PREFACE

Algebraic graph theory is the study of the interplay between algebra (both abstract structure as well as linear algebra) and graph theory. On one hand, various concepts of abstract algebra have facilitated the construction of graphs and are explored as tools in computer science, while conversely, graph theory has also helped to characterize certain algebraic properties of abstract algebraic structures. Over the last decade, there has been an increased contribution toward the extensive investigation of graphs obtained from groups.

In this thesis, we highlight the rich interplay between the two topics viz groups and graphs obtained from finite groups, particularly the Complement of a conjugate graph and Superpower graphs. We begin by introducing all the required notions and definitions from group theory and graph theory, followed by a brief literature survey of results that motivated this thesis.

It is well established that multi-partite graphs have their own significance in graph theory. Joseph Varghese [1] has proved the necessary and sufficient condition for a complete multi-partite graph to be Hamiltonian. As a first work, we extend Joseph Varghese's results by studying the Hamiltonian and Eulerian properties of a graph generated with the group-theoretic concept of conjugacy relation on a finite group, referred to as the complement of a conjugate graph $\overline{C(G)}$. We also explore the other variations of the Hamiltonian property, namely, 1-Hamiltonian, pancyclicity, etc. Further, we present several structural characterizations of the considered graph class.

Next, we focus on the superpower graphs, $S(G)$, defined on finite Abelian groups. We begin by first characterizing the structure of such graphs and then we study the relationship between the structure of tensor product of superpower graphs and the superpower graph of the direct product of underlying finite Abelian groups. Also, we find the tight bounds for the vertex connectivity of $S(G)$ and identify those groups for which the obtained bounds are attained by $S(G)$. In addition, we also establish the conditions for this class of graphs to be Hamiltonian along with some structural properties such as maximal dominating set, 1-Hamiltonian, Hamiltonian-connected, pancyclic and panconnected to name a few.

Among the class of non-Abelian groups, we first target the class of the dihedral and dicyclic groups to explore the properties of their superpower graphs, namely $S(D_{2n})$ and $S(T_{4n})$. It is well-known that these groups hold a fundamental place in group theory and they also appear as a subgroup of many groups. Our motive is to study the structural properties of the superpower graph of these groups and see how they differ from their corresponding power graphs. In this process, we compute sharp bounds for the vertex connectivity of $S(D_{2n})$ and $S(T_{4n})$ and identify the value of n for which the superpower graphs of respective groups attains the bound. In another attempt, we also obtain bounds for the edge connectivity of $S(D_{2n})$ and compute the edge connectivity of $S(D_{2n})$ for some special values of n through the minimum degree $S(D_{2n})$. We also investigate and characterize Hamiltonian-like properties for the groups and their superpower graphs under consideration.

Next, we discuss the structural characterizations of superpower graph $S(G)$ defined on any finite group G. In particular, explore the properties such as the perfectness, Eulerian graph and separating sets. We also discuss the order graph and comparability graph to their connection with the superpower graph. Next, we focus on generalizing the connectivity and Hamiltonian-like results for superpower graphs. To avoid repetition we will target on finite non-Abelian groups, particularly, those groups having an element of exponent order. We first begin by finding the dominating sets for this class and then we structurally characterizing this collection of superpower graphs. We establish tight bounds for vertex connectivity and identify those groups and respective superpower graphs that attain the obtained bounds. Further, we also explore various Hamiltonian-like properties for this collection of superpower graphs.

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