of iterations for synthesis of the proposed 8×8-element RSPA array.

0	0.99	0.72	0.65	0.51	0.65	0.72	0.99
0.51	0.64	0.54	0.51	0.51	0.54	0.64	0.51
0.69	0.62	0.51	0.51	0.51	0.51	0.62	0.69
0.68	0.62	0.53	0.51	0.51	0.53	0.62	0.68
0.68	0.62	0.53	0.51	0.51	0.53	0.62	0.68
0.69	0.62	0.51	0.51	0.51	0.51	0.62	0.69
0.94	0.64	0.54	0.51	0.51	0.54	0.64	0.94
0.51	0.72	0.65	0.51	0.51	0.65	0.72	0.51

Table 4-6: Variation of inter-element spacing along X-axis normalized to wavelength

 for 8×8-element RSPA array obtained by the proposed method.

Table 4-7: Variation of inter-element spacing along Y-axis normalized to wavelengthfor 8×8-element RSPA array obtained by the proposed method.

0	0.91	1	0.88	0.88	1	0.91	0.79
0.79	0.87	0.99	0.62	0.62	0.99	0.87	0.73
0.73	0.66	0.89	0.57	0.57	0.89	0.66	0.72
0.72	0.51	0.51	0.51	0.51	0.51	0.51	0.51
0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51
0.72	0.66	0.89	0.57	0.57	0.89	0.66	0.72
0.73	0.87	0.99	0.62	0.62	0.99	0.87	0.73
0.79	0.91	1	0.88	0.88	1	0.91	0.79



Fig. 4-15: Antenna elements distribution at the aperture of 8×8-element RSPA array operating at 3.3 GHz.



Fig. 4-16: Radiation pattern of the proposed 8×8-element RSPA array at 3.3 GHz in azimuth ($\phi = 0^0$) plane.



Fig. 4-17: Radiation pattern of the proposed 8×8-element RSPA array at 3.3 GHz in elevation ($\phi = 90^{\circ}$) plane.



Fig. 4-18: 3D radiation pattern of the proposed 8×8-element RSPA array operating at 3.3 GHz.



Fig. 4-19: Intensity plot of the proposed 8×8-element RSPA array operating at 3.3 GHz.



Fig. 4-20: Antenna elements distribution at the aperture of 4×4- and 8×8-elements multi-functional arrays operating at 1.5 GHz and 3.3 GHz.



Fig. 4-21: Radiation patterns of the proposed 4×4 - and 8×8 -elements multi-functional arrays operating at 1.5 GHz and 3.3 GHz in azimuth ($\phi = 0^0$) plane.



Fig. 4-22: Radiation patterns of the proposed 4×4 - and 8×8 -elements multi-functional arrays operating at 1.5 GHz and 3.3 GHz in elevation ($\phi = 90^{\circ}$) plane.

Table 4-8: Numerically evaluated PSLLs for the proposed 4×4- and 8×8-elementsRSPA arrays.

Parameters	RSPA array (1.5 GHz)	RSPA array (3.3 GHz)
PSLL (dB) in Azimuth ($\phi = 0^0$) plane	-12.4	-18.27
PSLL (dB) in Elevation ($\phi = 90^{\circ}$) plane	-13.3	-19.3

4.4 Summary and Conclusion

An MBC-GA based synthesis strategy for the design of MFA arrays consisting of differently sized radiating elements arranged in sparse configurations has been successfully presented. Initially, synthesis of RSPA array with reduced PSLL in both the planes has been successfully implemented. The performance of proposed method has been verified by numerically analysing an 8×16-element RSPA array. It can be observed from the presented results that significant amount of reduction in PSLL has occurred with uniform amplitude excitation. This authenticates the validity of the proposed method to synthesize RSPA arrays. Next, multi-functionality based on shared aperture concept has been demonstrated by two sub-arrays operating at 1.5 GHz and 3.3 GHz in the interleaved form at the aperture of single antenna system. Therefore, it can be concluded that a new solution for conceptualising multi-functional antenna array based on the concept of interleaving RSPA sub-arrays having radiating elements operating at different frequencies has been successfully achieved through this study.