8. Wind Directional Analysis

Wind direction is equally important characteristics of wind resources as wind speed for harnessing the wind energy potential at a given site. Accurate assessment of wind direction is essential prerequisite at the planning and development stage of the wind farm. The wind farm should be installed in such a zone as to get most frequent wind flow pattern in that zone and the overhead transmission lines should be laid in such a direction as to get maximum heat dissipation effect. The analysis of wind direction leads to estimation of most frequent wind flow direction that enables in estimation of desirable zone for the installation of the wind farms. To take the advantage of maximum heat dissipation effect the overhead transmission line should be installed perpendicular to the most frequent wind flow direction. As because winds blowing parallel to the transmission lines result in 60% lower convective heat loss than winds blowing perpendicular to the conductor **[233, 234]**. Therefore, the wind direction assessment helps in estimation of most frequent wind flow direction, the perpendicular to which helps in installation of the overhead transmission lines of the wind farm. **Carta, *et al.* [10]** discussed the importance of the wind direction in the installation of the wind turbine. However, most papers neglect the importance of wind direction along with the wind speed for the analysis of wind power potential. Perhaps, this is because of difficulties involved in the analyses, and more complexity involved in statistical modelling of wind direction compared to the wind speed. Besides wind speed, wind direction is also an important pillar in wind resources assessment for maximization of the wind power output.

The analysis of wind direction requires the use of particular statistical models, namely distributions on the circle. Indeed, a direction in the plane has no length, hence all observations can be considered to have length 1 and to lie on the same circle.

Razali, *et al.* [235] compared different circular distributions for wind direction analyses, namely the von Mises distribution, the generalized von Mises distribution and the wrapped Cauchy distribution (see Chapter 2 of **[236]** for a description of circular distributions). They stated that the von Mises distribution fits the data better compared to the other distributions. Thus, the researchers concluded that the 2-parameter von Mises distribution is the most suitable distribution for wind direction analysis. Based on conclusion **Razali, *et al.* [235]** later on researchers extend the use of this single distribution to a finite mixture distribution. **Carta, *et al.* [11]** uses finite mixture distribution of von Mises distribution (*mvM.pdf*) to represent the directional wind speed. They concluded that as the number of components (N) of the mixture distribution increases the value of R^2 increases. However, the variations in R^2 are not significant for N greater than six. **Masseran, *et al.* [13]** also uses the finite mixture of *vM.pdf*s to fit average hourly wind direction data for nine stations of Malaysia. They stated that the finite mixture of the *vM.pdf* with $N = 6$, was the best distribution to describe the wind direction distributions in Malaysia. Recently**, Kato, and Jones [237]** proposed a new 4-parameter distribution for circular data which is much more versatile than the von Mises distribution. It enjoys numerous good properties such as, for example, unimodality, simple parameter interpretability as location, concentration, skewness, and kurtosis parameters, a wide range of circular skewness and kurtosis, and straightforward parameter estimation methods. Moreover, it includes a number of attractive sub-models such as the wrapped Cauchy and cardioid distributions.

Above-mentioned literature survey clearly reveals that *vM.pdf*, *K-J.pdf*, and mixture *vM.pdf* have the potential to fit the wind direction data. However, their comparative assessment has rarely been done in any kinds of literature. Moreover, wind directional analysis for Indian climatology has rarely been studied.

In the Indian codal provision the importance of wind direction has been neglected, and the assessment of wind direction is absent from that code. Therefore, in this study, with an aim to determine the model that is best suitable for Indian subcontinent for wind directional analysis three locations namely Trivandrum, Ahmedabad and Calcutta have been taken for the comparative study of above-mentioned distributions (see Table 4.1). The wind direction data supplied is 3 hourly average. The wind direction is analyzed by conventional *vM.pdf*, *K-J.pdf*, and mixture von Mises ($N = 2$) distribution. The von Mises is a 2-parameter distribution, Kato-Jones is a 4-parameter distribution, and the mixture von Mises is a 5-parameter distribution. The wind direction also represented by the wind rose to evaluate zones of most frequent wind speed.

The directional data analyses are based on the geological definition of azimuths, which are cyclical in nature, with 0^0 and 360^0 representing the same direction, due north. Azimuths increase clockwise, with 90^0 representing east. In this study, the calm hours are not considered, as it might believe that during calm hours, the wind vanes could not measure the wind speed and wind direction accurately. Since the direction data were recorded with 16-point compass, the appropriate choice of class width is 22.5^0.

8.1 Mathematical Model

8.1.1 von Mises Distribution (vM.pdf)

If a random variable θ follows the von Mises distribution, Eq. (8.1) shows the expression for probability density function of von Mises distribution **[10, 13, 235, 238, 239]**:

$$vM(\theta;\kappa_j,\mu_j)=\frac{1}{2\pi I_0(\kappa_j)}\exp\left[\kappa_j\cos(\theta-\mu_j)\right] \tag{8.1}$$

$$0\le\theta\le 2\pi$$

where $\kappa \geq 0$ and $0 \leq \mu \leq 2\pi$ are parameters. The distribution is unimodal and is symmetrical about $\theta = \mu$. The parameter μ is the mean direction, and the parameter κ is the concentration parameter. Here $I_0(\kappa)$ is the modified Bessel function of the first kind of order zero. Eq. (8.2) shows the expression for modified Bessel function:

$$I_0(\kappa_j)=\frac{1}{\sqrt{2\pi}}\int_0^{2\pi}\exp[\kappa_j\cos\theta]d\theta$$

$$=\sum_{k=0}^{\infty}\frac{1}{(k!)^2}\left(\frac{\kappa_j}{2}\right)^{2k} \tag{8.2}$$

8.1.2 Mixture von Mises Distribution (mvM.pdf)

The *mvM.pdf* comprises of a weighted sum of *N* von Mises probability densities *vM* ($\theta;\kappa_j,\mu_j$). The probability density function of *mvM.pdf* is expressed as **[10, 13]**:

$$mvM(\theta;\kappa;\mu)=\sum_{j=1}^{N}\omega_j vM_j(\theta;\kappa,\mu) \tag{8.3}$$

The corresponding cumulative distribution function of *mvM* ($\theta;\kappa,\mu$) is given by:

$$MvM(\theta;\kappa_j,\mu_j,\omega_j)=\sum_{j=1}^{N}\frac{\omega_j}{2\pi I_0(\kappa_j)}\int_0^{\theta}\exp[\kappa_j\cos(\theta-\mu_j)]d\theta \tag{8.4}$$

The log likelihood of given distribution is expressed as:

$$LL = \sum_{j=1}^{N} \ln\{\omega_j vM_j(\theta;\kappa,\mu)\} \tag{8.5}$$

where ω_j is the j^{th} mixture weight, the summations of which is equal to one. The nonlinear programming technique has been used to solve the maximization of log-likelihood function. **Carta, *et al.* [10]** suggested technique used to guess the initial value of the parameters of N mixture distribution. First, the wind direction data are classified into N sectors, where N is same as that of the number of components in mixture distribution. For each sector, separate μ_j and κ_j are calculated by the equation given below, where the initial value of μ_j are:

$$\mu_j = \begin{cases} \arctan\left(\dfrac{\bar{s}_j}{\bar{c}_j}\right); \bar{s}_j \geq 0, \bar{c}_j > 0 \\ \dfrac{\pi}{2}; \bar{s}_j > 0, \bar{c}_j = 0 \\ \pi + \arctan\left[\dfrac{\bar{s}_j}{\bar{c}_j}\right]; \bar{c}_j < 0 \end{cases}$$

$$\mu_j = \begin{cases} \dfrac{3\pi}{2}; \bar{s}_j < 0, \bar{c}_j = 0 \\ \pi; \bar{s}_j = 0, \bar{c}_j = -1 \\ 2\pi + \arctan\left[\dfrac{\bar{s}_j}{\bar{c}_j}\right]; \bar{s}_j < 0, \bar{c}_j > 0 \end{cases} \tag{8.6}$$

where $\bar{s}_j$ and $\bar{c}_j$ are expressed as:

$$\bar{s}_j = \frac{\sum_{i=1}^{n_j} \sin\theta_i}{n_j}; \bar{c}_j = \frac{\sum_{i=1}^{n_j} \cos\theta_i}{n_j} \tag{8.7}$$

n_j is the number of directional wind speed data pertaining to sector j.

The coefficient κ_j is determined by the maximum likelihood solution of the equation:

$$A(\kappa_j) = \frac{I_1(\kappa_j)}{I_0(\kappa_j)} = \left[\bar{s}_j^{\,2} + \bar{c}_j^{\,2}\right]^{1/2} \tag{8.8}$$

$$I_1(\kappa_j) = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} \cos\theta \exp\left[\kappa_j \cos\theta\right] d\theta \;= \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+2)\Gamma(k+1)} \left(\frac{\kappa_j}{2}\right)^{2k+1} \tag{8.9}$$

where $A(\kappa_j)$ is the ratio of two Bessel functions, $I_0(\kappa_j)$ and $I_1(\kappa_j)$ are modified Bessel functions of the first kind of order 'zero' and 'one' respectively. However, solving the above equations is difficult. In this study, **Mardia [9]** suggested equations have been used to estimate the initial value of κ_j. Initially, one can calculate the mean vector length by:

$$\bar{R} = \left[\bar{s}_j^{\,2} + \bar{c}_j^{\,2}\right]^{1/2} \tag{8.10}$$

Then based on this $\bar{R}$, the initial values of κ_j are estimated as:

$$\begin{aligned}
&\textit{when} \\
&\bar{R} < 0.53 \\
&\kappa_j = 2\bar{R} + \bar{R}^3 + \frac{5}{6}\bar{R}^5 \\
&0.53 \le \bar{R} < 0.85 \\
&\kappa_j = -0.4 + 1.39\bar{R} + \frac{0.43}{(1-\bar{R})} \\
&\bar{R} \ge 0.85 \\
&\kappa_j = \frac{1}{\{2(1-\bar{R}) - (1-\bar{R})^2 - (1-\bar{R})^3\}}
\end{aligned} \tag{8.11}$$

8.1.3 Kato-Jones Distribution (K-J.pdf)

Kato, and Jones [237] suggested a four-parameter distribution for circular data that are unimodal and possess a wide range of skewness and kurtosis. The expression for the probability density function is given as:

$$g(\theta)=\frac{1}{2\pi}\left[1+2\gamma^2\frac{\gamma\cos(\theta-\mu)-\overline{\alpha_2}}{\gamma^2+\overline{\alpha_2}^2+\overline{\beta_2}^2-2\gamma\left\{\overline{\alpha_2}\cos(\theta-\mu)+\overline{\beta_2}\sin(\theta-\mu)\right\}}\right], \quad (8.12)$$

where $-\pi\le\theta<\pi$, $-\pi\le\mu<\pi$, $0\le\gamma<1$, and $\left(\overline{\alpha_2},\overline{\beta_2}\right)\ne(\gamma,0)$ satisfy $(\overline{\alpha}_2-\gamma^2)^2+\overline{\beta}_2^2\le\gamma^2(1-\gamma)^2$ The parameters μ , γ , $\overline{\alpha_2}$, and $\overline{\beta_2}$ are the location, concentration, kurtosis, and skewness parameters, respectively. The *K-J.pdf* is much more versatile as compared to the von Mises distribution, which is why it has been used here to model the wind direction; to the best of the authors knowledge, no other environmental research paper has used it before. The *MLM* method has been employed to estimate the parameters of this circular distribution.

8.2 Results and Discussion

For a complete harnessing of the wind energy potential, the wind direction needs to be taken into account along with the wind speed. The information about wind direction in compliance with wind speed not only enables drawing conclusions about the wind power potential at a particular location, but it also allows modelling the effective power delivery system.

Figures 8.1-8.3. show the histogram of wind direction distribution. From Figures 8.1-8.3, it has been observed that for Trivandrum the North-West direction has highest pdf followed by West direction, for Ahmedabad is South-West direction followed by West and North-West direction, and for Calcutta is South direction. The three different distributions namely von Mises, Kato-Jones and mixture von Mises distributions have been used to fit the wind direction histogram and are compared with each other. Among these distributions, the *vM.pdf* is the conventional distribution and is commonly being employed to fit win direction histogram. Table 8.1 shows the estimated statistical quantities of the *vM.*, *K-J.*, and *mvM.pdfs*. Table 8.2 shows their *GOF* of the circular distribution namely R^2, *RMSE*, and *AIC*. The R^2 and *RMSE* reveal how well the given distribution fit the observed histogram. On the other hand, *AIC* reveals the most suitable distribution among the three that can fit the wind direction histogram, higher value of R^2, and lower values of *RMSE* and *AIC* are desirable. As seen from Table 8.2 that *K-J.pdf* distribution has lowest *AIC* value followed by *mvM.pdf* and highest for *vM.pdf* for both Trivandrum and Ahmedabad. The highest value of R^2 confirms that the *K-J.pdf* performs better than the other two distribution to model the wind direction data for both Trivandrum and Ahmedabad. However, for Calcutta, the *mvM.pdf* has lowest *AIC* followed by *vM.pdf* and *K-J.pdf*. The R^2 value confirms the conclusion made by *AIC* value. From above observation, it can be concluded that the *K-J.pdf* performs better for the Ahmedabad and Trivandrum, and less good for the

Calcutta station. The *K-J.pdf* thus proves to be a suitable (and even preferable) alternative to the classical von Mises distribution when the goal is to estimate the wind direction. Moreover, the clear nature of its parameters (location, scale, skewness, kurtosis) makes the obtained parameter estimates easy to interpret.

Table 8.1: Estimated parameters for circular probability density function.

Station	**von Mises Distribution**		**2- von Mises Distribution**				
	κ	$\bar{\theta}$ **(degree)**	w	κ_1	$\bar{\theta}_1$ **(degree)**	κ_2	$\bar{\theta}_2$ **(degree)**
Tvm.	1.754	-55.08	0.208	0.819	0.020	2.561	-61.26
Ahmed.	0.6320	-84.64	0.554	0.713	0.00	2.341	-118.43
Calcutta	0.592	167.72	0.549	0.498	-0.1500	5.935	178.1

Station	**Kato-Jones Distribution**			
	μ (degree)	**γ**	$\bar{\alpha}_2$	$\bar{\beta}_2$
Tvm.	-54.408	0.6578	0.3141	-0.0166
Ahmed.	-84.64	0.3012	0.0526	-0.1335
Calcutta	167.72	0.2838	0.2624	0.0908

Table 8.2: Measure of goodness of fit for circular probability density functions.

Station	von Mises distribution			2- von Mises distribution		
	R^2(%)	*RMSE*	*AIC*	*R^2(%)*	*RMSE*	*AIC*
Trivandrum	97.74	0.051	2.138 x10^5	97.95	0.524	2.1274 x10^5
Ahmedabad	98.95	0.039	3.617 x10^5	99.07	0.037	3.5745 x10^5
Calcutta	97.33	0.058	2.652 x10^5	98.69	0.046	2.4848 x10^5
	Kato-Jones distribution					
	R^2(%)	*RMSE*	*AIC*			
Trivandrum	98.42	0.0520	2.1197 x10^5			
Ahmedabad	99.05	0.0314	3.553 x10^5			
Calcutta	74.77	0.1627	2.752 x10^5			

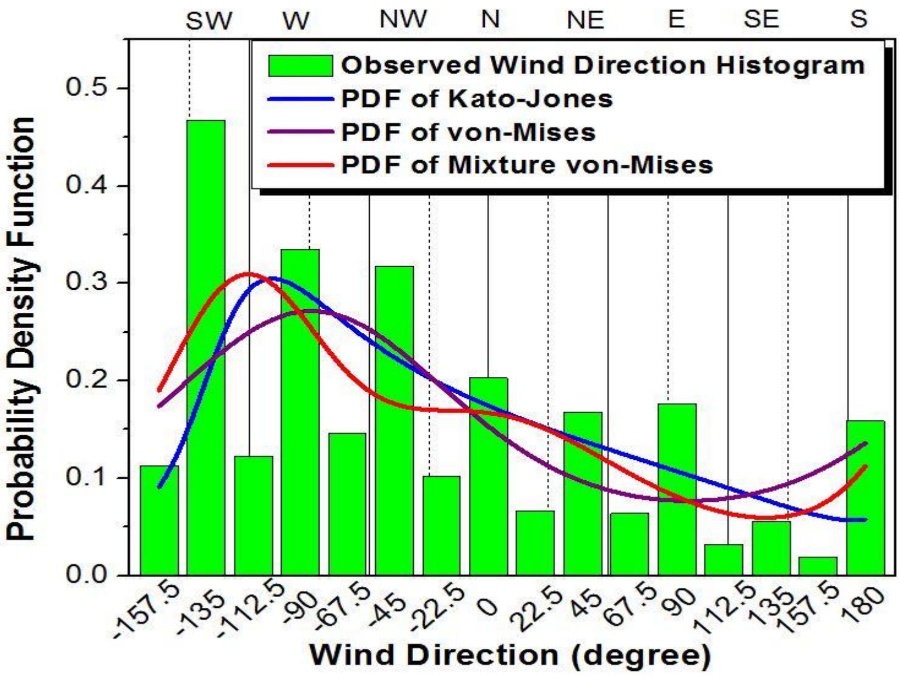

Figure 8.1: Histogram of wind direction together with the corresponding von Mises and Kato-Jones densities, at the Ahmedabad station.

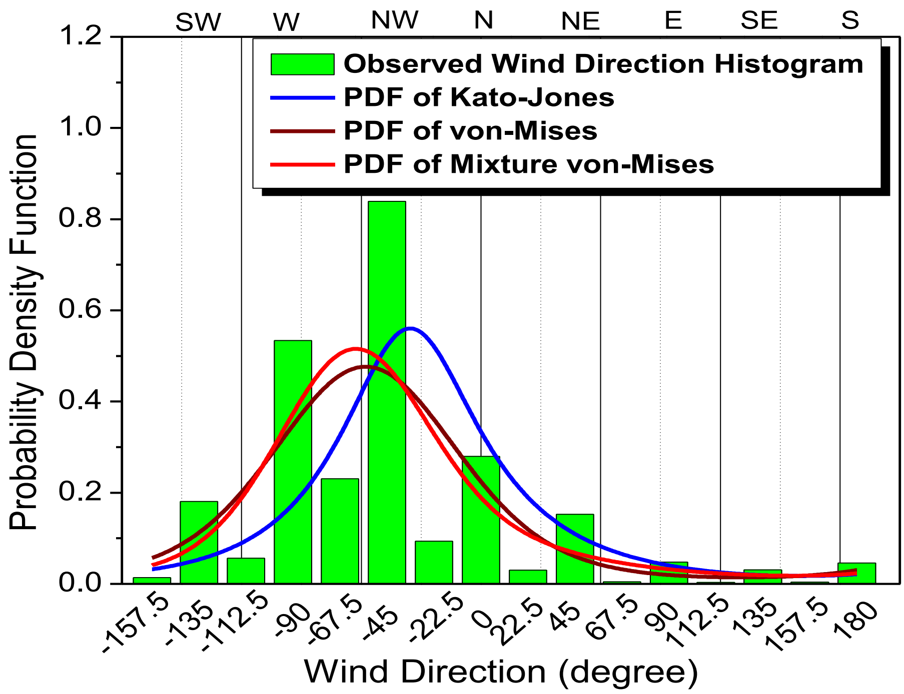

Figure 8.2: Histogram of wind direction together with the corresponding von Mises and Kato-Jones densities, at the Trivandrum station.

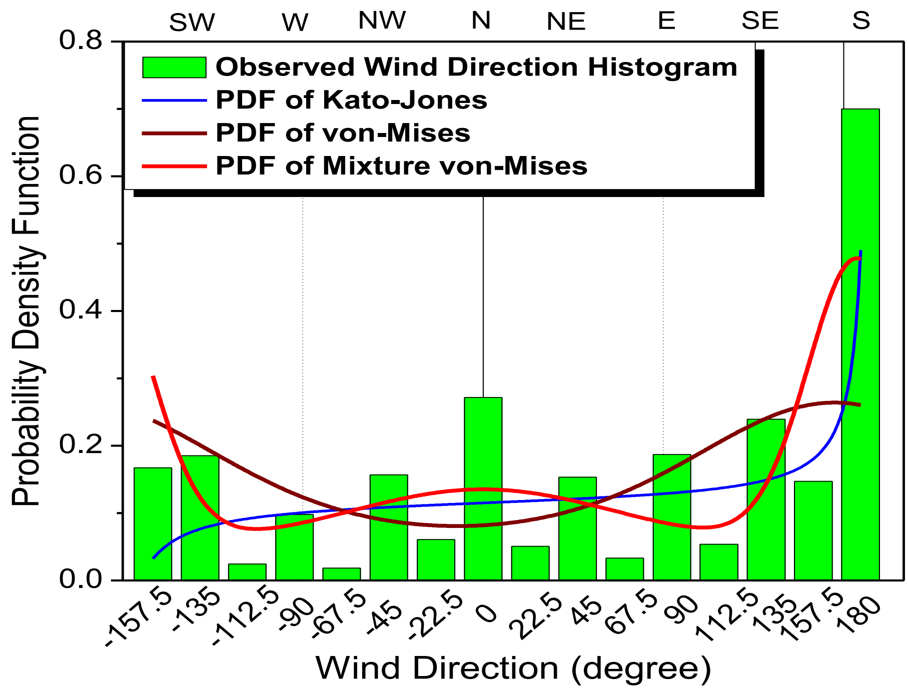

Figure 8.3: Histogram of wind direction together with the corresponding von Mises and Kato-Jones densities, at the Calcutta station.

Figure 8.4 demonstrates the physical map of India with wind rose located at the three stations. The wind roses at each location truly depict the zones of most frequent wind speed according to the location-specific physical terrain. The longest tail at each location clearly shows the local preferable alignment of wind turbines. The majority of the strong wind comes along with the onset of the southwest monsoon in India. Trivandrum and Ahmedabad are situated on the southwestern and western parts of India, respectively. The Indian Ocean and the Arabian Sea are on the southwest side of these stations. The southwest winds flowing over the ocean strike these two stations without much of land obstructions. Therefore, the probability of getting high wind speeds on the western part of India is more pronounced.

For Ahmedabad station, 15% of the wind comes from South-West direction, which is due to the Arabian Sea located at South-West of Ahmedabad followed by West with 12.0% and North-West with 11.5%. Therefore, the wind power grid should be aligned in South-west zone, and transmission lines are constructed in the axis of Northwest-Southeast direction. For Trivandrum station, 28% of wind comes from North-West direction because of the Indian Ocean and Trivandrum is surrounded by sea all around except an arc from North till East clockwise direction. The reason for north-west being the dominant direction for Trivandrum is that hills of Western Ghats (Ponmudi) is located at the North-East of Trivandrum and are parallel to the Arabian sea. These hills obstruct the wind flow from the Arabian sea and as a consequence, north-west is the dominant direction for Trivandrum. Therefore, the wind power grid should be in North-West zone of Trivandrum station, and transmission lines be along Northeast-Southwest direction. The above claims could be justified by already installed wind farm by the Government of India at several locations. These are Lamda (21.90N, 69.28E) and Amreli (21.62N, 71.23E) which are two locations situated at the South-West of Ahmedabad station. Muppandal (8.15N, 77.32E) is the world’s second-largest onshore wind farm situated in the direction of South-East zone of Trivandrum station. Calcutta is situated on the eastern coastal side of India.

The southwesterly wind has to cover a long ground over Indian peninsula, full of obstructions before it reaches Calcutta. As Calcutta is situated near the Bay of Bengal, chances of wind flow from the Bay of Bengal are rather more pronounced. However, the probability of getting relatively lower wind speeds is more prominent in the eastern region of India as compared to the western region. Since Calcutta is situated to the north of the Bay of Bengal, the prominent direction of the wind flow is southwardly for Calcutta station. For Calcutta station, the most prominent wind direction is South with 24% of wind coming from this direction. To avail maximum wind energy production the tower of the wind farm should be erected in the south zone, and transmission lines construction be aligned in the East-West direction. The West Bengal Renewable Energy Department is also planning to install two wind farm at Sagar Island and Fraserganj located at the south of Calcutta. Another conclusion can be drawn while observing the plot of wind rose which shows the most probable wind direction for Trivandrum is North-West followed by West direction. A similar observation has been found from the wind direction histogram which clearly reveals the North-West direction has highest pdf followed by West direction. Thus, wind direction histogram verifies the conclusion made by the wind rose diagram. For other two stations, a similar conclusion can be drawn, i.e., for Ahmedabad, the most probable wind direction is South-West followed by West and North-West direction, and for Calcutta is South direction. Thus the estimated parameters can directly be used in deciding the most probable wind direction for the installation of the wind farm and overhead transmission lines.

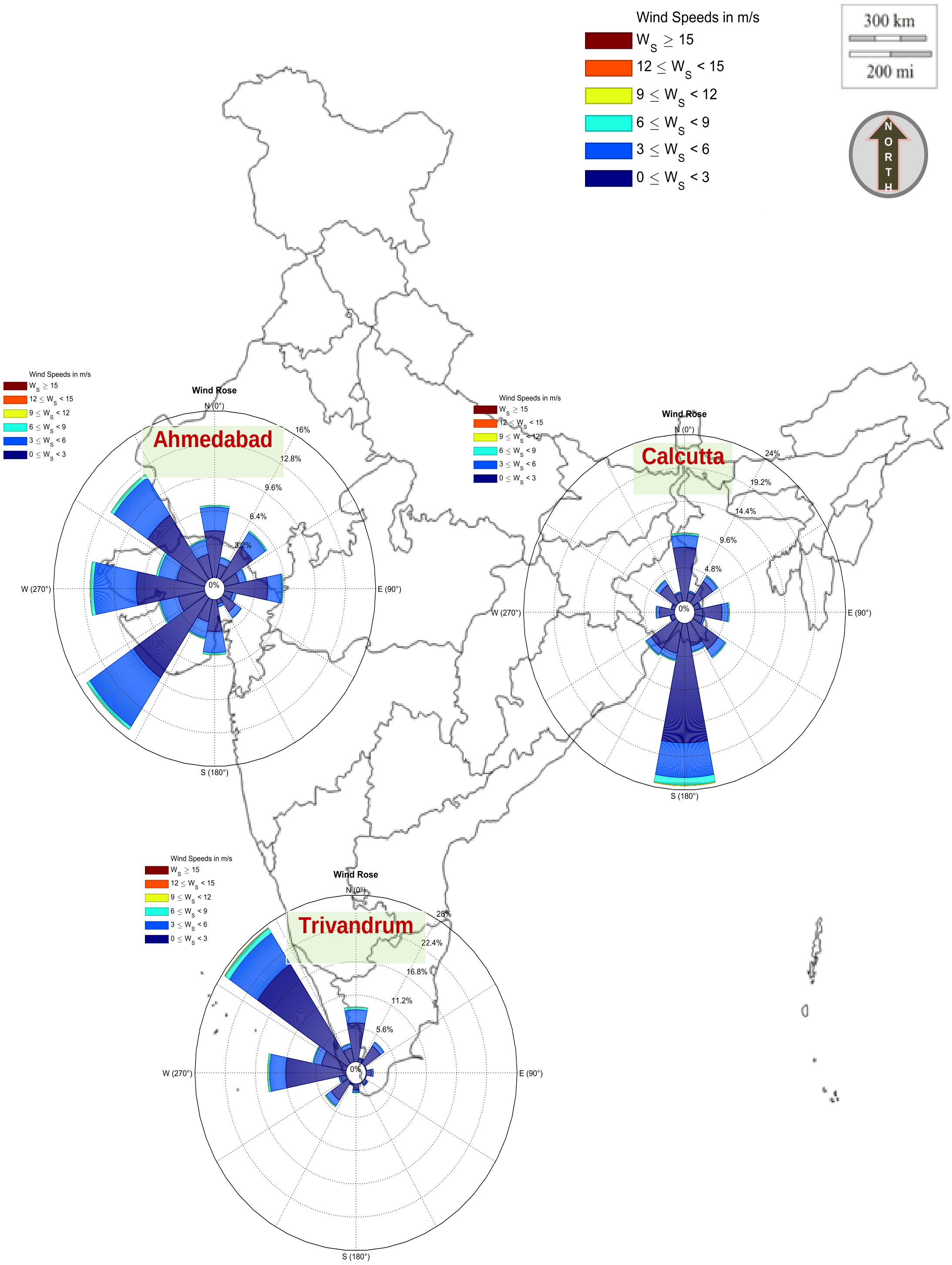

Figure 8.4: Map of India with wind roses at three different stations in (a) Trivandrum, (b) Ahmedabad, and (c) Calcutta.

8.3 Summary

1. For wind direction analysis the new *K-J.pdf* is a viable (and often better) alternative to the classical *vM.pdf* for describing the wind direction.
2. The southwest monsoon plays a dominant role in deciding the installation of the wind farm in India for generating electricity. The leading path for installation of the wind farm for Ahmedabad is the southwest direction; for Trivandrum, it is the northwest direction; and for Calcutta, it is the south direction. Among these three stations, taken as a reference at three onshore corners of India, Ahmedabad appeared as the most suitable site for installation of the wind farm followed by Trivandrum and Calcutta.
3. Both wind rose and wind direction histogram, confirm the same conclusion made at point 2, thereby, parameterizing the wind direction data can turn out to be a fruitful option for wind directional analysis.