Chapter 6

Overall conclusion & scope for the future work

6.1 Conclusion

In this section, the entire research work has been summarized. This thesis includes four different problems in which three are related to the non-classical phase change problems and the last problem is associated with diffusion logistic population model governing the processes of spreading and vanishing of the species. All these problems involve the parabolic partial differential equations in a domain in which a part of boundaries moves as time proceeds. Due to the presence of this moving boundary, these problems are always of non-linear nature, and the exact solutions to these problems are possible only for limited cases. Therefore, several approximate and numerical solutions have been utilized to solve these problems, like heat balance integral techniques, homotopy perturbation method, Adomian decomposition methods, finite element schemes, finite element methods, etc.

The main goal of this thesis is to discuss some one-dimensional non-classical mathematical models of the phase change problems and a diffusion logistic population model including moving interface (moving boundary problems). These models are solved successfully by different techniques, viz. finite difference method, heat balance integral techniques, and operational matrix method based on Genocchi polynomials and the collocation scheme. The accuracy of each used technique is discussed in the chapters of this thesis. The analytical solutions to the proposed models are also established for some specific cases. Based on the solution of the model, the dependence of parameters on the movement of the moving interface and the processes are shown in each chapter through graphs and tables. The summary of some of the important finding and conclusions from the whole thesis is given below:

- Chapter 1 introduced the thesis with literature survey and the schemes used. In this chapter, it is shown that how the moving boundary problems related to the phase change problems evolve and how the researchers established their solutions by using different kinds of approximate and numerical methods.

- In chapter 2, a mathematical model of a melting problem with moving phase change material and size-dependent thermal conductivity is discussed under a periodic boundary condition. The finite difference scheme is successfully applied to present the numerical solution of this model. The accuracy, consistency and stability of the proposed solution/scheme are also analyzed. The effects of size-dependent thermal conductivity and moving phase change material on the melting process are also investigated in this chapter.

- Chapter 3 presents a model of a population logistic diffusion model with fractional order space derivative and density-dependent dispersal rate. This problem is solved by the finite difference scheme to present the effect of the density-dependent dispersal rate and order of the space-fractional derivative are analyzed for the population density and expanding front.

- The chapter 4 presents a time-fractional melting problem with mixed boundary condition and variable thermal conductivity, and its solution is obtained by using the heat balance integral method for quadratic and exponential temperature profiles. It is seen that the solution of the problem obtained by quadratic and exponential approximations are nearly equal for the fractional case, and the melting process becomes fast as we increase the fractional order α after a short time interval but the reverse trend is seen initially.

- chapter 5 investigates a mathematical model of a non-classical Stefan problem that includes the convective boundary condition, variable latent heat and space-dependent thermal conductivity. The collocation method with the aid of the operational matrix of Genocchi polynomials is applied to solve the proposed problem. This solution is compared with finite difference method, and it is observed that the proposed algorithm is a simple, reliable and accurate scheme to solve the non-classical Stefan problems.

6.2 Scope for future study

From the science and industrial points of view, the moving boundary problems involving phase change processes are an exciting and important area of research, and looking into future, the following insights are the motivation to continue the research work in this area:

- Mathematical formulations: A mathematical model of a physical problem helps to understand how to study and control the physical system. The mathematical models related to the various processes are being formulated under some specific assumptions. Therefore, the field of mathematical modelling is constantly evolving, and new modifications in the models are being made to address various real physical situations. In this direction, the researchers can improve the existing models related to the phase change processes or any other moving boundary problems to present more reliable and accurate models of the actual physical situations.

- Computation: In other part of the research, an efficient and accurate algorithm is needed to solve the complicated phase change problems or moving boundary problems. In this direction, several techniques have been developed like finite difference method, finite element method, integral methods, enthalpy schemes, heat balance integral method, etc. Still, there is the scope to establish more accurate, efficient and simple algorithm to solve these nonlinear problems. Recently, various machine learning intelligence methods, particularly, Artificial Neural Networks (ANN) are being used to solve initial and boundary value problems. The researcher can try to solve the moving boundary problems by applying the approach of Neural Network.

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