# CHAPTER 2

# MATERNAL HEALTHCARE FACILITY PLANNING WITH FULL AVAILABILITY WITHIN REACH

## 2.1 Introduction

This chapter discusses the planning problem of locating different types of maternal healthcare facilities by addressing the issues related to 'availability' and 'accessibility' in India. A mixed-integer linear programming (MILP) model of this problem has been developed. The model aims to minimize the total cost of establishing various kinds of maternal healthcare facilities and the travel cost of MTBs to these facilities, including that for referrals. The developed facility location model is hierarchical (i.e., a system of different types of interacting facilities) and successively inclusive (i.e., a higher level will also provide all lower-level services). The model has been further improved in terms of its computational requirements by including a set of additional valid inequalities. An administrative unit (such as Tehsil) in India may have thousands of villages. Thus, getting a good quality solution in a reasonable time will be an arduous task as the proposed model will involve too many variables and constraints in solving the location and allocation problem simultaneously. The sequential approach proposed in this chapter helps to cut down the computational requirements sizeably without practically compromising the solution quality. The effectiveness and efficiency aspects of the proposed solution approaches are discussed and described through the computational experiments carried out using several problem instances with wide variations. The results of the sensitivity analyses with respect to coverage distance, capacity, referral proportion and fixed cost and also carried out to provide important and practical insights related to the mix of the facilities to be established.

The chapter is organized as follows. In Section 2.2, the essential features of the considered maternal healthcare problem are presented. The mathematical programming formulation of the problem is provided in Section 2.3. An illustrative example problem is illustrated in Section 2.4. Various approaches to solve the proposed mathematical model are presented in Section 2.5. The results of computational experiments are presented in Section 2.6. Section 2.7 investigates the effect of various problem parameters on the mix of the facilities to be established. Finally, Section 2.8 concludes the chapter.

## 2.2 The Problem

In India, strategic planning of healthcare facilities is traditionally carried out region-wise. These regions are marked administratively. In a region, there will be certain locations, with each having some MTBs. Since MTBs will be at different phases of their pregnancy, they would require different types of services. During the first and second trimesters of the pregnancy, the essential requirement is of primary care as routine check-ups, vaccination, and diagnostic examinations. An MTB approaching the due date of the delivery would require a routine or cesarean delivery. MTBs need neonatal assistance whenever there are complications during pregnancy or when the delivery is complicated. Besides, some MTBs may require an unplanned cesarean delivery on an emergency basis and subsequent neonatal assistance. If such service cannot be provided at the allocated facility, they will be referred to a higher level of facilities capable of providing such assistance. It is so because all the healthcare facilities are not identical in terms of the services provided by them, particularly in the Indian context. In this way, the services related to maternal care are broadly categorized as service types 1, 2, and 3 for primary care, regular and scheduled cesarean deliveries, and complicated deliveries with neonatal assistance, respectively. Depending upon the requirement and time, pregnant women require any of three types of services. It is assumed that on any day proportion of the requirement of these three types of services remains the same. Healthcare facilities are categorized into different types according to the level of service offered. A healthcare facility providing only type 1 service is referred to as a Sub-Center (SC), while those providing type 1 and type 2 services as Primary Healthcare Center (PHC), and all the three (type 1, type 2, and type 3) as Community Health Center (CHC). These are also represented by respective indices as I, II, and III in the mathematical formulation. While establishing these centres, it is desired that these be within a reasonable reach of each MTBs, not asking them to travel beyond a prespecified coverage distance. This consideration is important in limiting the maximum distance travelled by a mother-to-be. The centres to be established of a type are kept identical for administrative convenience. Their service capacity is taken to be limited and fixed. In case the coverage distance and the capacity become bottlenecks, it will necessitate establishing additional centres. It is assumed that the capacity for each service type for each facility type at a location is at least equal to the local demand of MTBs at that location. Allocation of MTBs to a facility type has to be made such that it does not exceed the capacity available therein for the required service type.

In case of an emergency or complication at a lower-level centre, the MTBs will be referred to a higher-level facility. For example, MTBs getting service of type 1 in an SC can be referred to a PHC or a CHC to avail type 2 or type 3 services as the need may be. Referrals can be handled at the same centre for higher-level services subject to the availability and capacity of the service type at that centre. The notion of the coverage distance is also applicable for referral cases. The maximum coverage distance for referral visits need not be the same as that for a non-referral visit.

In India, the facilities are planned at the government land, and thus the cost is to be incurred only for the superstructure and the other amenities. The facility of a type is going to be the same irrespective of the location. Since payment for construction is paid by the government according to its rate schedule, the establishment cost for a facility would not change with the location. Thus, the fixed cost incurred for establishing a facility at a location will depend upon its type and not the location. Because each type of facility has a distinct type of infrastructure and amenities, the cost of the establishment will vary. Further, A travel cost will be incurred by an MTB at location *i* to visit a facility at location *j*. It will generally be in proportion to the distance between locations *i* and *j*. Travel cost is also incurred during the referral visits. The objective considered is to minimize the total cost of establishing and the total cost incurred on visiting these facilities, including referrals. As the facilities (and the services provided by them) are nested, the problem is referred to as a Hierarchical Capacitated Facility Location-Allocation (HCFL) problem in subsequent elaborations and discussions.

The framework of the maternal healthcare planning undertaken in the present work assumes the establishment of separate and dedicated facilities for MTBs as the governments are focusing more on women health (Janani Suraksha Yojana, 2005; National Rural Health Mission, 2005; Pradhan Mantri Surakshit Matritva Abhiyan, 2016) in an unprecedented manner. It is assumed that the existing maternal healthcare facility, whether independent or part of general healthcare, shall be provided to hardpressed and infrastructure-deprived medical services.

## 2.3 The Mathematical Model

The following notations have been used in the problem formulation.

**Sets and indices**

$I$ : set of locations of MTBs, $I = \{1, 2, ..., m\}$, indexed by $i$

$J$ : set of locations of potential facilities, $J = I$, indexed by $j$, $k$ and $n$

$L$ : set of service types offered, $L = \{1, 2, 3\}$, indexed by $l$, $l'$ and $l''$

$F$ : set of types of facility, $F = \{I, II, III\}$, indexed by $f$ and $u$

**Parameters**

$d_1$ : a limit on the maximum distance to be covered by an MTB during a non-referral visit

$d_2$ : a limit on the maximum distance to be covered by an MTB during the referral visit

$d_{ij}$ : distance between locations $i \in I$ and $j \in J$

$d_{jk}$ : distance between facility locations $j \in J$ and $k \in J$

$C_{ij}$ : travel cost incurred by an MTB for visiting the facility at a location $j \in J$ from its current location $i \in I$

$C_{jk}$ : travel cost incurred by an MTB for referral visit from current facility location $j \in J$ to a referral facility at location $k \in J$

$F_j^f$ : fixed cost on establishing a facility of type $f \in F$ at location $j \in J$

$Q^{lf}$ : capacity of service type $l \in L$ available with facility type $f \in F$ ; $Q^{2I} = Q^{3I} = Q^{3II} = 0$

$W_i^l$ : number of MTBs at a location $i \in I$ requiring service type $l \in L$

$\theta^{ll'}$ : proportion of referrals for service type $l' \in L$ from service type $l \in L$, where $l' > l$

$$\alpha_{ij} = \begin{cases} 1, & \text{if a facility for non-referral visit at a location } j \in J \text{ is within the coverage distance of an MTB} \\ & \text{at a location } i \in I \text{ (i.e., } d_{ij} \le d_1\text{),} \\ 0, & \text{otherwise} \end{cases}$$

$$\beta_{jk} = \begin{cases} 1, & \text{if a referral facility at a location } k \in J \text{ is within the coverage distance of a lower level facility} \\ & \text{at location } j \in J \text{ (i.e., } d_{jk} \le d_2\text{),} \\ 0, & \text{otherwise} \end{cases}$$

**Decision variables**

$x_{ij}^{l}$ : number of MTBs from location $i \in I$ allocated to a facility at location $j \in J$ to receive service type $l \in L$

$x_{jk}^{ll'}$ : number of MTBs allocated under referral for availing a higher service type $l' \in L$ located at $k \in J$ who were availing service type $l \in L$ at a facility in location $j \in J$ $(l' > l)$

$$y_j^f = \begin{cases} 1, & \text{if a facility of type } f \in F \text{ is located at } j \in J, \\ 0, & \text{otherwise} \end{cases}$$

The mathematical model of the Hierarchical Capacitated Facility Location-Allocation (HCFL) problem is presented hereunder using the above notations.

**Minimize** $\sum_{j \in J} \sum_{f \in F} F_j^f y_j^f + \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} c_{ij} x_{ij}^l + \sum_{k \in J} \sum_{j \in J} \sum_{l \in L} \sum_{l \in L} c_{jk} x_{jk}^{ll'}$ (2.1)

**Constraints**

$$\sum_{j \in J} x_{ij}^l = W_i^l, \qquad \forall i \in I, l \in L \tag{2.2}$$

$$\sum_{k \in J} x_{jk}^{ll'} = \theta^{ll'} \left\{ \sum_{i \in I} x_{ij}^l + \sum_{l'' \in L} \sum_{n \in J} x_{nj}^{l''l} \right\}, \qquad \forall j \in J, l \in L, l', l'' \in L, \text{where } l'' < l < l' \tag{2.3}$$

$$\sum_{i \in I} x_{ij}^l + \sum_{k \in J} \sum_{l' \in L} x_{kj}^{ll'} \le \sum_{f \in F} Q^{lf} y_j^f, \qquad \forall j \in J, l \in L \tag{2.4}$$

$$\sum_{f \in F} y_j^f \le 1, \qquad \forall j \in J \tag{2.5}$$

$$x_{ij}^l \le \sum_{f \in F} \alpha_{ij} y_j^f Q^{lf}, \qquad \forall i \in I, j \in J, l \in L \tag{2.6}$$

$$x_{jk}^{ll'} \le \sum_{f \in F} \beta_{jk} y_k^f Q^{l'f}, \qquad \forall\, j \in J, k \in J, l \in L, l' \in L \tag{2.7}$$

$$x_{ij}^{l}, x_{jk}^{ll'} \ge 0, \qquad \forall\, i \in I, j \in J, l \in L, l' \in L \tag{2.8}$$

$$y_j^f \in \{0,1\}, \qquad \forall\, j \in J, f \in F. \tag{2.9}$$

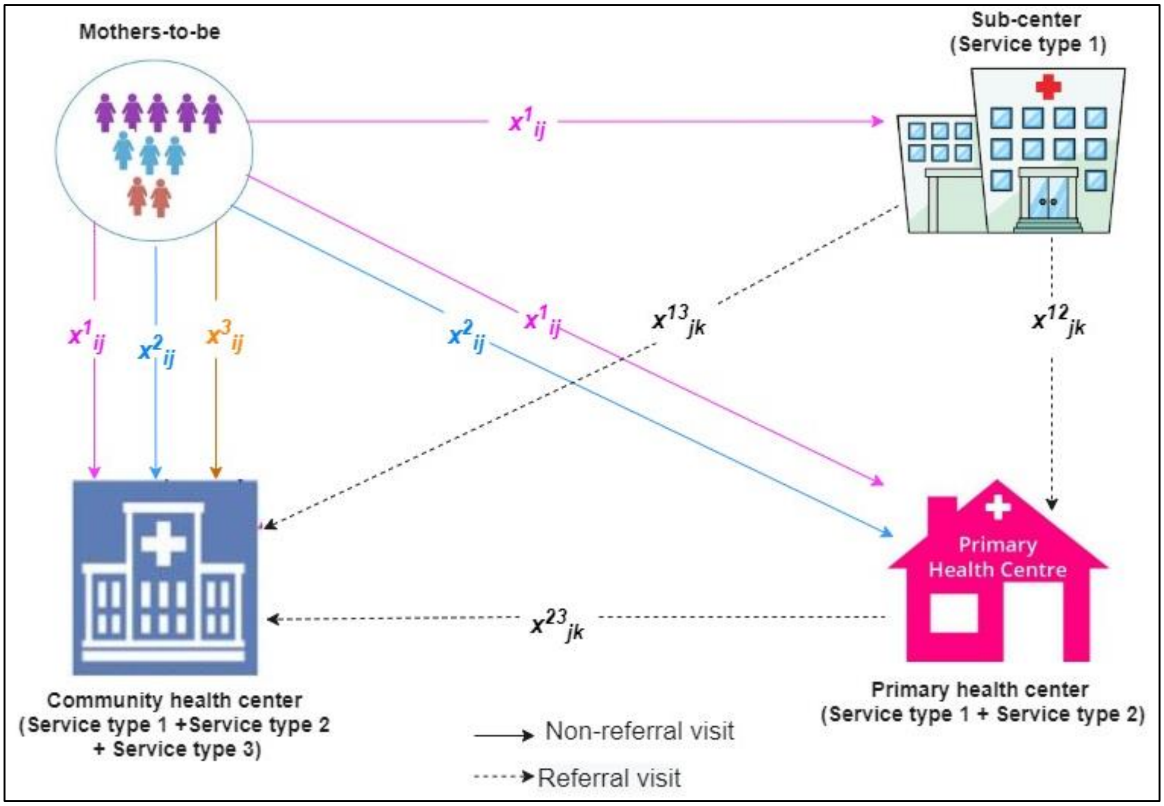



**Figure 2.1: Allocation of Mothers-to-be in a different types of facilities**

The mathematical model can be easily understood with the help of Figure 2.1, that shows a part of the regional flows of MTBs to various facility types. Three mathematical expressions in the objective function (2.1) respectively represent the fixed cost of establishing the healthcare facilities, the cost of travel for visiting a facility, and the cost of travel upon referral. The formulation takes care of the stated assumptions related to the cost for the establishment of facilities and the service demand worked out appropriately on a daily basis. Constraint (2.2) ensures complete fulfilment of the demand of various service types of all the MTBs. Constraint

(2.3) determines the amount of referral cases from a lower-level facility to a higher-level facility. Constraint (2.4) ensures that the total allocation of MTBs, including referrals, should not be more than the available capacity of the required service type at the facility type. Constraint (2.5) ensures that only one type of maternal healthcare facility can be established at a location. Constraint (2.6) states that the MTB at location $i \in I$ can be assigned to a facility at location $j \in J$ if the required service type is available at the facility within the coverage distance. Constraint (2.7) states that the MTBs at facility $j \in J$ can be referred for a higher level of service type to a facility at location $k \in J$ if the facility is located within the coverage distance from location *i*. Constraints (2.8) and (2.9) define the nature of decision variables.

## 2.4 An Illustrative Example

For illustration purposes, a small region having 18 locations (Figure 2.2) is taken. Each location has a number of MTBs requiring different types of services, categorized into three types: MTB_1, MTB_2, and MTB_3, requiring services of types 1, 2, and 3, respectively. The population of each type of MTBs has been shown in Table 2.1. According to the type of service required, the services available in healthcare facilities have also named as Service type 1, Service type 2 and Service type 3. The healthcare facility, which has all the three type of services is being referred as Community Health Center (CHC); Primary Healthcare Center (PHC) provide service type 1 and service type 2; and Sub Center (SC) provides only service type 1. The capacity of each service type in each healthcare facility is fixed, as shown in Table 2.2. There is a provision for a referral from the lower level facility to a higher level facility in case of unavailability of the required referral service type. The referral proportion is shown in Table 2.2. The cost of travelling between locations is considered directly proportional to the distance travelled. The distance between the locations is shown in Table 2.3. The maximum coverage distance between MTB location to a healthcare facility is taken as 5 km, and the same between a healthcare facility to a healthcare facility on referrals is taken as 7 km. To better understand this concept, one should refer Figure 2.2. In this Figure, it is assumed that nodes 2, 3, 8, 9 and 10 are within a distance of 5 km from node 1.

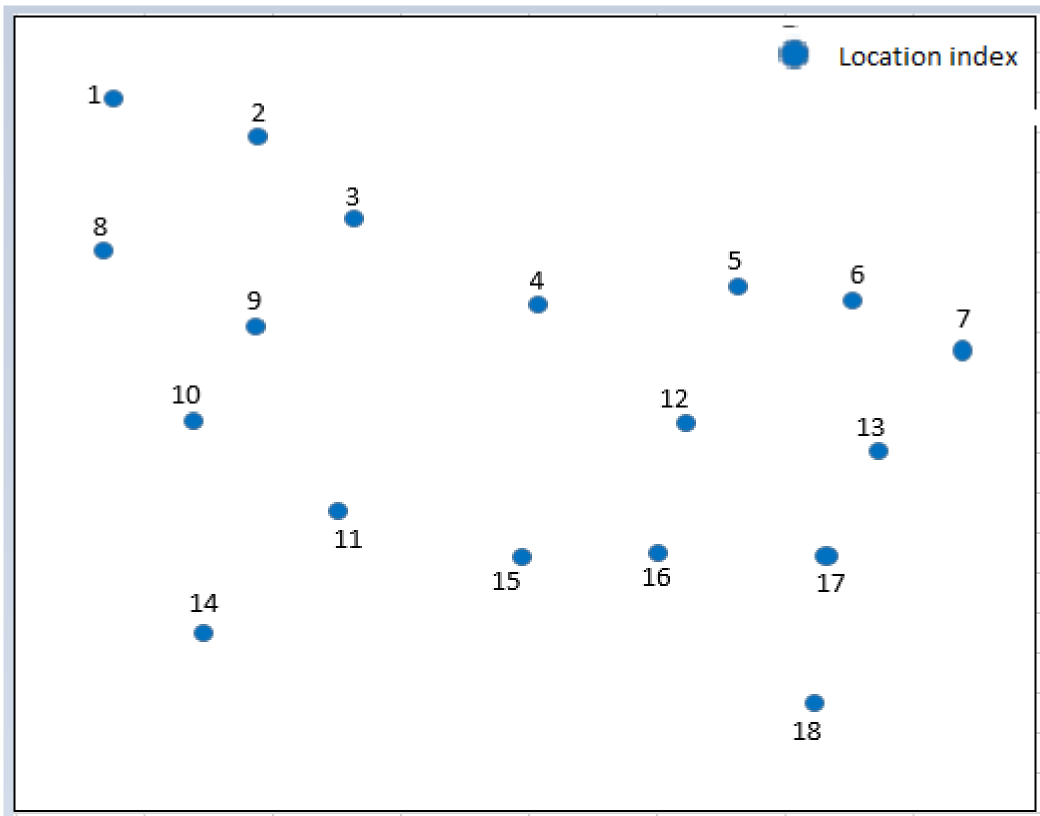



**Figure 2.2 : Locations of mothers-to-be**

In addition to these nodes, nodes 11 and 14 can also be included if the coverage distance limit is raised to 7 km. Rest other nodes are assumed to be far and beyond the coverage distance. Under this circumstance, MTB at location 1 can only directly visit the facilities available at locations 1, 2, 3, 8, 9 and 10. It implies that there should at least one facility should be available within a 5 km radius of location 1. Assuming that there is a healthcare facility available at location 1 (facility type I or II), then there must be at least one high level facility available within a 7 km distance, i.e., at either of the locations 2, 3, 5, 8, 9, 10, 11 and 14. The fixed cost of opening the facility type SC, PHC, and CHC is taken as Rs.100000, Rs.500000, and Rs.1000000, respectively.

**Table 2.1: Number of MTBs at various locations**

| Locations | MTB_1 | MTB_2 | MTB_3 | Locations | MTB_1 | MTB_2 | MTB_3 |
|---|---|---|---|---|---|---|---|
| 1 | 347 | 139 | 7 | 10 | 442 | 177 | 9 |
| 2 | 478 | 191 | 10 | 11 | 459 | 184 | 10 |
| 3 | 445 | 178 | 9 | 12 | 248 | 99 | 5 |
| 4 | 459 | 184 | 10 | 13 | 462 | 185 | 10 |
| 5 | 405 | 162 | 9 | 14 | 200 | 80 | 4 |
| 6 | 247 | 99 | 5 | 15 | 100 | 40 | 2 |
| 7 | 445 | 178 | 9 | 16 | 596 | 239 | 12 |
| 8 | 131 | 53 | 3 | 17 | 368 | 148 | 8 |
| 9 | 368 | 147 | 8 | 18 | 250 | 100 | 5 |

**Table 2.2: Referral proportion and assumed capacity for the illustrative example**

| | Referral Proportion | | | | Capacity | | |
|---|---|---|---|---|---|---|---|
| | Service type | | | | Facility type | | |
| | **1** | **2** | **3** | | **SC** | **PHC** | **CHC** |
| **1** | 0 | 0.1 | 0.03 | **1** | 500 | 800 | 1,000 |
| **2** | 0 | 0 | 0.05 | **2** | 0 | 500 | 700 |
| **3** | 0 | 0 | 0 | **3** | 0 | 0 | 300 |

**Table 2.3: Distance Matrix (Distances in km)**

| From \ To | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 2.1 | 3.0 | 7.1 | 6.8 | 9.1 | 9.8 | 1.3 | 2.9 | 3.9 | 6.5 | 8.3 | 9.8 | 6.3 | 8.4 | 8.5 | 9.7 | 9.4 |
| **2** | 2.1 | 0 | 1.0 | 3.5 | 4.8 | 5.6 | 6.2 | 1.2 | 1.4 | 2.3 | 3.7 | 4.8 | 6.2 | 4.6 | 5.8 | 5.5 | 6.7 | 6.3 |
| **3** | 3.0 | 1.0 | 0 | 2.5 | 3.8 | 4.6 | 52 | 2.2 | 1.7 | 2.2 | 2.7 | 3.8 | 5.2 | 4.1 | 4.7 | 4.5 | 5.7 | 5.4 |
| **4** | 7.1 | 3.5 | 2.5 | 0 | 1.9 | 2.1 | 27 | 5.9 | 4.4 | 3.4 | 1.0 | 1.3 | 2.7 | 3.2 | 2.4 | 2.0 | 3.2 | 2.9 |
| **5** | 6.8 | 4.8 | 3.8 | 1.9 | 0 | 1.1 | 17 | 5.8 | 6.3 | 5.2 | 2.9 | 1.3 | 2.3 | 5.1 | 4.1 | 3.0 | 3.2 | 3.3 |
| **6** | 9.1 | 5.6 | 4.6 | 2.1 | 1.1 | 0 | 7 | 7.9 | 6.5 | 5.4 | 3.0 | 0.8 | 1.3 | 5.2 | 3.4 | 2.4 | 2.1 | 2.2 |
| **7** | 9.8 | 6.2 | 5.2 | 27 | 1.7 | 0.7 | 0 | 8.6 | 7.1 | 6.1 | 3.7 | 1.5 | 1.1 | 5.9 | 4.0 | 2.8 | 2.2 | 2.9 |
| **8** | 1.3 | 1.2 | 2.2 | 59 | 5.8 | 7.9 | 86 | 0 | 1.4 | 2.7 | 5.3 | 7.1 | 8.6 | 5.0 | 7.4 | 7.3 | 8.5 | 8.2 |
| **9** | 2.9 | 1.4 | 1.7 | 44 | 6.3 | 6.5 | 71 | 1.4 | 0 | 1.0 | 3.8 | 5.7 | 7.1 | 3.3 | 5.8 | 5.8 | 7.1 | 6.7 |
| **10** | 3.9 | 2.3 | 2.2 | 34 | 5.2 | 5.4 | 61 | 2.7 | 1.0 | 0 | 2.8 | 4.7 | 6.1 | 2.4 | 4.8 | 4.8 | 6.1 | 5.7 |
| **11** | 6.5 | 3.7 | 2.7 | 10 | 2.9 | 3.0 | 37 | 5.3 | 3.8 | 2.8 | 0 | 2.3 | 3.7 | 2.3 | 2.2 | 2.1 | 3.4 | 3.0 |
| **12** | 8.3 | 4.8 | 3.8 | 13 | 1.3 | 0.8 | 15 | 7.1 | 5.7 | 4.7 | 2.3 | 0 | 1.5 | 4.5 | 2.6 | 1.6 | 2.1 | 2.4 |
| **13** | 9.8 | 6.2 | 5.2 | 27 | 2.3 | 1.3 | 11 | 8.6 | 7.1 | 6.1 | 3.7 | 1.5 | 0 | 5.9 | 2.9 | 1.7 | 1.1 | 1.4 |
| **14** | 6.3 | 4.6 | 4.1 | 32 | 5.1 | 5.2 | 59 | 5.0 | 3.3 | 2.4 | 2.3 | 4.5 | 5.9 | 0 | 2.9 | 4.3 | 5.6 | 5.2 |
| **15** | 8.4 | 5.8 | 4.7 | 24 | 4.1 | 3.4 | 40 | 7.4 | 5.8 | 4.8 | 2.2 | 2.6 | 2.9 | 2.9 | 0 | 1.3 | 2.5 | 2.2 |
| **16** | 8.5 | 5.5 | 4.5 | 20 | 3.0 | 2.4 | 28 | 7.3 | 5.8 | 4.8 | 2.1 | 1.6 | 1.7 | 4.3 | 1.3 | 0 | 1.3 | 0.9 |
| **17** | 9.7 | 6.7 | 5.7 | 32 | 3.2 | 2.1 | 22 | 8.5 | 7.1 | 6.1 | 3.4 | 2.1 | 1.1 | 5.6 | 2.5 | 1.3 | 0 | 0.7 |
| **18** | 9.4 | 6.3 | 5.4 | 29 | 3.3 | 2.2 | 29 | 8.2 | 6.7 | 5.7 | 3.0 | 2.4 | 1.4 | 5.2 | 2.2 | 0.9 | 0.7 | 0 |

The above-stated problem is formulated by using the HCFL model, a highly generalized model. For simplicity and proper understanding, here Constraints (2.3) and (2.4) are rewritten for each service type, which are as follows.

$$\sum_{k \in J} x_{jk}^{12} = \theta^{12} \left\{ \sum_{i \in I} x_{ij}^{1} \right\}, \qquad \forall j \in J \tag{2.10}$$

$$\sum_{k \in J} x_{jk}^{13} = \theta^{13} \left\{ \sum_{i \in I} x_{ij}^{1} \right\}, \qquad \forall j \in J \tag{2.11}$$

$$\sum_{k\in J} x_{jk}^{23} = \theta^{23}\left\{\sum_{i\in I} x_{ij}^{2} + \sum_{n\in J} x_{nj}^{12}\right\}, \qquad \forall j \in J \tag{2.12}$$

$$\sum_{i\in I} x_{ij}^{1} \le \sum_{t\in T} Q^{1f} y_j^f, \qquad \forall j \in J \tag{2.13}$$

$$\sum_{i\in I} x_{ij}^{2} + \sum_{k\in J} x_{kj}^{12} \le \sum_{t\in T} Q^{2f} y_j^f, \qquad \forall j \in J \tag{2.14}$$

$$\sum_{i\in I} x_{ij}^{3} + \sum_{k\in J} x_{kj}^{13} + \sum_{k\in J} x_{kj}^{23} \le \sum_{t\in T} Q^{3f} y_j^f, \qquad \forall j \in J \tag{2.15}$$

Constraints (2.10), (2.11) and (2.12) follows Constraint (2.3) to determine the number of referral from each lower-level service type. Similarly, Constraint (2.13), (2.14) and (2.15) follows the notion of Constraint (2.4) to respect the capacity of the facility.

After solving the example problem using the proposed HCFL model using the state-of-the-art Gurobi solver, it is observed that 3 SCs, 4 PHCs, and 2 CHCs are required to cater to all the demands, and the locations of these facilities are shown in Figure 2.3 and Table 2.4. The total cost is Rs.4.36E+06, in which Rs.4.30E+06 is towards the fixed cost, and Rs 5.55E+04 towards the travel cost including that for referrals. The allocation of MTBs for service types 1 and 2 is shown in Figure 2.4, and the allocation of MTBs for service type 3 and referrals to service type 3 are shown in Figure 2.5. The number of MTBs travelling from one location to another location to get the required service type is shown in Table 2.5. Due to the capacity constraint, MTBs from the same location can be allocated to facilities available at different locations. This can be seen for MTBs at location 9 from Table 2.5. For the same type 1 service, 10 MTBs have to visit the facility available at location 2, and 358 MTBs have to visit the facility available at location 10. This seems to be rational because the difference between the distance from location 9 to 2 and that from location 9 to 10 is not much high. In real practice also, one cannot restrict the population from visiting a particular facility. In the case of referrals, the number of

MTBs travelling from one healthcare facility to referred healthcare facility is shown in Table 2.6. There can be referral within the same healthcare facility also and the cost of referral will be zero.

**Table 2.4: Location of facilities**

| City | Type of facility |
|---|---|
| 1 | SC |
| 2 | CHC |
| 4 | SC |
| 6 | CHC |
| 10 | PHC |
| 11 | PHC |
| 13 | PHC |
| 16 | PHC |
| 17 | SC |

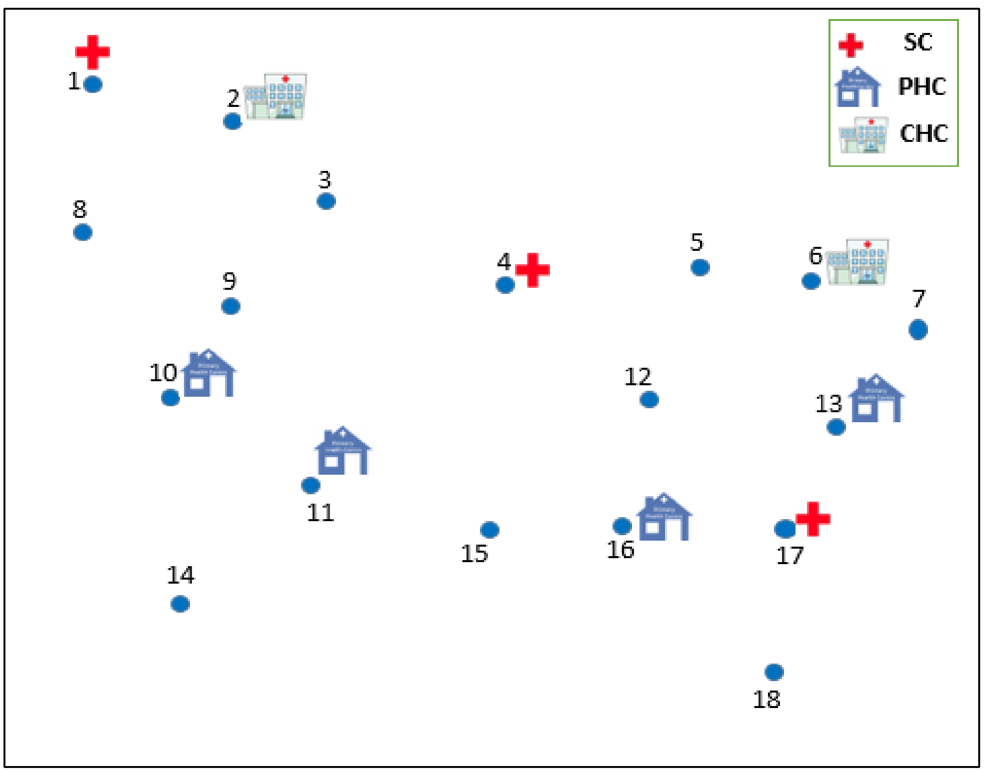



**Figure 2.3: Location solution for healthcare facilities to be established**

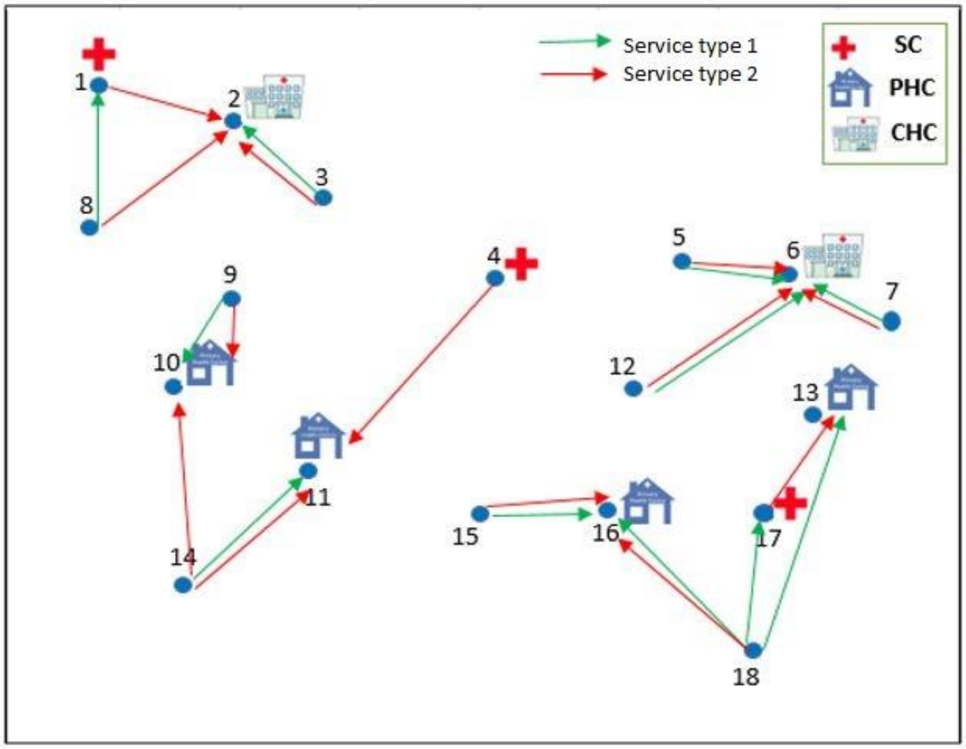



**Figure 2.4: Allocation of mothers-to-be to various facilities for service types 1 and 2**

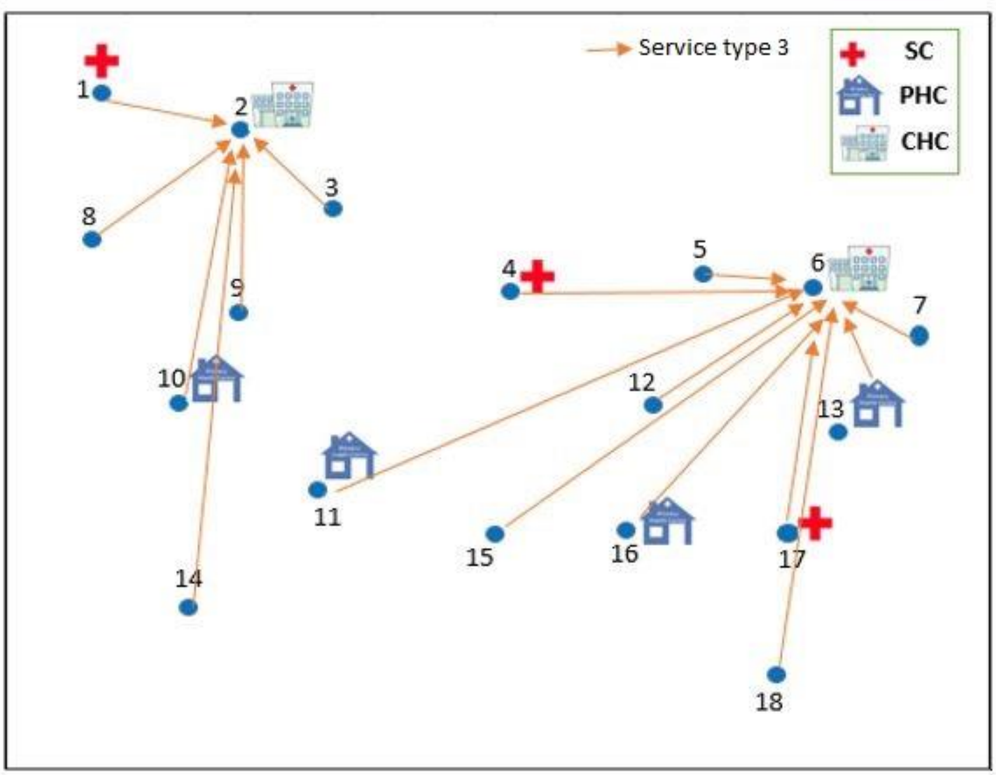



**Figure 2.5: Allocation of mothers-to-be for service type 3**

**Table 2.5: Non-referral allocations of mothers-to-be**

| MTBs Movement | | Type of Service | Number of MTBs | MTBs Movement | | Type of Service | Number of MTBs | MTBs Movement | | Type of Service | Number of MTBs | MTBs Movement | | Type of Service | Number of MTBs |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| From | To | | | From | To | | | From | To | | | From | To | | |
| 1 | 1 | 1 | 347 | 6 | 6 | 1 | 247 | 10 | 10 | 1 | 442 | 15 | 6 | 3 | 2 |
| 1 | 2 | 2 | 139 | 6 | 6 | 2 | 99 | 10 | 10 | 2 | 177 | 15 | 16 | 1 | 100 |
| 1 | 2 | 3 | 7 | 6 | 6 | 3 | 5 | 11 | 6 | 3 | 10 | 15 | 16 | 2 | 40 |
| 2 | 2 | 1 | 478 | 7 | 6 | 1 | 121 | 11 | 11 | 1 | 459 | 16 | 6 | 3 | 12 |
| 2 | 2 | 2 | 191 | 7 | 6 | 2 | 178 | 11 | 11 | 2 | 184 | 16 | 16 | 1 | 596 |
| 2 | 2 | 3 | 10 | 7 | 6 | 3 | 9 | 12 | 4 | 1 | 21 | 16 | 16 | 2 | 239 |
| 3 | 2 | 1 | 445 | 7 | 13 | 1 | 324 | 12 | 6 | 1 | 227 | 17 | 6 | 3 | 8 |
| 3 | 2 | 2 | 175 | 8 | 1 | 1 | 64 | 12 | 6 | 2 | 99 | 17 | 13 | 2 | 148 |
| 3 | 2 | 3 | 9 | 8 | 2 | 1 | 67 | 12 | 6 | 3 | 5 | 17 | 17 | 1 | 368 |
| 3 | 10 | 2 | 2 | 8 | 2 | 2 | 53 | 13 | 6 | 3 | 10 | 18 | 6 | 3 | 5 |
| 4 | 4 | 1 | 459 | 8 | 2 | 3 | 3 | 13 | 13 | 1 | 462 | 18 | 13 | 1 | 14 |
| 4 | 6 | 3 | 10 | **9** | **2** | **1** | **10** | 13 | 13 | 2 | 185 | 18 | 16 | 1 | 104 |
| 4 | 11 | 2 | 184 | 9 | 2 | 3 | 8 | 14 | 2 | 3 | 4 | 18 | 16 | 2 | 100 |
| 5 | 6 | 1 | 405 | **9** | **10** | **1** | **358** | 14 | 10 | 2 | 62 | 18 | 17 | 1 | 132 |
| 5 | 6 | 2 | 162 | 9 | 10 | 2 | 147 | 14 | 11 | 1 | 200 | | | | |
| 5 | 6 | 3 | 9 | 10 | 2 | 3 | 9 | 14 | 11 | 2 | 19 | | | | |

**Table 2.6: Referrals allocations mothers-to-be**

| Referrals from 1 to 2 | | | Referrals from 1 to 3 | | | Referrals from 2 to 3 | | |
|---|---|---|---|---|---|---|---|---|
| MTBs Movement | | Number of MTBs | MTBs Movement | | Number of MTBs | MTBs Movement | | Number of MTBs |
| From | To | | From | To | | From | To | |
| 1 | 2 | 41.1 | 1 | 2 | 12.33 | 2 | 2 | 35 |
| 2 | 2 | 100 | 2 | 2 | 30 | 6 | 6 | 31.9 |
| 4 | 11 | 48 | 4 | 6 | 14.4 | 10 | 2 | 23.4 |
| 6 | 6 | 100 | 6 | 6 | 30 | 11 | 6 | 25 |
| 10 | 10 | 80 | 10 | 2 | 24 | 13 | 6 | 23.15 |
| 11 | 11 | 65.9 | 11 | 2 | 15.17 | 16 | 6 | 22.95 |
| 13 | 13 | 80 | 11 | 6 | 4.6 | | | |
| 16 | 16 | 80 | 13 | 6 | 24 | | | |
| 17 | 13 | 50 | 16 | 6 | 24 | | | |
| | | | 17 | 6 | 15 | | | |

## 2.5 Solution Approaches

For solving the illustrative example described in the previous section, the related HCFL formulation was coded in Python 3.8 (Rossum & Drake, 2011) and linked with Gurobi 9.0.3. The application was run on a personal computer with an Intel Core i7 processor with a speed of 3.40 GHz and 8GB RAM. While experimenting with varying size of problems using Gurobi, it was observed that the solver hardly took a few minutes in providing optimal solution for a small size problem with fewer variables and constraints. For a large size problem (e.g., for the Tehsil level planning in India where a Tehsil can have more than 1000 villages as potential locations), the number of variables and constraints increases drastically (exponentially). For such problems, the solver could not find the optimal solution even in couple of days on the system mentioned above. Since the developed MILP model is NP-hard, it poses a computational challenge in obtaining a good quality solution in a reasonable amount of time. To address this issue, various strategies have been proposed here to solve the model in a computationally efficient manner as the Branch and Bound (B&B) method used by Gurobi solver becomes intractable for a reasonable size problem. The strategies proposed either use some additional constraints or handle the HCFL model in a phase-wise manner to arrive at the optimal or reasonably good solution. The additional constraints do not modify the characteristics of the problem. These are based on the understanding of the features of the problem and to help in reducing the computational effort. The two proposed strategies have been named as (1) VI-HCFL simultaneous approach and (2) Sequential approach. In the first approach, the problem of location and allocation is solved by considering the issue of locating all types of facilities (SC, PHC, CHC) simultaneously. In the sequential approach, it is carried out differently. First, the determination of location and allocation for CHC is carried out.

Taking this as input, the location and allocation problems related to PHC are addressed next and the last for SC. These solution approaches are detailed below.

### 2.5.1 Valid inequalities

As discussed earlier, the mathematical model, proposed in Section 2.3, is NP-hard and is computationally difficult to solve in a reasonable amount of time because of its inherent combinatorial nature and the used Branch and Bound (B & B) method by Gurobi solver. B & B method causes the associated tree to expand drastically with the increase in the number of integer variables. In order to reduce the computational time, it is important to reduce the size of the B & B tree by reducing the number of B & B nodes required to solve the model (Pochet and Wolsey, 2006). For this purpose, some additional valid inequalities have been added to the proposed model. Some of the proposed valid inequalities are redundant constraints. Anjos and Vieira, (2017) have reported the valid inequalities, in the form of redundant constraints, to be very useful in the use of the B & B method to improve the lower bound for a minimization problem even if they do not help in reducing the set of feasible solutions. These valid inequalities provide a better lower bound and faster converge3e. These additional valid inequalities are presented and discussed below.

**(i) Valid inequality for ensuring the availability of the established facilities within the coverage distance for each service type for each location of MTBs**

Constraints (2.6) relate flow variables ( $x_{ij}^{l}$ ) with the facility establishment variables ( $y_j^f$ ) with coverage distance issue in perspective. This constraint will cause the flow variables to assume any value from zero to the capacity level value of the service types available at the corresponding facility type in the absence of constraints (2.4). These constraints relate flow variables with the facility establishment variables along with the available capacity but do not address the maximum coverage distance issue. Because of these, there is a room for working on a large number of not so useful combinations of values for flow and facility establishment variables. To curtail these meaningless combinations, it is desired to ensure that the facilities are located within the maximum coverage distance and have sufficient capacity to meet the requirements of the allocated MTBs for all the service types. This is achieved by adding the following valid inequality to the HCFL model.

$$\sum_{j \in J} \sum_{f \in F} \alpha_{ij} y_j^f Q^{lf} \geq W_i^l, \qquad \forall i \in I, \forall l \in L \tag{2.16}$$

The above inequality can be seen to be related to constraints (2.2) and (2.6).

**(ii) Valid inequality for ensuring the availability of higher-level facilities within the maximum coverage distance of lower-level facility for referral cases**

The constraint set of the mathematical model proposed in Section 2.3 does not directly address the issue of the availability of referral facilities within the maximum coverage distance of the lower-level facilities. From this perspective, the following valid inequalities are added to the proposed mathematical model.

$$y_j^I \leq \sum_{k \in J} \beta_{jk} y_k^{II}, \qquad \forall j \in J \tag{2.17}$$

$$y_j^I \leq \sum_{k \in J} \beta_{jk} y_k^{III}, \qquad \forall j \in J \tag{2.18}$$

$$y_j^{II} \leq \sum_{k \in J} \beta_{jk} y_k^{III}, \qquad \forall j \in J \tag{2.19}$$

Constraints (2.17) and (2.18) will respectively and directly ensure the availability of PHC (indexed by *II*) and CHC (indexed by *III*) within the coverage distance of SC (indexed by *I*).

Similarly, constraint (2.19) will ensure the establishment of a referral facility as CHC (indexed by *III*) within the coverage distance of PHC (indexed by *II*).

**(iii) Valid inequality for ensuring the availability of the capacity of each service type on an overall basis**

The constraint set of the mathematical model proposed in Section 2.3 does not directly address the issue of the required total capacity of the healthcare system as a whole with the demand. The following valid inequalities added to the proposed mathematical model will ensure that the capacity of the facilities to be established should be at least equal to the overall requirement of each service type.

$$\sum_{i\in I} W_i^l + \sum_{i\in I}\sum_{p\in L, p<l} \theta^{pl} \left\{ W_i^p + \sum_{n\in L, n<p} \theta^{np} W_i^n \right\} \le \sum_{j\in J}\sum_{f\in F} y_j^f Q^{lf}, \qquad \forall l \in L \tag{2.20}$$

The constraint (2.20) is a combination of constraints (2.2) and (2.4). It is a generalized inequality and valid for any level of hierarchy. For three levels of hierarchy, for each service type 1, 2 and 3, this valid inequality can be rewritten as follows.

$$\sum_{i\in I} W_i^1 \le \sum_{j\in J}\sum_{f\in F} y_j^f Q^{1f} \tag{2.21}$$

$$\sum_{i\in I} W_i^2 + \sum_{i\in I} \theta^{12} \left\{ W_i^1 \right\} \le \sum_{j\in J}\sum_{f\in F} y_j^f Q^{2f} \tag{2.22}$$

$$\sum_{i\in I} W_i^3 + \sum_{i\in I} \theta^{13} \left\{ W_i^1 \right\} + \sum_{i\in I} \theta^{23} \left\{ W_i^2 + \theta^{12} W^1 \right\} \le \sum_{j\in J}\sum_{f\in F} y_j^f Q^{3f} \tag{2.23}$$

Constraints (2.21), (2.22) and (2.23) show that the total capacity of each type of service in the entire system should be at least equal to the total number of MTBs seeking the respective services directly and referrals for higher-level services.

The result of the effectiveness of all the above valid inequalities in reducing the computation time is presented in Section 2.6.3. The proposed mathematical model with these valid inequalities is referred as VI-HCFL in further elaboration and discussions. Because of the possibility of bringing down the time, all the above valid inequalities are incorporated in the mathematical model proposed in Section 2.3 to be used in the various stages of the approach discussed in the next sub-section.

### 2.5.2 Sequential approach

This approach is basically composed of three stages. It draws basic motivation from Relax-and-Fix (RF) and Fix-and-Optimize (FO) heuristics. RF method was developed by Dillenberger et al. (1994) to find an initial feasible solution for lot seque3ing problems. Subsequently, many authors have used the RF approach to find the initial solution to large-scale problems, such as lot sizing and scheduling problem (Absi and Heuvel, 2019; Araujo et al., 2007; Beraldi et al., 2008), production and distribution planning problem (Bilgen, 2014), travelling umpire problem (Oliveira et al., 2014) and routing problem (Oliveira and Scarpin, 2020). RF decomposes a large MILP problem into smaller sub-problems that can be solved easily. All the binary variables are relaxed initially. The partially relaxed MILPs are solved iteratively by fixing the variables in each stage. The feasible solution obtained by RF is then fed to FO to improve the solution quality further. FO was used by Gintner et al. (2005) for bus scheduling problems. It was formally introduced by Helber and Sahling (2010) to solve a multi-level capacitated lot-sizing problem. FO also decomposes the MILP problem into smaller sub-problems, with each having a lesser number of binary variables, some free and the rest fixed. After solving the sub-problem iteratively, the solution quality was found to improve. FO has been found to be efficient for large scale MIP problems, such as production planning problem

(Bilgen, 2014; Wolter and Helber, 2015), inventory problems (Tanksale and Jha, 2019) and facility location-network design problems (Ghaderi and Jabalameli, 2013; Moreno et al., 2015), and high school timetabling problem (Dorneles et al., 2014). The three-stage sequential approach proposed to solve the HCFL model is based on these co3epts. The three stages are (i) construction stage, (ii) improvement stage, and (iii) refinement stage. The actions to be taken at these stages are presented in respective Figure 2.6, Figure 2.7 and Figure 2.8, and the same is further discussed as follows.

**2.5.2.1 Construction stage**

At this stage, binary establishment variables are taken sequentially for each service type. Because of a few binary establishment variables taken, the computational requirements are expected to reduce. In this process, the allocation problem of CHCs is focused first. The corresponding $y_j^{III}$ variables are taken as binary, while this requirement is relaxed for $y_j^{II}$ and $y_j^{I}$ variables. With a relaxed condition, the proposed mathematical model with valid inequalities is used to obtain the optimal location of CHCs. With this optimal location of CHCs and taking now $y_j^{II}$ variables as binary, the mathematical model is again used to find out optimal locations for PHCs. The optimal solution thus obtained is ready with decided locations for PHCs and CHCs. Now, with $y_j^{I}$ variables as binary and the current optimal locations for the two higher-level facilities, the mathematical model will be used to solve the problem of finding the optimal location of SCs. While solving the location problems, allocation problems are also solved. In the construction stage, the locations of CHCs are worked out first as it generates opportunities for service types 1 and 2 as well. This amount of availability for service types 1 and 2 will be taken into consideration while deciding on the establishment of its lower-level facilities. Going otherwise from SCs to CHCs, a lot of spare capacity will unnecessarily be left as lower-level requirements will be taken care of while deciding on lower-level facilities. The higher-level facilities decided later will leave extra capacity for lower-level service types. This will make the establishment cost to rise high unnecessarily.

---

**I. Construction Stage**

0: Initialize $(x_{ij}^{l}, x_{jk}^{lm}, y_j^f \geq 0; \overline{Z} = 0)$

1: Rearrange the set of facility type $F$ as $\overline{F} = \{\text{III,II,I}\}$

2: For $f == III$ :

3: Set $y_j^{III} \in \{0,1\}, \forall j \in J$

4: Solve model [HCFL] by the solver

5: Set $\overline{Z} \leftarrow Z_{\text{HCFL}}$ and $\overline{y}_j^{III} \leftarrow y_j^{III}, \forall j \in J$

6: For $f == II$ :

7: Set $y_j^{II} \in \{0,1\}, \forall j \in J$

8: Add a constraint $\overline{y}_j^{III} = y_j^{III}, \forall j \in J$ to model [HCFL]

9: Solve model [HCFL] by the solver

10: Update $\overline{Z} \leftarrow Z_{\text{HCFL}}$ and $\overline{y}_j^{II} \leftarrow y_j^{II}, \forall j \in J$

11: For $f == I$ :

12: 0 Set $y_j^{I} \in \{0,1\}, \forall j \in J$

13: Add a constraint $\overline{y}_j^{II} = y_j^{II}, \forall j \in J$ to model [HCFL]

14: Solve model [HCFL] by the solver

15: Update $\overline{Z} \leftarrow Z_{\text{HCFL}}$ and $\overline{y}_j^{I} \leftarrow y_j^{I}, \forall j \in J$

16: Set $\overline{\overline{y}}_j^f \leftarrow \overline{y}_j^f, \forall j \in J, \forall f \in F$

17: Remove constraints added in Steps 8 and 13 from the model [HCFL]

---

**Figure 2.6: Pseudo-code for the construction stage of sequential approach**

$\overline{Z}$ = *Objective function value of the incumbent*

In the sequential facility allocation approach discussed above, the decisions taken earlier will govern the decisions taken later. Thus, the locations determined may not be the best ones. It is with this concern the improvement stage will be followed, and the same is discussed in the next sub-section

### 2.5.2.2 Improvement stage

For large size problems, B&B method used by the Gurobi solver was not yielding the optimal solution even after elapse of considerable amount of CPU time. This can be seen from the experimentations carried out in Section 2.6. It is this solution that is the best solution obtained in the specified amount of CPU time and this is not necessarily the optimal solution. In such cases, the follow-up of improvement stage is found to generally yield an improved solution.

---

**II. Improvement Stage**

18: For $f == III$ :<br>
19: Add a constraint $y_j^{II} = \bar{\bar{y}}_j^{II}, \forall j \in J$ to model [HCFL]<br>
20: Add a constraint $y_j^{I} = \bar{\bar{y}}_j^{I}, \forall j \in J$ to model [HCFL]<br>
21: Solve model [HCFL] by the solver<br>
22: If $Z_{\text{HCFL}} \leq \bar{Z}$ , update $\bar{Z} \leftarrow Z_{\text{HCFL}}, \bar{\bar{y}}_j^{III} \leftarrow \bar{y}_j^{III}, \forall j \in J$<br>
23: Remove the constraint added in Step 19 from the model [HCFL]<br>
24: For $f == II$ :<br>
25: Add constraint $y_j^{III} = \bar{\bar{y}}_j^{III}, \forall j \in J$ to model [HCFL]<br>
26: Solve model [HCFL] by the solver<br>
27: If $Z_{\text{HCFL}} \leq \bar{Z}$ , update $\bar{Z} \leftarrow Z_{\text{HCFL}}, \bar{\bar{y}}_j^{II} \leftarrow \bar{y}_j^{II}, \forall j \in J$<br>
28: Remove constraint added in Step 20 from the model [HCFL]<br>
29: For $f == I$ :<br>
30: Add constraint $y_j^{II} = \bar{\bar{y}}_j^{II}, \forall j \in J$ to model [HCFL]<br>
31: Solve model [HCFL] by the solver<br>
32: If $Z_{\text{HCFL}} \leq \bar{Z}$ , update $\bar{Z} \leftarrow Z_{\text{HCFL}}, \bar{\bar{y}}_j^{I} \leftarrow \bar{y}_j^{I}, \forall j \in J$<br>
33: Remove constraints added in Steps 25 and 30 from the model [HCFL]

---

**Figure 2.7: Pseudo-code for the improvement stage of the sequential approach**

At this stage, new optimal locations of CHCs are determined using the current optimal locations of SCs and PHCs to do away with the bias resulting from the construction stage, particularly for large size problems. With this new optimal location of CHCs and the existing locations of SCs, the optimal locations for PHCs will be discovered next. In the last, with the current optimal locations of PHCs and CHCs, the optimal locations of SCs will be determined. It should be noted that most of the establishment and allocation variables will get fixed value and thus reducing the size of problem to be solved using HCFL formulation.

### 2.5.2.3 Refinement stage

In the earlier two stages, the optimal locations for each facility were obtained sequentially, one after the other, and never simultaneously. This sequential determination of optimal locations will have an inherent bias, which on removal may result in a better solution. This stage helps in doing so and is detailed below. At this stage, a fresh assignment of CHCs, PHCs and SCs is worked out at the locations identified in the improvement stage and using the HCFL model to find optimal allocation decisions as well. The mathematical model used at this stage, with the valid inequalities, will be of much-reduced size due to known locations for the facilities. The problem size reduces due to zero value assigned to location and flow variables corresponding to those locations that did not find any facilities located in them at the improvement stage.

**III. Refinement Stage**

```
Provide a solution to a black-box solver as a starting solution
Solve model [HCFL] by the solver
If Z_HCFL ≤ Z̄ , update Z̄ ← Z_HCFL
Report solution Z* ← Z̄
```

**Figure 2.8: Pseudo-code for the refinement stage of the sequential approach**

$Z^*$ = *final* o*bjective function value*

Due to the reduction in the number of integer variables and constraints on account of prefixing of values for a good number of the decision variables, the computational requirement decreases drastically, and the solution obtained is generally better as compared to the solution obtained after the improvement stage. The reduction in the number of establishment variables causes an exponential reduction in the total number of decision variables.

## 2.6 Computational Experiments

Computational experiments have been carried out to:

1. analyze the usefulness of the proposed valid inequalities,
2. check the effectiveness and computational efficiency of the proposed sequential approach, and
3. perform sensitivity analyses to understand the impact of variation in the problem parameters on the solution.

The extensive computational experiments, based on randomly generated datasets, are detailed in the following sub-sections.

### 2.6.1 Generation of test instances

Seven problem classes were taken, with the number of locations varying from 10 to 400. These problem classes are designated as D10, D20, D50, D100, D200, D300, and D400. For each problem class, five instances were generated. The demand nodes (or the locations of MTBs) are generated randomly by taking them to be uniformly distributed over a grid of 100 by 100 for problem classes D10, D20, and D50, and 200 by 200 for the remaining problem classes. It is noteworthy that the proposed model and solution methodologies are not limited to the consideration of Euclidean distance alone. However, the distance between the demand nodes is taken as Euclidean distance simply for the purpose of experimentation. For example, for a 50 node problem (of class D50), 50 coordinates were generated randomly on a grid of 100 by 100. All the 50 coordinates were uniformly distributed over the grid. The distance matrix was prepared considering the Euclidean distance between each of them. The cost of visiting a facility was taken to be proportional to the distance. Demands originating from various locations for service types 1, 2, and 3 were taken as random numbers uniformly distributed over (50, 1000), (20, 400), (1, 20), respectively. The numbers taken were representative of the true condition where the requirement of service type 1 is the highest and that of service type 3 is the lowest. The capacity taken for various service types at the three facility types is shown in Table 2.7. The proportion of referrals from service types 1 to 2, 1 to 3, and 2 to 3 were taken as 0.1, 0.03, and 0.05, respectively. It is estimated using the data available in the Indian context. The maximum distance that an MTB can travel to reach a facility from her origin (i.e., coverage distance) was taken as 50 units, while in the case of referral as 70 units. The fixed costs incurred on establishing a single unit of SC, PHC, and CHC were taken as 100,000, 500,000, and 1,000,000 units, respectively.

**Table 2.7: Capacity (units) of the facilities**

| **Service types** | **Problem class** | | | | | |
|---|---|---|---|---|---|---|
| | **D10, D20** | | | **D50, D100, D200, D300, D400** | | |
| | **SC** | **PHC** | **CHC** | **SC** | **PHC** | **CHC** |
| **1** | 300 | 500 | 800 | 1000 | 1200 | 1500 |
| **2** | 0 | 300 | 500 | 0 | 1000 | 1200 |
| **3** | 0 | 0 | 100 | 0 | 0 | 300 |

The reduction in the number of establishment variables causes an exponential reduction in the total number of decision variables, and this can be verified from Figure 2.9, which has been drawn for the problem instances undertaken for experimentation.

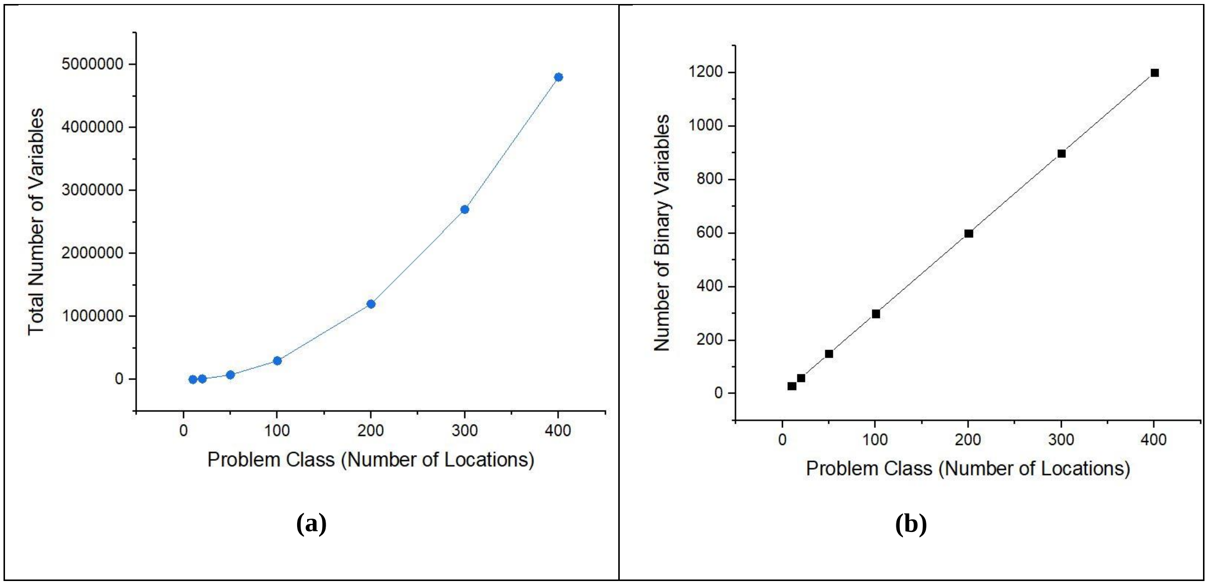



**Figure 2.9: Increase in (a) total number of variables (b) number of establishment variables**

### 2.6.2 Experimental results on the suitability of the proposed approaches

The experimental results related to the efficiency and efficacy of the solution approaches presented in Sections 2.5.1 and 2.5.2 are provided in Table 2.8,Table 2.9, and Table 2.10. In these tables, columns labelled as 'D2' and 'D1' present the percentage by which the upper bound (UB) values obtained by the sequential approach (Section 2.5.2) are more than that from HCFL or VI-HCFL formulations, respectively. The comparison was made based on the upper bound value as the problem seeks to minimize the overall cost. Further, the upper bound value will correspond to a feasible solution, while the lower bound may not be. Columns labelled as 'Gap' measure the gap between upper bound (UB) and lower bound (LB) values as a percentage of UB.

Table 2.8 contains the results for those problems for which all the three approaches (HCFL, VI-HCFL and Sequential approach) yielded optimal solutions. Naturally, the LB and UB values for these cases are equal. For all these problems (except problem number 1), the inclusion of all the valid inequalities into the mathematical model shows clear improvement by way of lesser CPU times taken for resulting in the optimal solutions. The improvement, even though it is smaller for the VI-HCFL approach for problem class D10, becomes remarkably very high for D20 problem class. Naturally, the use of valid inequalities has paid its dividend in reducing the CPU time requirement or the computational effort drastically. The sequential approach is better than the VI-HCFL approach for D20 problem class but not for the D10 problem class in terms of the CPU time requirement. The sequential approach is found much better in terms of CPU time requirements than the HCFL and VI-HCFL for the D20 problem class. The quality of the solution obtained from the sequential approach is very good, and the same can be witnessed from columns D1 and D2.

Table 2.9 contains the results for those problem instances for which VI-HCFL formulation has yielded the optimal solution. Gurobi solver did not converge while working with HCFL formulation even after spending in a high CPU time of 7200 seconds for problems 13, 14, and 15. This table also finds the sequential approach to be taking the least amount CPU time, and VI-HCFL formulation to be better than HCFL formulation from this perspective. Thus, the sequential approach turns out to be the best from the CPU time requirement perspective. The sequential approach could result in the optimal solution for one of the problems (problem number 14) taking just 14 seconds, whereas VI-HCFL took more than seven times of CPU time to yield the optimal solution, and HCFL could not yield the solution even in 7200 sec (514 times of the CPU time for the sequential approach). Even though the sequential approach is quite fast, but is marginally poor. It is evident from entries in columns 'D1' and 'D2'. However, even the highest gap of 0.113% (for problem number 11) is a very small number. This table evidences VI-HCFL to be superior to HCFL both in terms of efficiency and efficacy.

On an overall basis, the sequential approach can be observed to be the best, with the resulting solution quality being practically the same as from VI-HCFL.

Table 2.10 contains the results for the remaining problem instances. Since the HCFL or VI-HCFL approaches were taking too much CPU time to arrive at the optimal solution, the rest of the analysis on efficacy and efficiency for large size problems was carried out while limiting the CPU time to 7200 seconds. Looking into the UBs for HCFL and VI-HCFL approaches, it is found that the two are equivalent for problems 18 and 19. In the rest cases, the VI-HCFL approach is superior not only in terms of UB value but also in terms of the gap between UBs and LBs (a tighter band). From this perspective, the sequential approach is found to be very marginally inferior to the HCFL and VI-HCFL approaches but for D400 problem class. For problem classes such as D100, D200, or D300 (which are close to the Indian context), the sequential approach has concluded itself quite early and yielded reasonably good solutions as well. It is obvious from the entries in the columns related to 'D1' and 'D2'. Wilcoxon signed-rank test was applied to the CPU time for each of the proposed approaches. The test found the sequential approach to be better than both the HCFL ($p$ = 0.00001) and VI-HCFL formulations ($p$ = 0.00001). VI-HCFL formulation was found to be better than HCFL formulation with $p$ = 0.0008.

Based on the above observations, it can be concluded that the addition of valid inequalities helps in reducing the computational effort, and the proposed sequential approach is practically a better approach compared to the other approaches for resulting in a good solution (quite close to the optimal one).

**Table 2.8: Optimal solutions obtained by HCFL and VI-HCFL approaches for some problems**

| Problem class | Problem number | Instance number | HCFL formulation | | | | VI-HCFL formulation | | | | Sequential approach | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | UB | LB | Gap (%) | Time (sec) | UB | LB | Gap (%) | Time (sec) | UB | Time (sec) | D1 (%) | D2 (%) |
| D10 | 1 | 1 | 3655273 | 3655273 | 0.00 | 0.24 | 3655273 | 3655273 | 0.00 | 0.26 | 3655273 | 0.39 | 0.00 | 0.00 |
| | 2 | 2 | 3245764 | 3245764 | 0.00 | 0.27 | 3245764 | 3245764 | 0.00 | 0.20 | 3245764 | 0.37 | 0.00 | 0.00 |
| | 3 | 3 | 3647216 | 3647216 | 0.00 | 0.36 | 3647216 | 3647216 | 0.00 | 0.18 | 3648201 | 0.35 | 0.03 | 0.03 |
| | 4 | 4 | 3229844 | 3229844 | 0.00 | 0.38 | 3229844 | 3229844 | 0.00 | 0.13 | 3229844 | 0.30 | 0.00 | 0.00 |
| | 5 | 5 | 3233145 | 3233145 | 0.00 | 0.56 | 3233145 | 3233145 | 0.00 | 0.25 | 3233145 | 0.27 | 0.00 | 0.00 |
| D20 | 6 | 1 | 3957152 | 3957152 | 0.00 | 132 | 3957152 | 3957152 | 0.00 | 0.85 | 3957152 | 0.81 | 0.00 | 0.00 |
| | 7 | 2 | 3366991 | 3366991 | 0.00 | 258 | 3366991 | 3366991 | 0.00 | 2.18 | 3366991 | 1.37 | 0.00 | 0.00 |
| | 8 | 3 | 4363713 | 4363713 | 0.00 | 100 | 4363713 | 4363713 | 0.00 | 1.43 | 4363713 | 1.02 | 0.00 | 0.00 |
| | 9 | 4 | 3985069 | 3985069 | 0.00 | 165 | 3985069 | 3985069 | 0.00 | 1.55 | 3985069 | 1.40 | 0.00 | 0.00 |
| | 10 | 5 | 4474788 | 4474788 | 0.00 | 34 | 4474788 | 4474788 | 0.00 | 2.24 | 4475439 | 1.59 | 0.02 | 0.02 |

**Table 2.9: Problems in which the same UB is obtained by HCFL and VI-HCFL formulations**

| Problem class | Problem number | Instance number | HCFL formulation | | | | VI-HCFL formulation | | | | Sequential approach | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | UB | LB | Gap (%) | Time (sec) | UB | LB | Gap (%) | Time (sec) | UB | Time (sec) | D1 (%) | D2 (%) |
| D50 | 11 | 1 | 6198274 | 6198274 | 0.000 | 790 | 6198274 | 6198274 | 0.000 | 204 | 6205247 | 14 | 0.113 | 0.113 |
| | 12 | 2 | 6191730 | 6191730 | 0.000 | 3095 | 6191730 | 6191730 | 0.000 | 55 | 6193841 | 13 | 0.034 | 0.034 |
| | 13 | 3 | 6280104 | 6201740 | 0.012 | 7200 | 6280104 | 6280104 | 0.000 | 161 | 6283516 | 18 | 0.054 | 0.054 |
| | 14 | 4 | 5797499 | 5717716 | 0.014 | 7200 | 5797499 | 5797499 | 0.000 | 104 | 5797499 | 14 | 0.000 | 0.000 |
| | 15 | 5 | 6802824 | 6791343 | 0.002 | 7200 | 6802824 | 6802824 | 0.000 | 158 | 6803499 | 17 | 0.010 | 0.010 |

**Table 2.10: Results for other problem instances**

| Problem class | Problem number | Instance number | HCFL formulation | | | | VI-HCFL formulation | | | | Sequential approach | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | UB | LB | Gap (%) | Time (sec) | UB | LB | Gap (%) | Time (sec) | UB | Time (sec) | D1 (%) | D2 (%) |
| D100 | 16 | 1 | 11783409 | 11536985 | 2.09 | 7200 | 11782473 | 11771613 | 0.09 | 7200 | 11784249 | 629 | 0.007 | 0.015 |
| | 17 | 2 | 11785575 | 11474852 | 2.64 | 7200 | 11784725 | 11772991 | 0.10 | 7200 | 11786967 | 623 | 0.012 | 0.019 |
| | 18 | 3 | 11891016 | 11753485 | 1.16 | 7200 | 11891016 | 11879440 | 0.10 | 7200 | 11895361 | 627 | 0.037 | 0.037 |
| | 19 | 4 | 11772656 | 11504419 | 2.28 | 7200 | 11772656 | 11759353 | 0.11 | 7200 | 11781110 | 452 | 0.072 | 0.072 |
| | 20 | 5 | 13479673 | 13095001 | 2.85 | 7200 | 13479087 | 13466221 | 0.10 | 7200 | 13484309 | 171 | 0.034 | 0.039 |
| D200 | 21 | 1 | 22320525 | 21953476 | 1.64 | 7200 | 22325435 | 22028220 | 1.33 | 7200 | 22360590 | 2679 | 0.179 | 0.157 |
| | 22 | 2 | 23320474 | 22817655 | 2.16 | 7200 | 23308370 | 22894610 | 1.78 | 7200 | 23326391 | 1824 | 0.025 | 0.077 |
| | 23 | 3 | 23001255 | 22573663 | 1.86 | 7200 | 22988457 | 22876101 | 0.49 | 7200 | 23011816 | 2551 | 0.046 | 0.102 |
| | 24 | 4 | 21910910 | 21769827 | 0.64 | 7200 | 21914078 | 21784573 | 0.59 | 7200 | 21931671 | 2190 | 0.095 | 0.080 |
| | 25 | 5 | 22402508 | 22100827 | 1.35 | 7200 | 22397013 | 22296504 | 0.45 | 7200 | 22409148 | 1940 | 0.030 | 0.054 |
| D300 | 26 | 1 | 34202506 | 33816967 | 1.13 | 7200 | 34190936 | 34018507 | 0.50 | 7200 | 34210107 | 3011 | 0.022 | 0.056 |
| | 27 | 2 | 33088955 | 32674682 | 1.25 | 7200 | 33078453 | 32804918 | 0.83 | 7200 | 33137687 | 3380 | 0.147 | 0.179 |
| | 28 | 3 | 33604363 | 33000545 | 1.80 | 7200 | 33581420 | 33425026 | 0.47 | 7200 | 33632356 | 3680 | 0.083 | 0.152 |
| | 29 | 4 | 34122862 | 33317291 | 2.36 | 7200 | 34102981 | 33963793 | 0.41 | 7200 | 34135816 | 3312 | 0.038 | 0.096 |
| | 30 | 5 | 34726111 | 34033008 | 2.00 | 7200 | 34720646 | 34244296 | 1.37 | 7200 | 34726252 | 2955 | 0.000 | 0.016 |
| D400 | 31 | 1 | 50725329 | 44518013 | 12.24 | 7200 | 45099260 | 44772705 | 0.72 | 7200 | 44977721 | 3815 | -11.331 | -0.269 |
| | 32 | 2 | 59412153 | 44810000 | 24.58 | 7200 | 45691319 | 45016508 | 1.48 | 7200 | 45477897 | 4345 | -23.454 | -0.467 |
| | 33 | 3 | 56123253 | 44960221 | 19.89 | 7200 | 45430940 | 45120680 | 0.68 | 7200 | 45586773 | 4606 | -18.774 | 0.343 |
| | 34 | 4 | 54752318 | 43836145 | 19.94 | 7200 | 45220943 | 44275504 | 2.09 | 7200 | 44822761 | 4606 | -18.135 | -0.881 |
| | 35 | 5 | 56451591 | 44812241 | 20.62 | 7200 | 45638762 | 45020484 | 1.35 | 7200 | 45522335 | 4605 | -19.360 | -0.255 |

### 2.6.3 Experimental results on the efficacy of the proposed valid inequalities

Section 2.6.2 demonstrated the usefulness of valid inequalities. In view of this, it is desired to further investigate the effectiveness of these inequalities individually. This section is devoted to this end. Table 2.11 shows the results from the use of valid inequalities for the various problem classes when added individually to the HCFL model. 'Gap' in this table refers to the gap between UB and LB values as a percentage of UB value. For a fair comparison, a maximum CPU time of 7200 seconds was allocated to the GUROBI solver.

It can be observed from this table that all the versions of the formulation yielded optimal solutions for problem classes D10 and D20. Only the inclusion of valid inequality 3 yielded optimal solutions for all instances of problem class 50. Not only this, it has taken the least CPU time for all D10, D20, and D50 classes of problems. For the other remaining problem classes (D100, D200, D300, and D400), CPU time was limited to 7200 seconds. For these problem classes, the gap between UB and LB values for HCLF formulation was high in most of the cases compared to when the proposed inequalities were included in it individually. Wilcoxon signed-rank test was applied to the gap values for each variation of HCFL formulation. The result of the test is shown in Table 2.12. This table shows that the inclusion of valid inequality 3 is quite useful and effective. Valid inequality 3 can be observed to perform better compared to the other two valid inequalities from the solution quality perspective. No statistically significant difference was found in the use of either valid inequality 1 or inequality 2 with HCFL formulation. When all the valid inequalities are included in the HCFL formulation, it remains to be seen whether the effectiveness is mostly governed by valid inequality 3 or by the other two inequalities as well. For this, the results with the inclusion of valid inequality 3 (shown in Table 2.11) were compared to the results from the VI-HCFL formulation shown in

Table 2.8, Table 2.9 and Table 2.10. Wilcoxon signed-rank test on CPU time resulted in a *p*-value of 0.13888, meaning thereby that the difference in CPU time requirement is insignificant. However, this test performed on the 'Gap' value resulted in a *p*-value of 0.00001. It clearly shows that the use of all the three valid inequalities proposed in this chapter is more effective than the use of valid inequalities 3 alone.

The above experimentation demonstrates the usefulness of the proposed inequalities for the problem considered in this chapter.

**Table 2.11: Comparison of HCFL formulation with valid inequalities**

| Problem class | Problem | HCFL formulation | | | | Valid Inequality 1 (Section 5.2.1) | | | | Valid Inequality 2 (Section 5.2.2) | | | | Valid Inequality 3 (Section 5.2.3) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | UB | LB | Gap (%) | Time (sec) | UB | LB | Gap (%) | Time (sec) | UB | LB | Gap (%) | Time (sec) | UB | LB | Gap (%) | Time (sec) |
| D10 | 1 | 3655273 | 3655273 | 0.00 | 0.24 | 3655273 | 3655273 | 0.00 | 3.40 | 3655273 | 3655273 | 0.00 | 0.27 | 3655273 | 3655273 | 0.00 | 0.16 |
| | 2 | 3245764 | 3245764 | 0.00 | 0.27 | 3245764 | 3245764 | 0.00 | 3.71 | 3245764 | 3245764 | 0.00 | 0.26 | 3245764 | 3245764 | 0.00 | 0.17 |
| | 3 | 3647216 | 3647216 | 0.00 | 0.36 | 3647216 | 3647216 | 0.00 | 1.78 | 3647216 | 3647216 | 0.00 | 0.48 | 3647216 | 3647216 | 0.00 | 0.20 |
| | 4 | 3229844 | 3229844 | 0.00 | 0.38 | 3229844 | 3229844 | 0.00 | 0.94 | 3229844 | 3229844 | 0.00 | 0.59 | 3229844 | 3229844 | 0.00 | 0.17 |
| | 5 | 3233145 | 3233145 | 0.00 | 0.56 | 3233145 | 3233145 | 0.00 | 1.89 | 3233145 | 3233145 | 0.00 | 0.67 | 3233145 | 3233145 | 0.00 | 0.17 |
| D20 | 6 | 3957152 | 3957152 | 0.00 | 132 | 3957152 | 3957152 | 0.00 | 712 | 3957152 | 3957152 | 0.00 | 375 | 3957152 | 3957152 | 0.00 | 1.05 |
| | 7 | 3366991 | 3366991 | 0.00 | 258 | 3366991 | 3366991 | 0.00 | 125 | 3366991 | 3366991 | 0.00 | 289 | 3366991 | 3366991 | 0.00 | 2.22 |
| | 8 | 4363713 | 4363713 | 0.00 | 100 | 4363713 | 4363713 | 0.00 | 184 | 4363713 | 4363713 | 0.00 | 183 | 4363713 | 4363713 | 0.00 | 9.02 |
| | 9 | 3985069 | 3985069 | 0.00 | 165 | 3985069 | 3985069 | 0.00 | 168 | 3985069 | 3985069 | 0.00 | 146 | 3985069 | 3985069 | 0.00 | 2.06 |
| | 10 | 4474788 | 4474788 | 0.00 | 34 | 4474788 | 4474788 | 0.00 | 46 | 4474788 | 4474788 | 0.00 | 41 | 4474788 | 4474788 | 0.00 | 1.53 |
| D50 | 11 | 6198274 | 6198274 | 0.00 | 790 | 6198274 | 6198274 | 0.00 | 1038 | 6198274 | 6198274 | 0.00 | 511 | 6198274 | 6198274 | 0.00 | 129 |
| | 12 | 6191730 | 6191730 | 0.00 | 3095 | 6191730 | 6191730 | 0.00 | 3449 | 6191730 | 6191730 | 0.00 | 1261 | 6191730 | 6191730 | 0.00 | 53 |
| | 13 | 6280104 | 6201740 | 1.25 | 7200 | 6280104 | 6202994 | 1.23 | 7200 | 6280104 | 6217998 | 0.99 | 7200 | 6280104 | 6280104 | 0.00 | 85 |
| | 14 | 5797499 | 5717716 | 1.38 | 7200 | 5797499 | 5719738 | 1.34 | 7200 | 5797499 | 5721474 | 1.31 | 7200 | 5797499 | 5797499 | 0.00 | 68 |
| | 15 | 6802824 | 6791343 | 0.17 | 7200 | 6802824 | 6802824 | 0.00 | 7200 | 6802824 | 6747027 | 0.82 | 7200 | 6802824 | 6802824 | 0.00 | 97 |
| D100 | 16 | 11783409 | 11536985 | 2.09 | 7200 | 11782473 | 11536975 | 2.08 | 7200 | 11782510 | 11538051 | 2.07 | 7200 | 11782473 | 11771358 | 0.09 | 7200 |
| | 17 | 11785575 | 11474852 | 2.64 | 7200 | 11785575 | 11519277 | 2.26 | 7200 | 11785575 | 11518946 | 2.26 | 7200 | 11785121 | 11773742 | 0.10 | 7200 |
| | 18 | 11891016 | 11753485 | 1.16 | 7200 | 11891016 | 11756496 | 1.13 | 7200 | 11891856 | 11753853 | 1.16 | 7200 | 11891005 | 11879309 | 0.10 | 7200 |
| | 19 | 11772656 | 11504419 | 2.28 | 7200 | 11774342 | 11502188 | 2.31 | 7200 | 11773878 | 11501924 | 2.31 | 7200 | 11772656 | 11762574 | 0.09 | 7200 |
| | 20 | 13479673 | 13095001 | 2.85 | 7200 | 13479115 | 13095068 | 2.85 | 7200 | 13479087 | 13098492 | 2.82 | 7200 | 13479087 | 13467479 | 0.09 | 7200 |
| D200 | 21 | 22320525 | 21953476 | 1.64 | 7200 | 22335763 | 21954026 | 1.71 | 7200 | 22333464 | 21955130 | 1.69 | 7200 | 22321977 | 22028432 | 1.32 | 7200 |
| | 22 | 23320474 | 22817655 | 2.16 | 7200 | 23340027 | 22555403 | 3.36 | 7200 | 23312522 | 22554997 | 3.25 | 7200 | 23308838 | 22900445 | 1.75 | 7200 |
| | 23 | 23001255 | 22573663 | 1.86 | 7200 | 23000573 | 22852383 | 0.64 | 7200 | 23007616 | 22856128 | 0.66 | 7200 | 22986891 | 22926397 | 0.26 | 7200 |
| | 24 | 21910910 | 21769827 | 0.64 | 7200 | 21925477 | 21631193 | 1.34 | 7200 | 21923493 | 21634603 | 1.32 | 7200 | 21924579 | 21850921 | 0.34 | 7200 |
| | 25 | 22402508 | 22100827 | 1.35 | 7200 | 22398202 | 22091246 | 1.37 | 7200 | 22407109 | 22092295 | 1.40 | 7200 | 22392277 | 22299714 | 0.41 | 7200 |
| D300 | 26 | 34202506 | 33816967 | 1.13 | 7200 | 34210945 | 33818531 | 1.15 | 7200 | 34223244 | 33818860 | 1.18 | 7200 | 34193563 | 34017860 | 0.51 | 7200 |
| | 27 | 33088955 | 32674682 | 1.25 | 7200 | 33078632 | 32678101 | 1.21 | 7200 | 33111218 | 32676864 | 1.31 | 7200 | 33082617 | 32803184 | 0.84 | 7200 |
| | 28 | 33604363 | 33000545 | 1.80 | 7200 | 33612411 | 33001050 | 1.82 | 7200 | 33631041 | 33001437 | 1.87 | 7200 | 33597512 | 33423550 | 0.52 | 7200 |
| | 29 | 34122862 | 33317291 | 2.36 | 7200 | 34116394 | 33315314 | 2.35 | 7200 | 34153279 | 33315737 | 2.45 | 7200 | 34100633 | 33674276 | 1.25 | 7200 |
| | 30 | 34726111 | 34033008 | 2.00 | 7200 | 34727290 | 34034461 | 2.00 | 7200 | 34772342 | 34034204 | 2.12 | 7200 | 34703570 | 34242378 | 1.33 | 7200 |
| D400 | 31 | 50725329 | 44518013 | 12.24 | 7200 | 59423462 | 44514788 | 25.09 | 7200 | 60681311 | 44515650 | 26.64 | 7200 | 44939674 | 44774595 | 0.37 | 7200 |
| | 32 | 59412153 | 44810000 | 24.58 | 7200 | 52397030 | 44811786 | 14.48 | 7200 | 56482929 | 44809854 | 20.67 | 7200 | 45375618 | 45019215 | 0.79 | 7200 |
| | 33 | 56123253 | 44960221 | 19.89 | 7200 | 52876735 | 44960927 | 14.97 | 7200 | 75324206 | 44955178 | 40.32 | 7200 | 45434007 | 45113672 | 0.71 | 7200 |
| | 34 | 54752318 | 43836145 | 19.94 | 7200 | 55913366 | 43836270 | 21.60 | 7200 | 58531805 | 43836739 | 25.11 | 7200 | 44740039 | 44275170 | 1.04 | 7200 |
| | 35 | 56451591 | 44812241 | 20.62 | 7200 | 57911578 | 44812882 | 22.62 | 7200 | 81289578 | 44803821 | 44.88 | 7200 | 45377451 | 45018494 | 0.79 | 7200 |

**Table 2.12: Results of Wilcoxon signed-rank test**

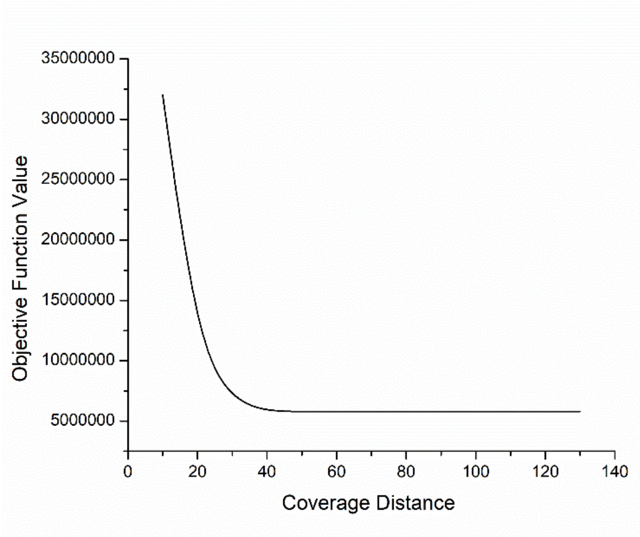
| Effectiveness of inclusion of | *p*-value | Observation (statistically significant) |
|---|---|---|
| Valid inequality 1 over HCFL | 0.79486 | No |
| Valid inequality 2 over HCFL | 0.06876 | No |
| Valid inequality 3 over HCFL | 0.00001 | Yes |
| Valid inequality 2 over valid inequality 1 | 0.04884 | Yes |
| Valid inequality 3 over valid inequality 1 | 0.00001 | Yes |
| Valid inequality 3 over valid inequality 2 | 0.00001 | Yes |
| All the valid inequalities together over HCFL | 0.00001 | Yes |

## 2.7 Sensitivity Analysis

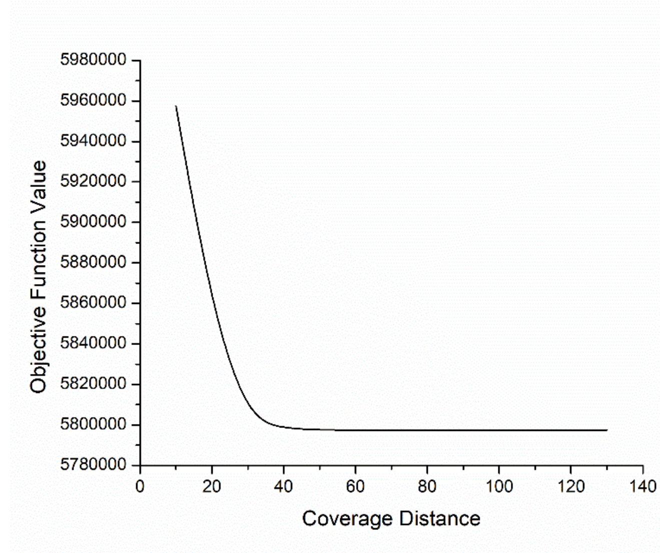
Sensitivity analyses have been carried out to analyze the effect of the various parameters, such as coverage distance, fixed cost, referral proportion, and capacity. For this purpose, problem number 14 has been taken. The results of the analyses are provided in the following sub-sections.

### 2.7.1 Impact of coverage distance

Location problems with a prespecified coverage distance are basically covering problems (Mark S. Daskin, 2009). In these problems, the choice of coverage distance generally has a significant impact on the solution. The problem chosen in the current work considers two types of distances: one for allocation of MTBs to a facility from their origin and the other for referrals to another facility. As mentioned in Section 2.6.1, the values of these distances are 50 and 70, respectively. For the sensitivity analyses, the coverage distances are varied one at a time in the range from 10 to 130 with a step size of 10. The impact of the coverage distance on the overall cost is summarised in Figure 2.10. In general, a reduction in the maximum coverage distance should cause more facilities to be created to meet the demand. In extreme cases (when the maximum coverage distance is specified to be very low), every location will then have at least one CHC to meet the requirement of all three types of services. With the decrease in the maximum coverage distance, the travel cost is likely to come down while the facility establishment cost to go up. Since the facility establishment cost is expected to be high compared to the travel cost, a net increase in the overall cost is to be experienced. This characteristic is clearly evident from Figure 2.10(a) and Figure 2.10(b). Going in the reverse direction (increasing the maximum coverage distance), a reduction in the overall cost is to be expected. However, this kind of trend is not evident in Figure 2.10 throughout. After a particular maximum coverage distance (40 for the non-referral case and 50 for referrals), no reduction in the overall cost is observed. It is so because when the requirement on the maximum coverage distance is relaxed (a higher value of the maximum coverage distance), the model will try to take cost advantage by establishing more lower-level facilities at a lower cost compared to the increase in the travel cost. This advantage can be continuously gained until the maximum distance travelled is not constrained by the permissible maximum coverage distance. It is this limit beyond which, if the maximum coverage distance increases, no change in the solution is observed as the best trade-off between the cost of establishing the facilities and the travel cost has already been sought. There is neither any change in the number of facilities of each type nor in the location and allocation of MTBs at this stage. It can be noticed from Figure 2.11 that shows no change beyond the maximum coverage distance of 40 for the non-referrals.




**(a)**




**(b)**

**Figure 2.10: Effect on the overall cost due to variation in maximum coverage distance (a) a non-referral visit and (b) for a referral visit**

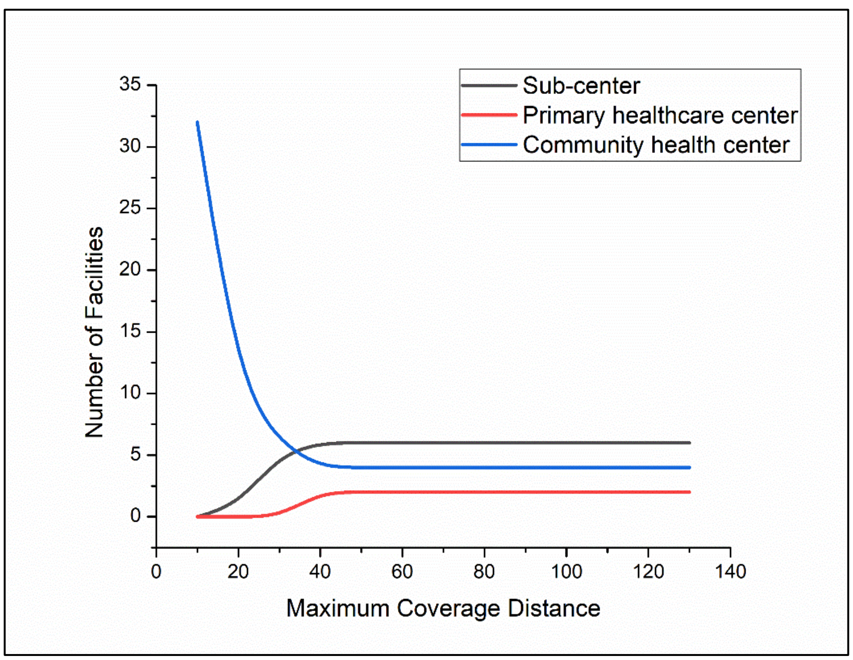



**Figure 2.11: Effect of maximum coverage distance for non-referral visits on the number of various facility types established**

### 2.7.2 Impact of change in the referral proportion

It is expected that the increase in the referral proportion may require an increase in the number of the higher level of facilities to be established. For example, in the case of configurations numbered as 1 and 9 in Table 2.13, the referral proportion for service type 1 to service type 2 $(\theta^{12})$ is going up from a value of 0.05 to 0.15. With the increase in the referral proportion for service type 2, it is natural to have a possible increase in the number of facilities of type 2 (PHCs) and possibly of type 3 (CHCs) due to the cascading effect resulting from referrals from service type 2 to service type 3. However, a reverse of the above is witnessed in Table 2.13. Even though the number of facilities of type 2 increases from 2 $(\theta^{12} = 0.05)$ to 3 $(\theta^{13} = 0.15)$, the cascading effect going to facilities of type 3 is observed to be missing but to the number of SCs (going down from 6 to 4). This reduction in the number of lower-level facilities with an increase in the higher-level facility type 2 can be observed in the case of configurations 3 and 6, but now coupled with the decrease in the number of health facility type 3 (CHC). In case the referral proportion of service type 1 to service type 3 $(\theta^{13})$ is increased from 0.01 (configuration 3) to 0.05 (configuration 5), it causes an increase in the number of the facilities of type 3 (CHCs) from 3 to 5 while causing a reduction in the number of lower-level facility of type 2 (PHCs) by 3. Configuration 8 in comparison to configuration 1 finds $\theta^{12}$ and $\theta^{23}$ getting doubled. It shows a 100% increase in referrals. Under this circumstance, the number of each of SCs and PHCs is brought down by 1 instead of remaining stationary. It is due to an increase in the number of CHCs from 4 to 5, and also because CHCs provides facilities for both service types 1 and 2. In the cases discussed so far, establishing an additional facility of higher-level has resulted in a reduction in the number of next lower-level facilities. The cascading effect, in fact, can go in either direction, forward and/or backward. This can be observed from configurations 1, 9, 10, 11, and 12. No change in the number of the various facility types is observed when $\theta^{12}$ is moved up from 0.05 to 0.10. When $\theta^{12}$ is increased to 0.15 (configuration number 9), the number of PHCs increases by one while bringing down the number of SCs by 2. A similar trend is observed going further to configurations 10 and 11. In the case of configuration 12, it is observed that the increase in $\theta^{12}$, now from 0.3 to 0.4, does not cause any change in the number of PHCs. But the number of CHCs goes up from 4 to 5, and the number of SCs comes down from 2 to zero.

The above analysis shows that the effect of referral proportion is not responded to by the model in a typical way, sometimes following our intuition and sometimes otherwise, all because of the highly combinatorial nature of the problem.

**Table 2.13: Effect of change in the referral proportion**

| **Configuration number** | $\theta^{12}$ | $\theta^{13}$ | $\theta^{23}$ | **Number of established** | | |
|---|---|---|---|---|---|---|
| | | | | **SC** | **PHC** | **CHC** |
| 1 | 0.05 | 0.03 | 0.05 | 6 | 2 | 4 |
| 2 | 0.10 | 0.03 | 0.05 | 6 | 2 | 4 |
| 3 | 0.10 | 0.01 | 0.05 | 5 | 4 | 3 |
| 4 | 0.10 | 0.03 | 0.05 | 6 | 2 | 4 |
| 5 | 0.10 | 0.05 | 0.05 | 5 | 1 | 5 |
| 6 | 0.10 | 0.03 | 0.01 | 5 | 4 | 3 |
| 7 | 0.10 | 0.03 | 0.05 | 6 | 2 | 4 |
| 8 | 0.10 | 0.03 | 0.1 | 5 | 1 | 5 |
| 9 | 0.15 | 0.03 | 0.05 | 4 | 3 | 4 |
| 10 | 0.20 | 0.03 | 0.05 | 3 | 4 | 4 |
| 11 | 0.30 | 0.03 | 0.05 | 2 | 5 | 4 |
| 12 | 0.40 | 0.03 | 0.05 | 0 | 5 | 5 |

### 2.7.3 Impact of change in the capacity of maternal healthcare facilities

The result of the sensitivity analysis with respect to the capacity available for each service type at different facility types has been presented in Table 2.14, Table 2.15 and Table 2.16. In all these tables, the base capacity value is kept at what was provided in Table 2.7. The impact of change in the capacity of a service type for a particular facility type has been analyzed without modifying the capacity of remaining facility types, be it of the same service type or not. The experimentation is carried out with variations both on the lower and the higher sides of the capacity values for problem number 14 (Section 2.6.1).

In Table 2.14, configurations numbered as 1 to 6 show variation in the capacity for service type 3, configurations 7 to 15 for service type 2, and configurations 16 to 23 for service type 1. Configurations 1 to 6 show that the number of CHCs to be established decreases with the increase in its capacity. It is what was expected. The total demand for service type 3 is 1035, including the cases of referrals. Configuration number 1 shows that this demand for service type 3 is completely met by 11 CHCs, each with a capacity of 100 for service type 3. While going further from configuration 2 to configuration 4, it is observed that the number of required CHCs decreases as the combined capacity available for service type 3 is good enough to handle the total demand for service type 3. For configuration 5, the past trend of reduced number discontinues, and the value of the number remains stationary. Going with the trend of the reduced number and taking 2 CHCs to be established will not serve the purpose as the combined available capacity of 2 X 500 = 1000 units will be less than the required demand of 1035. Going with 2 CHCs in configuration number 6, the demand of 1035 visits for service type 3 could have been handled as the total available capacity will be 1200. But the optimal solution is asking for the establishment of 3 CHCs. It is because of the restriction on the maximum coverage distance to be covered by an MTBs. While going from configuration 4 to 6, a small improvement in the objective function value is observed. It is because of the local adjustment of referral cases causing savings in the travel cost. Otherwise, a sizeable change in the objective function value is noticed whenever the number of facilities to be established changes. A similar trend can be noticed from the other configurations. It can be noticed that the objective function value for configuration number 14 is less by 20 units compared to that for configuration number 13 but has a value equal to that for configuration number 15. Increasing the capacity value for service type 2, from 1800 (configuration number 13) to 2000 (configuration number 14), helps in carrying out adjustment by locally accommodating referral cases to result in a lower objective function value. But increasing this capacity to 2200 (configuration number 15) does not help as the fullest advantage of the capacity enhancement had been exploited earlier itself. This characteristic can also be observed from configurations 21 to 23.

**Table 2.14: Effect of variation in the capacity of various service types in CHC**

| Configuration number | Service type 1 | Service type 2 | Service type 3 | Number of established | | | Objective function value |
|---|---|---|---|---|---|---|---|
| | | | | SCs | PHCs | CHCs | |
| 1 | 1500 | 1200 | 100 | 0 | 0 | 11 | 11135400 |
| 2 | 1500 | 1200 | 200 | 5 | 0 | 6 | 6696890 |
| 3 | 1500 | 1200 | 300 | 6 | 2 | 4 | 5797500 |
| 4 | 1500 | 1200 | 400 | 5 | 4 | 3 | 5697330 |
| 5 | 1500 | 1200 | 500 | 5 | 4 | 3 | 5696880 |
| 6 | 1500 | 1200 | 600 | 5 | 4 | 3 | 5696810 |
| 7 | 1500 | 600 | 300 | 2 | 5 | 4 | 6876730 |
| 8 | 1500 | 800 | 300 | 3 | 4 | 4 | 6491460 |
| 9 | 1500 | 1000 | 300 | 4 | 3 | 4 | 6103210 |
| 10 | 1500 | 1200 | 300 | 6 | 2 | 4 | 5797500 |
| 11 | 1500 | 1400 | 300 | 6 | 2 | 4 | 5788350 |
| 12 | 1500 | 1600 | 300 | 7 | 1 | 4 | 5399330 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 13 | 1500 | 1800 | 300 | 8 | 0 | 4 | 5025370 |
| 14 | 1500 | 2000 | 300 | 8 | 0 | 4 | 5025350 |
| 15 | 1500 | 2200 | 300 | 8 | 0 | 4 | 5025350 |
| 16 | 1000 | 1200 | 300 | 8 | 2 | 4 | 5990390 |
| 17 | 1200 | 1200 | 300 | 7 | 2 | 4 | 5891570 |
| 18 | 1500 | 1200 | 300 | 6 | 2 | 4 | 5797500 |
| 19 | 1800 | 1200 | 300 | 4 | 2 | 4 | 5621010 |
| 20 | 2000 | 1200 | 300 | 4 | 2 | 4 | 5617260 |
| 21 | 2200 | 1200 | 300 | 3 | 2 | 4 | 5530430 |
| 22 | 2400 | 1200 | 300 | 3 | 2 | 4 | 5530430 |
| 23 | 2600 | 1200 | 300 | 3 | 2 | 4 | 5530430 |

A general observation from Table 2.14 is that the increase in the capacity helps in reducing the overall facility establishment cost and/or travel cost to a certain extent. Going beyond a particular limit, the enhancement of the capacity will not pay any dividend. Similar trends can be observed in Table 2.15 and Table 2.16 related to the capacity of service types 1 and 2 for SCs and PHCs.

**Table 2.15: Effect of variation in the capacity of various service types in PHC**

| Configuration number | Service type 1 | Service type 2 | Number of established | | | Objective function value |
|---|---|---|---|---|---|---|
| | | | SCs | PHCs | CHCs | |
| 24 | 1200 | 600 | 3 | 3 | 4 | 6479090 |
| 25 | 1200 | 800 | 4 | 3 | 4 | 6095160 |
| 26 | 1200 | 1000 | 6 | 2 | 4 | 5797500 |
| 27 | 1200 | 1200 | 6 | 2 | 4 | 5795270 |
| 28 | 1200 | 1400 | 6 | 2 | 4 | 5793140 |
| 29 | 1200 | 1600 | 6 | 2 | 4 | 5792000 |
| 30 | 800 | 1000 | 6 | 2 | 4 | 5805450 |
| 31 | 1000 | 1000 | 6 | 2 | 4 | 5801220 |
| 32 | 1200 | 1000 | 6 | 2 | 4 | 5797500 |
| 33 | 1400 | 1000 | 5 | 2 | 4 | 5707390 |
| 34 | 1600 | 1000 | 5 | 2 | 4 | 5706040 |
| 35 | 2000 | 1000 | 4 | 2 | 4 | 5620520 |
| 36 | 2200 | 1000 | 4 | 2 | 4 | 5620520 |
| 37 | 2400 | 1000 | 4 | 2 | 4 | 5620520 |

**Table 2.16: Effect of variation in the capacity of various service types in SC**

| Configuration number | Service type 1 | Number of established SCs | PHCs | CHCs | Objective function value |
|---|---|---|---|---|---|
| 38 | 600 | 9 | 2 | 4 | 6088570 |
| 39 | 800 | 7 | 2 | 4 | 5893020 |
| 40 | 1000 | 6 | 2 | 4 | 5797500 |
| 41 | 1200 | 5 | 2 | 4 | 5707520 |
| 42 | 1400 | 4 | 2 | 4 | 5623260 |
| 43 | 1600 | 4 | 2 | 4 | 5621200 |

### 2.7.4 Impact of change in the fixed cost

The results of this analysis are summarized in Table 2.17. This table shows the results for a few variations in the fixed cost values for the sake of brevity, even though this analysis was carried out for many other combinations as well. Configuration number 11 is basically the example problem (Section 2.6.1) without any change in the cost data. Configurations 1 to 10 represent the cases with a lower fixed cost value, while 12-14 for the higher values. Going from configuration number 11 to 5 (decreasing fixed cost value) or going from 11 to 14 (increasing fixed cost value), no change in the travel cost is observed and also in the number of SCs, PHCs, and CHCs to be established. The difference is in the overall fixed cost because of the modified value of the fixed cost associated with the establishment of SCs, PHCs, and CHCs. In configuration number 4, a decrease in the total fixed costs has been witnessed due to the readjustment of the establishment of a number SCs and PHCs, with the number of PHCs going up from 2 to 4 and that of SCs going down from 6 to 4. While going from configuration 4 to configuration 3, it is found that the model establishes more number of SCs to help in cutting down the travel cost by taking advantage of quite reduced fixed cost associated with SCs. This trend continues even going to configuration number 2 where fixed costs were reduced by a factor of 5 compared to that for configuration 3. However, this trend does not continue while going to configuration number 1 from configuration number 2 where the fixed costs were reduced by a factor of 2. The numbers of higher-level facilities, CHCs and PHCs, increase sizeably, while the number of SCs comes down. It is so because the model now aims to cut down the travel costs sizeably on both non-referrals and referral visits.

**Table 2.17: Effect of variation in fixed cost**

| Configuration number | Fixed cost of | | | Number of established | | | Objective function value | | |
|---|---|---|---|---|---|---|---|---|---|
| | SCs | PHCs | CHCs | SCs | PHCs | CHCs | Total fixed cost | Travel cost | Total cost |
| 1 | 100 | 500 | 1000 | 9 | 25 | 13 | 26400 | 9753 | 36153 |
| 2 | 500 | 2500 | 5000 | 29 | 13 | 5 | 72000 | 40693 | 112693 |
| 3 | 1000 | 5000 | 10000 | 20 | 9 | 4 | 105000 | 68313 | 173313 |
| 4 | 10000 | 20000 | 30000 | 4 | 4 | 4 | 240000 | 166841 | 406841 |
| 5 | 10000 | 30000 | 50000 | 6 | 2 | 4 | 320000 | 197500 | 517500 |
| 6 | 10000 | 30000 | 70000 | 6 | 2 | 4 | 400000 | 197500 | 597500 |
| 7 | 10000 | 50000 | 70000 | 6 | 2 | 4 | 440000 | 197500 | 637500 |
| 8 | 10000 | 50000 | 100000 | 6 | 2 | 4 | 560000 | 197500 | 757500 |
| 9 | 20000 | 100000 | 200000 | 6 | 2 | 4 | 1120000 | 197500 | 1317500 |
| 10 | 50000 | 250000 | 500000 | 6 | 2 | 4 | 2800000 | 197500 | 2997500 |
| 11 | 100000 | 500000 | 1000000 | 6 | 2 | 4 | 5600000 | 197500 | 5797500 |
| 12 | 200000 | 1000000 | 2000000 | 6 | 2 | 4 | 11200000 | 197500 | 11397500 |
| 13 | 500000 | 2500000 | 5000000 | 6 | 2 | 4 | 28000000 | 197500 | 28197500 |
| 14 | 1000000 | 5000000 | 10000000 | 6 | 2 | 4 | 56000000 | 197500 | 56197500 |

## 2.8 Conclusions

This chapter provides a framework to plan and locate the Community Health Centres, Primary Healthcare Centres and Sub-centers in the right mix and specifically for the mothers-to-be (MTBs). Determination of the right location of these facilities in a wide geographical area in India is a big and difficult task. It is because the healthcare service is to be provided at the least cost, even from the perspective of its users. To handle such a problem, a hierarchical facility location-allocation mathematical programming model is proposed. Besides determining the number and location of maternal healthcare facilities, the model allocates MTBs to these facilities for their varied service requirements, including referrals. The formulation provides optimal results while minimizing the total cost incurred in visiting and establishing the facilities. The proposed mathematical model is NP-hard and thus is hard to solve. In order to reduce the computational burden, some improvisations have been suggested. The addition of valid inequalities to the proposed model is found to be quite useful. Another strategy for reducing the computational effort has been developed for solving the problem sequentially and each time of a much smaller size. This sequential approach produces much better solutions in the same amount of computational time. The effectiveness and computational efficiency of the sequential approach were also established by the statistical analysis.

Sensitivity analysis was carried out by varying the coverage distance, referral proportion, capacity of the facilities, and fixed cost of establishing the facilities. During the sensitivity analyses, it was found that the coverage distance and the capacity of the facilities play an important role in minimizing the overall cost but up to a certain extent. The change in the referral proportion impacts not only the number of higher-level facilities but also of lower-level facilities. The fixed cost was found to impact the mix of numbers of SCs, PHCs, and CHCs to be established. When the same becomes very high compared to the travel cost, it does not cause any impact.

The proposed formulation believes that exclusive maternal healthcare facilities are to be planned. In case the planner wish to include the capacity of the three service types related to maternal healthcare with the existing healthcare facilities, it can simply be done by fixing the corrsponding facility location variable with value as 1 for the corresponding available facility types. In this chapter, demand is considered to be deterministic, but in the real operational environment, the demand from a location to a healthcare facility may vary randomly from day to day, which may affect the quality of service. Additionally, due to capacity and coverage distance-related constraints, it may so happen the model will open a facility even for a small number of MTBs leading to a very high total fixed cost and the solution may have over-capacitated facilities.