Chapter 4

THEORETICAL MODELLING

4.1 Introduction

Theoretical research on solar stills is completed and generalized for use on single slope solar stills with any fabrication material and climate condition. Furthermore, this mathematical modelling incorporates the influence of quantum dot coating by altering the convective heat transfer coefficient from basin-to-basin water. The solar analytical solution is written in MATLAB.

4.2 Assumptions

The mathematical modelling of solar still need some assumption to compute the results on MATLAB. Basically,

1. A regular supply of brackish water keeps the basin water level at a constant height.
2. One-dimensional heat flow-based analysis is performed.
3. The fabrication of solar still is leaked proof to water and vapor.
4. Initially, the inner glass temperature and outer glass temperature are the same.
5. The thickness of the glass cover is assumed to be thin enough so that temperature gradient can be conveniently neglected.
6. It is assumed that all condensed water droplets reached to trough collector without falling back into the basin.
7. Thermophysical property of basin water remain.
8. Conductive heat loss from side walls is assumed to be negligible.

9. Air velocity is assumed to be constant throughout the experiment.

10. Temperature gradient along the thickness of solar still is negligible.

Nomenclature	
m	Mass (kg)
C	Specific heat capacity (kJ/kg.K)
T	Temperature (K)
t	Time (s)
α	Absorptivity
μ	Dynamic viscosity (Pa.s)
A	Area (m^2)
Q	Heat Transfer (W)
I	Intensity
h	Heat transfer coefficient (W/m^2K)
P	Pressure (Pa)
Nu	Nusselt No.
Gr	Grashof No.
Pr	Prandtl No.
Ra	Rayleigh No.
Subscript	

w	Water (basin water)
g	Glass
bs	Basin
con	Convection heat Transfer
rad	Radiation Heat Transfer
evap	Evaporation heat transfer
cd	Conduction heat transfer
sky	Sky
amb	Ambient
w-g	water to glass
g-sky	glass to sky
g-amb	glass to ambient
bs-w	basin surface to basin water
bs-amb	basin to ambient
bs-in	Basin to insulation

4.3 Modelling

Considering different elements to analyze the overall solar still, energy balance is required, like Condenser surface, basin water, and basin absorber. Quantum dot material affects the inner basin surface, which absorbs more heat than without QD-dye coating. It happens due to the high solar absorptance property of QD that is observed by the rate of increase of basin water temperature. It is also seen that the convective heat transfer coefficient from the basin surface to the basin water is affected by a change in the concentration of QDs. To understand the heat transfers associated with SS thermal network diagram is drawn as shown in Figure 4.1. The nodes are defined as temperature points, and resistances are defined as the heat transfer coefficients.

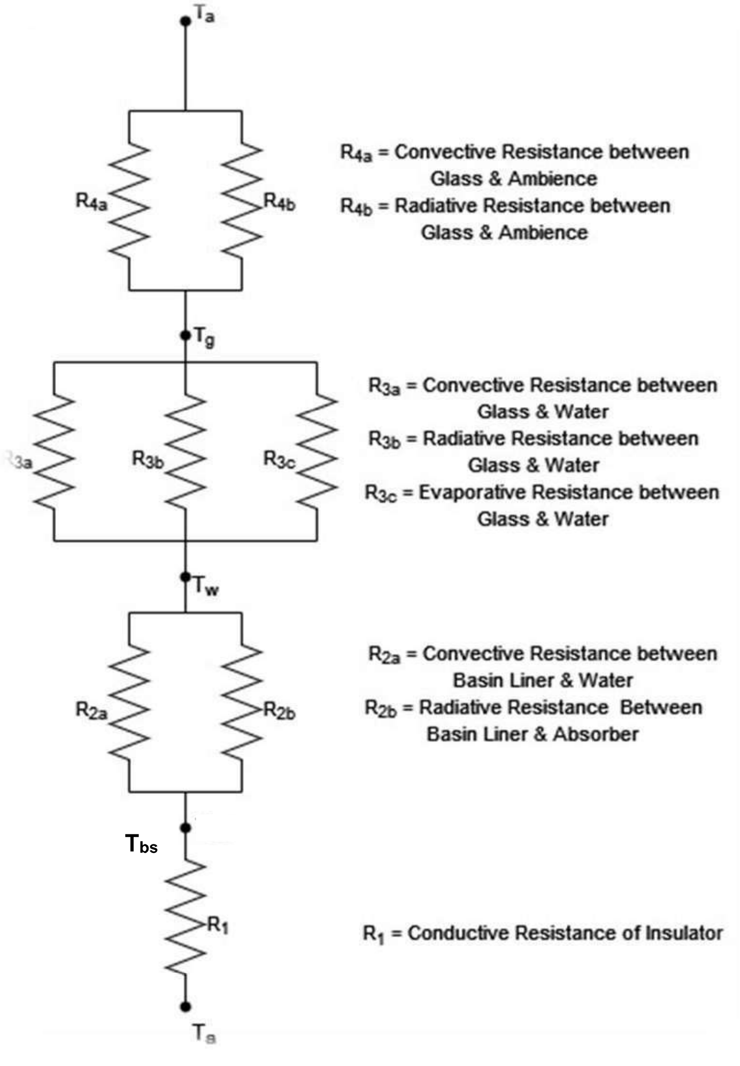
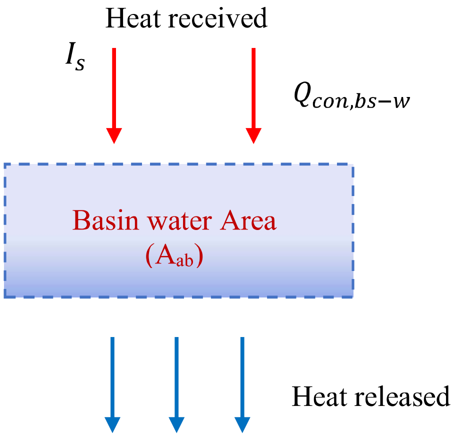


Figure 4. 1 Thermal network diagram of Solar still with QD

4.3.1 Energy balance on Basin water

Here, the rate of change of internal energy in the basin water will be equal to the energy received in the form of solar intensity (I_1) on basin water area (A_{ab}) convective heat transfer from the basin surface and energy liberated from the glass in the form of convection, radiation, and evaporation.

$$m_w C_w \frac{dT_w}{dt} = \tau_g \alpha_w I_s(t) A_{ab} + Q_{con,bs-w} - \left(Q_{con,w-g} + Q_{rad,w-g} + Q_{evap,w-g}\right) \dots\dots\dots\dots..(1)$$



$$Q_{con,w-g} = h_{con,w-g}(T_w - T_g)$$

$$Q_{rad,w-g} = h_{rad,w-g}(T_w - T_g)$$

$$Q_{evap,w-g} = h_{evap,w-g}(T_w - T_g)$$

The heat transfer coefficients have to calculate to find out before to calculate total heat transfer by Eq.1. The radiative heat transfer coefficient is calculated by Duffie and Beckman

$$h_{rad,w-g} = \varepsilon_{eff} \times \sigma \times \left({T_w}^2 + {T_g}^2\right) \times \left(T_w + T_g\right) \dots\dots\dots\dots\dots\dots(2)$$

Where, $$\varepsilon_{eff} = \frac{1}{\varepsilon_w} + \frac{1}{\varepsilon_g} - 1 \ldots\ldots\ldots\ldots\ldots\ldots.. (3)$$

$$h_{con,w-g} = 0.884 \times \left((T_w - T_g) + \left(\frac{(P_w - P_g) \times T_w}{268900 - P_w}\right)\right)^{\frac{1}{3}} \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Here, to find the partial pressure of water and glass at corresponding saturation temperature Dunkle's equation [121] is used as follow,

$$P_w = e^{25.317 - \left(\frac{5144}{T_w}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$P_g = e^{25.317 - \left(\frac{5144}{T_g}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (6)$$

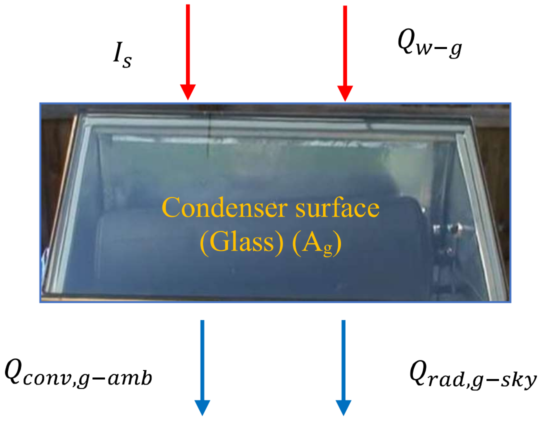
$$h_{evap,w-g} = 0.016276 \times h_{con,w-g} \times \frac{P_w - P_g}{T_w - T_g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$\boxed{h_{tot,w-g} = h_{rad,w-g} + h_{con,w-g} + h_{evap,w-g}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

4.3.2 Energy balance on Glass (Condenser)

Here, rate of change of internal energy the condenser surface (glass) will be equal to energy received in the form of solar intensity plus heat energy transferred from water to glass and energy released to ambient in the form of radiation and convention.

$$m_g C_g \frac{dT_g}{dt} = I_s(t) A_g + \left(Q_{con,w-g} + Q_{rad,w-g} + Q_{evap,w-g}\right)$$
$$- \left(Q_{rad,g-sky} + Q_{conv,g-amb}\right) \qquad \ldots\ldots\ldots.. (9)$$



Modelling of solar still in transient condition can be perform as,

$$T_{sky} = 0.0052 \times {T_{amb}}^{1.5} \dots \dots \dots \dots \dots \dots \dots (10)$$

Where, heat exchange from and to the glass surface can be considered as,

$$Q_{conv,g-amb} = h_{con,g-a}\left(T_g - T_{amb}\right) \dots \dots \dots \dots (11)$$

$$Q_{rad,g-sky} = h_{rad,g-sky}\left(T_g - T_{sky}\right) \dots \dots \dots \dots (12)$$

The heat transfer coefficients are calculated from following equations,

$$h_{rad,g-sky} = \varepsilon_g \times \sigma \times \left({T_g}^2 + {T_{sky}}^2\right) \times \left(T_g + T_{sky}\right) \dots \dots \dots (13)$$

The convective heat transfer from glass to ambient is a function of velocity.

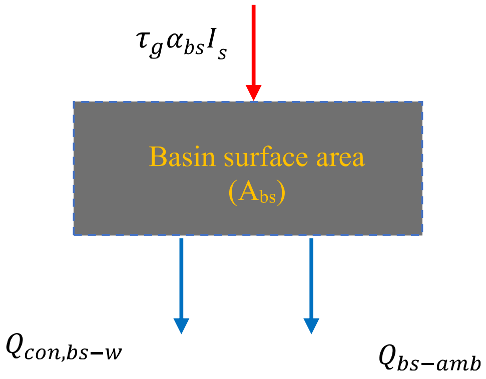
$$h_{con,g-amb} = 2.8 + 3 \times V \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (14)$$

$$\boxed{\boldsymbol{h_{tot,g-amb} = h_{con,g-amb} + h_{rad,g-sky}}}$$

4.3.3 Energy balance on Basin liner

The basin liner is the total basin surface area (A_{bs}) receiving solar energy (I_2). This surface has absorptivity (α_{bs}), heat capacity (C_{bs}), and mass (m_{bs}). The basin surface heat capacity is lower than basin water to primarily facilitate heat transfer from the surface to water via convective mode.

$$m_{bs}C_{bs}\frac{dT_{bs}}{dt} = \tau_g \boldsymbol{\alpha_{bs} I_s(t) A_{bs}} - \left(Q_{tot,bs-w} + Q_{conv,bs-amb} + Q_{cond,bs-ins}\right) \ldots\ldots\ldots(15)$$



The convective heat transfer coefficient from basin surface to basin water can be expressed as,

$$Nu = \frac{h_{conv,bs-w}L}{K_{bs}} = C\,(Gr.Pr)^n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

$$\boldsymbol{h_{conv,bs-w}} = \frac{C}{L} \times K_{bs} \times (Gr \times \mathrm{Pr})^n \ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

Here, ‘C’ and ‘n’ are assumed to be depends upon Gr. [122].

a) For $Gr < 10^3$, $C = 1$ and $n = 0$; (magnitude of convection is neglected)

b) For $10^5 < Gr < 2\times10^7$, $C = 0.54$ and $n = 1/4$; (Convection in laminar phase)

c) For $2\times10^7 < Gr < 3\times10^7$, $C = 0.14$ and $n = 1/3$; (Convection in Turbulent phase)

Heat loss from boundary wall insulation,

$$Q_{con,bs-ins} = K_{ins}A_{ins}\frac{dT}{dx} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(18)$$

4.3.4 Heat balance on Insulation

$$m_{in}C_{in}\frac{dT_{in}}{dt} = \left(Q_{cond,bs-in} - Q_{loss}\right) \ldots\ldots\ldots\ldots\ldots..(19)$$

Here, $$Q_{loss} = U_i(T_{ins} - T_{amb}) \ldots\ldots\ldots\ldots\ldots\ldots(20)$$

In this experiment Air velocity throughout the day was varying so at average velocity of 5 m/s different parameters are calculated.

The $h_{tot,bs-amb}$ represents total heat transfer coefficient between basin and air through mode of conduction via basin and insulation that is calculated by following expression,

$$\boxed{\boldsymbol{h_{tot,bs} = \frac{K_{bs}}{L_{bs}} + \frac{K_{in}\times h_{air}}{L_{in}\times h_{air} + K_{in}}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

Here, initially we assume water temperature is more than basin surface temperature. From water to basin heat transfer coefficient depends on the Grashof No. (Gr)

$$Gr = g \times \beta_w \times (T_w - T_b) \times \frac{{L_c}^3}{{v_w}^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(22)$$

$$L_c = \frac{Area}{Perimeter} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(23)$$

By putting all constant values, we get,

$$Gr = 10 \times 0.00041 \times (T_w - T_b) \times \frac{\frac{1}{64}}{0.00000089^2} \dots\dots\dots\dots (24)$$

$$Pr = C_w \times \frac{\mu_w}{K_w} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (25)$$

On putting all constant values, we get,

$$Pr = 4.18 \times \frac{0.00089}{0.62}$$

$$Ra_L = Pr \times Gr \dots\dots\dots\dots\dots\dots\dots\dots (26)$$

Here, Ra_L lies between $10^4\ to\ 10^7$ so, **value of C=0.54 and n=0.25**.

Conduction would be the main mode of heat transport when Ra $\leq 10^3$. At high Rayleigh number (Ra), convection currents dominate. The buoyant force increases and the viscous force decreases by raising the Rayleigh number.

It is assumed that inner glass temperature and outer glass temperature is same and heat absorption by the glass cover is negligible so heat come in and goes out from glass cover is equal.

$$h_{tot,w-g}\left(T_w - T_g\right) = h_{tot,g-amb}\left(T_g - T_{amb}\right) \dots\dots\dots\dots\dots\dots\dots\dots. (27)$$

$$\boldsymbol{T_g} = \frac{h_{tot,w-g} \times T_w + h_{tot,g-amb} \times T_{amb}}{h_{tot,w-g} + h_{tot,g-amb}} \dots\dots\dots\dots\dots\dots\dots\dots\dots (28)$$

Heat absorption by basin liner will be equal to the absorptivity (α_{bs}) times of solar heat intensity (I_s) that will play a crucial role to improve the yield of solar still. At particular concentration of QD dot on the surface, absorptivity would remain constant. Although with increase in the concentration, absorptivity increases results in higher temperature difference $(T_{bs} - T_w)$.

$$\tau_g \boldsymbol{\alpha_{bs}} \times I_s(t) = h_{tot,bs-w}(T_{bs} - T_w) + h_{conv,bs-amb}(T_{bs} - T_{amb}) \ldots\ldots\ldots (29)$$

From above equation, T_{bs} is expressed in term of T_w and T_{amb}.

$$\boldsymbol{T_{bs}} = \frac{h_{tot,bs-w} \times T_w + h_{conv,bs-amb} \times T_{amb} + \tau_g \boldsymbol{\alpha_{bs}} \times I_s(t)}{h_{tot,bs-w} + h_{conv,bs-amb}} \ldots\ldots\ldots. (30)$$

By putting these two Eqs. (28) and (30) in Eq. (1),

$$\tau_g \alpha_w \times I_s(t) + h_{tot,bs-w} \times (T_b - T_w) = m_w C_w \frac{dT_w}{dt} + h_{tot,w-g} \times (T_w - T_g) \ldots\ldots.. (31)$$

As we know heat equation in 1-D is written in the form of

$$\frac{dTw}{dt} + aT_w = C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

After putting value of T_{bs}, T_g in term of T_{amb}, T_w in energy balance equation of basin water after rearranging and comparing it with heat transfer equation in 1-D, we get expression of **a** and **C** as,

$$\boldsymbol{a} = \left(\frac{h_{tot,bs-w} \times h_{conv,bs-amb}}{h_{tot,bs-w} + h_{conv,bs-amb}} + \frac{h_{tot,w-g} \times h_{tot,g-amb}}{h_{tot,w-g} + h_{tot,g-amb}}\right)\left(\frac{1}{m_w C_w}\right) \ldots\ldots\ldots\ldots (33)$$

$$\boldsymbol{C} = \left(\frac{1}{m_w C_w}\right)\left(\left(\frac{h_{tot,bs-w} \times \tau_g \alpha_{bs} \times I_s(t) + h_{conv,bs-amb} \times h_{tot,bs-w} \times T_{amb}}{h_{conv,bs-amb} + h_{tot,bs-w}}\right) + \left(\frac{h_{tot,g-amb} \times h_{tot,w-g} \times T_{amb}}{h_{conv,bs-amb} + h_{tot,bs-w}}\right)\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (34)$$

So, **‘a’** and **‘C’** are constant value for first step hour and the solution of Eq. (32) is,

$$T_w^{i+1} = \frac{\left(T_w^i e^{-3600a} + C \times (1 - e^{-3600\times a})\right)}{a} \ldots\ldots\ldots\ldots\ldots\ldots.(35)$$

$$T_g^{i+1} = \frac{\left(h_{gw} \times T_w^{i+1} + h_{ga} \times T_a^{i+1}\right)}{\left(h_{gw} + h_{ga}\right)} \ldots\ldots\ldots\ldots\ldots\ldots..(36)$$

$$T_{bs}^{i+1} = \frac{\tau_g \alpha_{bs} I_s(t)_{avg}^i(t) + h_{bw} \times T_w^i + h_{bw} \times T_w^i}{h_{bw} + h_{ba}} \ldots\ldots\ldots\ldots\ldots\ldots(37)$$

Mass of water evaporated is given by below equation ,

$$\boldsymbol{m_{evaporation}} = \boldsymbol{h_{evap,w-g}} \times \left(\boldsymbol{T_w} - \boldsymbol{T_g}\right) \times \boldsymbol{A_{ab}} \times \frac{\mathbf{3600}}{\boldsymbol{Latent\ heat}} \ldots\ldots\ldots\ldots\ldots(38)$$

$$h_{evap,w-g} = 16.273 \times 10^{-3} h_{conv,w-g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(38.A)$$

To calculate the different heat transfer coefficients some constant values have to use which are taken from different literatures and books [123,124].

Constant Values

g=10	Gravity constant (m/s^2)
K_b=0.8	Thermal conductivity of basin (kW/mK)
C_w=4.18	Specific heat constant of water (kJ/kgK)
K_{in}=0.03	Thermal conductivity glass wool (W/mK)
L_{in}=0.07	Thickness of insulation material (m)
L_b=0.01	Thickness of basin (m)

$h_{air}=13.75$	Average free heat transfer coefficient of air (W/m^{2}K)
$V_{avg}=4$	Avg. velocity of air (m/s)
$A_{bs}=1$	Area of basin (m^2)
$K_w=0.60$	Thermal conductivity of water (W/mK)
$\alpha_w=0.85$	Absorptivity of water
$\alpha_{bs}=0.68$	Absorptivity of basin (FRP)
$\alpha_{bs}=0.99$	Absorptivity of basin (with QD)
$\nu_w=0.599\times10^{-6}$	Kinematic viscosity of water (m^2/s) @ 45 ºC
$\mu_w=5.99\times10^{-4}$	Dynamic viscosity of water (N.s/m^2) @ 45 ºC
$\beta_w=2.95\times10^{-4}$	Volumetric expansion coefficient of water (K^{-1})
$\varepsilon_w=0.9$	Emissivity of water
$\varepsilon_g=0.9$	Emissivity of glass
$K_{bs}=0.577$	Thermal conductivity of basin (FRP) (W/mK)
$K_{bs}=0.577$	Thermal conductivity of basin (FRP with QD) (W/mK)
$C_{bs}=1.5$	Specific heat capacity of FRP (kJ/kgK)
$\sigma=5.67\times10^{-8}$	Stephen-Boltzmann Constant (Wm^{-2}K^{-1})

4.3.5 Block diagram of algoritham

Various heat transfer coefficients must be calculated through the above formulations to find the yield from a solar still. After several iterations, the following algorithm can calculate the heat transfer coefficients, as shown in Figure 4.1.

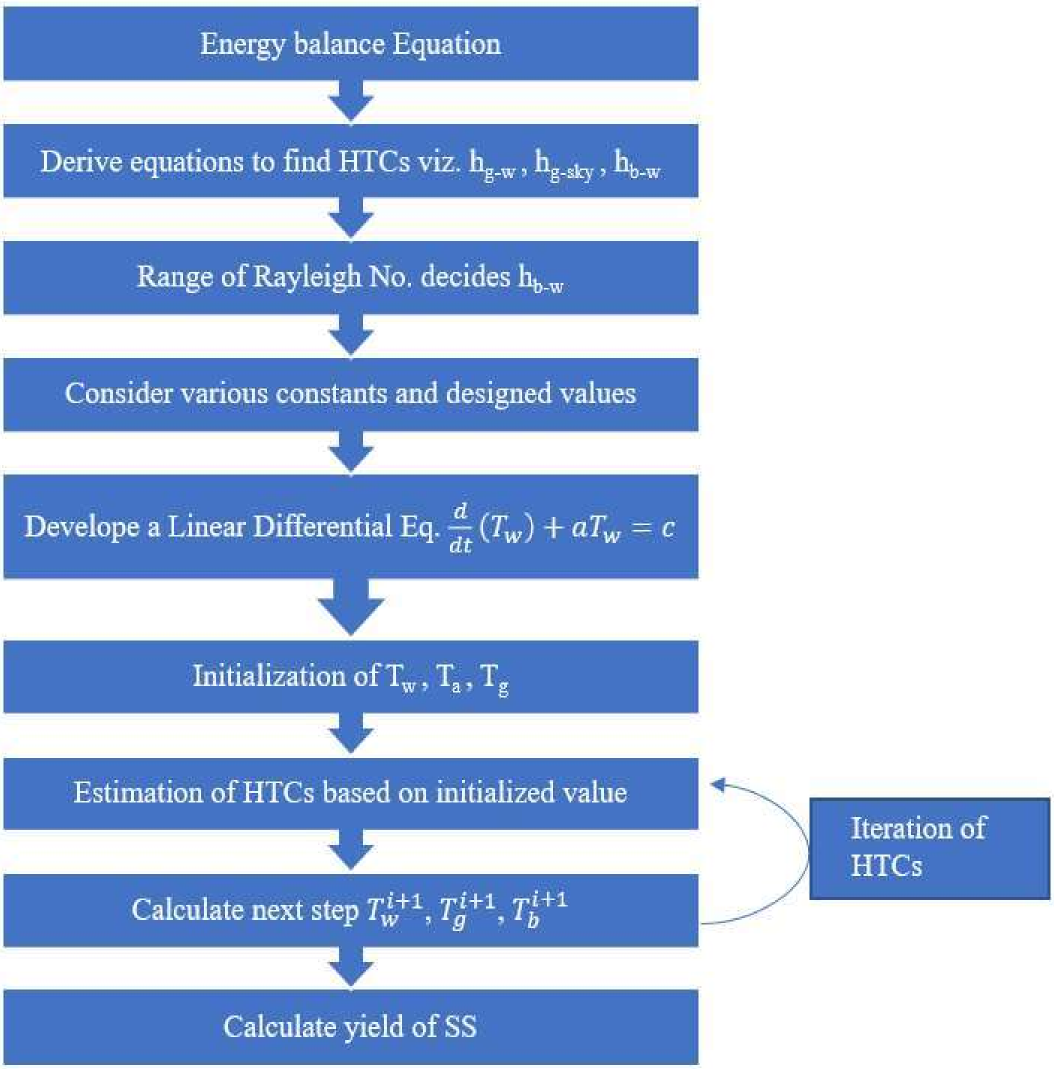


Figure 4. 2 Mathematical algorithm to find the yield and HTCs of modified solar stills