# Appendix A

## Modified Maximum Likelihood Method (Expectation Maximisation Algorithm)

The likelihood of obtaining observation which falls in *n* independent classes is given as:

$$L(v_1,.....,v_n;k,s) = f(v_1;k,s)^{P(v_1)} f(v_2;k,s)^{P(v_2)} .....f(v_n;k,s)^{P(v_n)} = \prod_{i=1}^{n} f(v_i;k,s)^{P(v_i)} \quad \text{(A.1)}$$

*L* is the likelihood function of wind speed data in frequency distribution format, $v_1$, $v_2$, ….,$v_n$ are the wind speed central to the each bins. The maximum likelihood estimator of *k*, and *s* will then be the particular values of *k* and *s* so that *L* or the probability of obtaining data within the classes will be maximised. Due to the multiplicative nature of *L* it is generally more convenient to maximise the logarithm of the likelihood function instead of simple likelihood function. The log-likelihood of the function can be written as:

$$ln\ L\ (v_1,\ \ldots\ldots,\ v_n;k,\ s) = \sum_{i=1}^{n} P(v_i)\ln f(v_i;k,s) \quad \text{(A.2)}$$

The maximization of the log-likelihood can be obtained by differentiating the log-likelihood function with respect to *k* and *s* and equate the expression to zero. The expressions for maximization of $lnL(v_1,v_2,\ldots\ldots,v_n;k,s)$ for *W.pdf* is given as

$$\frac{\partial \ln L(v_1,...,v_n;k,s)}{\partial s} = -\frac{k}{s}P(v>0) + \frac{k}{s^{k+1}}\sum_{i=1}^{n} P(v_i)v_i^{\,k} = 0 \quad \text{(A.3)}$$

Solving Eq. (A.3) yield the expression for *s* which has further been utilize in Eq. (A.4) to calculate the expression for *k*.

$$\frac{\partial \ln L(v_1,...,v_n;k,s)}{\partial k} = \frac{P(v>0)}{k} + \sum_{i=1}^{n} P(v_i)\ln(v_i) - P(v>0)\cdot\frac{\sum_{i=1}^{n} P(v_i)v_i^{k}\ln(v_i)}{\sum_{i=1}^{n} P(v_i)v_i^{k}} = 0 \qquad \text{(A.4)}$$

The above expression is difficult to solve; therefore, iteration technique namely Newton-Raphson has been adopted. The formulation of Newton-Raphson and the procedure adopted to calculate the expression for $k$ is given below:

$$k_{j+1} = k_j - \frac{f(k_j)}{f'(k_j)} \qquad \text{(A.5)}$$

where, $k_{j+1}$ is the value of $k$ in $(j+1)^{th}$ iterative step and $k_j$ is the value of $k$ in $j^{th}$ iterative step. From Eq. (A.4), $f(k_j)$ can be written as:

$$f(k_j) = \frac{P(v>0)}{k_j} + \sum_{i=1}^{n}\ln(v_i)P(v_i) - P(v>0)\cdot\frac{\sum_{i=1}^{n} v_i^{k_j}\ln(v_i)P(v_i)}{\sum_{i=1}^{n} v_i^{k_j}P(v_i)} \qquad \text{(A.6)}$$

Since $f'(k_j) = \frac{df(k_j)}{dk_j}$, so $f'(k_j)$ can be written as:

$$f'(k_j) = -\frac{P(v>0)}{k_j^{2}} - P(v>0)\cdot\frac{\sum_{i=1}^{n} v_i^{k_j}P(v_i)\sum_{i=1}^{n} v_i^{k_j}(\ln(v_i))^2 P(v_i) - \left[\sum_{i=1}^{n} v_i^{k_j}\ln(v_i)P(v_i)\right]^2}{\left[\sum_{i=1}^{n} v_i^{k_j}P(v_i)\right]^2} \qquad \text{(A.7)}$$

Eqs. (A.6) and (A.7) can be used in Eq, (A.5) to estimate the expression for $k_{j+1}$. The expression for $k_{j+1}$ is given as:

$$k_{j+1} = k_j + \frac{\dfrac{1}{k_j} + \dfrac{1}{P(v>0)}\displaystyle\sum_{i=1}^{n}\ln(v_i)P(v_i) - \dfrac{\displaystyle\sum_{i=1}^{n} v_i^{k_j}\ln(v_i)P(v_i)}{\displaystyle\sum_{i=1}^{n} v_i^{k_j}P(v_i)}}{\dfrac{1}{k_j^{2}} + \dfrac{\displaystyle\sum_{i=1}^{n} v_i^{k_j}P(v_i)\sum_{i=1}^{n} v_i^{k_j}(\ln(v_i))^2 P(v_i) - \left[\sum_{i=1}^{n} v_i^{k_j}\ln(v_i)P(v_i)\right]^2}{\left[\displaystyle\sum_{i=1}^{n} v_i^{k_j}P(v_i)\right]^2}} \quad \text{(A.9)}$$

Starting point play an important role in iteration technique. In the present scope of study, $k = 2$ is selected as a suitable initial guess which is used in Rayleigh distribution as because Rayleigh distribution is a special case of *W.pdf*. With this initial guess the final value of $k$ can be found iteratively from Eq. (A.9). The solution will converge to the true value with less number of iteration. The Maximum Likelihood Method also shares the same procedure for calculating the Weibull parameters. Slight modification in Eq. (A.9) ($P(v_i) = 1$, and $P(v > 0) = n$) yield the expression for $k_{j+1}$for Maximum Likelihood Method.

## Appendix B

For the derivation of the analytical expressions of Capacity Factor (*CF*), initially, as per the definition of *CF*, it is the ratio of average electrical power output to the rated power output. Using Eqs. (1.3) and (1.5) to represents the initial expression for *CF*. The initial expression for *CF* is given as:

$$CF = \frac{P_{avg}(v)}{P_{rated}} = \int_{V_c}^{V_r} P_n(v) f(v) dv + \int_{V_r}^{V_f} f(v) dv \quad \text{(B.1)}$$

In the present scope of study, the expression for normalized power is given as **[27, 64]**:

$$P_n(v) = \frac{v^3}{V_r^3} \quad \text{(B.2)}$$

In the present scope of study, the Weibull probability density function has been used for the analytical estimation of *CF*. The expression after incorporating normalised power Eq. (B.1) and pdf of *W.pdf* becomes:

$$CF = \frac{1}{V_r^3} \int_{V_c}^{V_r} v^3 f(v) dv + \int_{V_r}^{V_f} f(v) dv \quad \text{(B.3)}$$

$$CF = \frac{1}{V_r^3} \int_{V_c}^{V_r} v^3 \left(\frac{k}{s}\right)\left(\frac{v}{s}\right)^{k-1} \exp\left[-\left(\frac{v}{s}\right)^k\right] dv + \int_{V_r}^{V_f} \left(\frac{k}{s}\right)\left(\frac{v}{s}\right)^{k-1} \exp\left[-\left(\frac{v}{s}\right)^k\right] dv \quad \text{(B.4)}$$

Let

$$\left(\frac{v}{s}\right)^k = x \ \text{ or } \ v = sx^{1/k} \quad \text{(B.5)}$$

In its differential form Eq. (B.5) is given as:

$$\left(\frac{k}{s}\right)\left(\frac{v}{s}\right)^{k-1} dv = dx \tag{B.6}$$

Incorporating Eqs. (B.5) and (B.6) into Eq. (B.4) becomes

$$CF = \underbrace{\frac{s^3}{V_r^3} \int_{\left(\frac{V_c}{s}\right)^k}^{\left(\frac{V_r}{s}\right)^k} \left(x^{3/k}\right) \exp[-x]\, dx}_{I_1} + \underbrace{\int_{\left(\frac{V_r}{s}\right)^k}^{\left(\frac{V_f}{s}\right)^k} \exp[-x]\, dx}_{I_2} \tag{B.7}$$

The above expression resembles the lower incomplete Gamma function, whose expression are given as:

$$\begin{aligned} \gamma(c,x) &= \int_0^x t^{c-1} \exp(-t)\, dt \\ \gamma(c+1,x) &= s\gamma(c,x) - x^c \exp(-x) \end{aligned} \tag{B.8}$$

Solving Eq. (B.7) yields $I_1$ and $I_2$ are given as:

$$I_1 = \frac{3}{k}\left[\gamma\left(\frac{3}{k},\left(\frac{V_r}{s}\right)^k\right)\right] - \left[\left(\frac{V_r}{s}\right)^{k\cdot\frac{3}{k}} \exp\left(-\left(\frac{V_r}{s}\right)^k\right)\right] - \frac{3}{k}\left[\gamma\left(\frac{3}{k},\left(\frac{V_c}{s}\right)^k\right)\right] + \left[\left(\frac{V_c}{s}\right)^{k\cdot\frac{3}{k}} \exp\left(-\left(\frac{V_c}{s}\right)^k\right)\right]$$

$$I_2 = \exp\left(-\left(\frac{V_r}{s}\right)^k\right) - \exp\left(-\left(\frac{V_f}{s}\right)^k\right)$$

The final expression after incorporating the expression for $I_1$ and $I_2$ is given as:

$$CF = \left(\frac{V_c}{V_r}\right)^3 \exp\left(-\left(\frac{V_c}{s}\right)^k\right) + \frac{3}{k\left(\frac{V_r}{s}\right)^3}\left[\gamma\left(\left(\frac{V_r}{s}\right)^k, \frac{3}{k}\right) - \gamma\left(\left(\frac{V_c}{s}\right)^k, \frac{3}{k}\right)\right] - \exp\left(-\left(\frac{V_f}{s}\right)^k\right)$$

# Appendix C

Let mean probable design life of the structures be $N$

Exceedance Probability of the structure in 1 year = $1/N$

Non-Exceedance Probability of the structure in 1 year ($P$) = $1-1/N$

Non-Exceedance Probability of the structure in $N$ years = $(1-1/N)^N$

Exceedance Probability of the structure in $N$ years ($P_N$) = $1-(1-1/N)^N$

Exceedance Probability of the structure in 1000 years ($P_{1000}$) = $1-(1-1/1000)^{1000}$ = 0.6323

According to Indian standard **[179]**,

$$k_1 = \frac{X_{N,P_N}}{X_{50,0.63}} = \frac{A - B\left[\ln\left\{-\frac{1}{N}\ln\left(1-P_N\right)\right\}\right]}{A+4B} \quad \text{(C.1)}$$

where $A$ and $B$ are constant given in Indian standard **[179]**

*Table C.1. The calculated risk factors for 1000 years return period for three stations.*

| **Stations** | **Trivandrum** | **Ahmedabad** | **Calcutta** |
|---|---|---|---|
| $\boldsymbol{k_1}$ | 1.291 | 1.327 | 1.367 |
| $\boldsymbol{V_b}$ **(*m/s*)** | 39 | 44 | 50 |
| $\boldsymbol{V_d}$ **(*m/s*)** | 50.35 | 58.40 | 68.35 |