MATHEMATICAL MODELING ON VARIOUS STRUCTURES UNDER COUPLED DYNAMICAL THERMOELASTICITY

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Doctor of Philosophy

by

Bhagwan Singh

DEPARTMENT OF MATHEMATICAL SCIENCES
INDIAN INSTITUTE OF TECHNOLOGY
(BANARAS HINDU UNIVERSITY)
VARANASI - 221005
INDIA

Roll No. 18121011

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CHAPTER 7

SUMMARY OF THE THESIS AND SCOPE FOR FURTHER WORK

7.1 Summary

The coupled theory of thermoelasticity recognizes that changes in temperature affect the state of strain and stress of deformable bodies and vice versa. Hence, the coupled thermoelasticity theory aims to overcome the drawback in the classical "uncoupled theory of thermoelasticity" that the elastic changes have no effect on the temperature and vice versa. A wide range of applications have led to its becoming of an integral part of science, including machine structures, aircraft, building design, nuclear engineering, micro- and nano-electromechanical resonator systems, and many others. The present Thesis examines some of the theoretical aspects of the recently introduced Moore-Gibson-Thompson generalized thermoelasticity theory. As part of this thesis, the Euler-Bernoulli beam, Timoshenko beam, and Kirchhoff plate theories are employed to examine the thermoelastic damping, vibration analysis and bending characteristics of micro and nano resonators within the context of classical and nonclassical continuum theories as well as the Moore-Gibson-Thompson (MGT) thermoelasticity theory. The major findings out of the present work can be summarized as follows:

Chapter 2 deals with the Galerkin-type representation of solution for the system of equations of motion in the context of the thermoelasticity theory based on the

Moore-Gibson-Thompson heat conduction model as proposed by Quintanilla (2019). A theorem is established to obtain some theoretical analysis of MGT theory in terms of elementary functions. In addition, a theorem that provides the Galerkin-type solution of equations for steady-state oscillations is also established by assuming complex-valued periodic functions. Lastly, General solutions are achieved for the system of homogeneous equations of steady oscillations by ignoring external body forces and external heat sources. By using Galerkin-type representation, it is possible to obtain the fundamental solution of the MGT thermoelasticity theory.

Chapter 3 derives short-time approximated fundamental solutions for a system of field equations of motion in the context of the Moore-Gibson-Thompson thermoelasticity theory for two separate cases: concentrated body force and concentrated heat source. The Galerkin-type solution of MGT thermoelasticity theory provides the building blocks to obtain the fundamental solution for homogeneous and isotropic material. The fundamental solutions for displacement and temperature change are derived for the coupled thermoelastic system of partial differential equations by Laplace inversion. Lastly, the fundamental solution is obtained for steady vibration in terms of elementary functions.

Chapter 4 attempts to investigate size-dependent TED and thermoelastic vibration in micro/nano-beam resonators utilizing the Euler-Bernoulli beam and Timoshenko beam theories under the MCST and the Moore-Gibson-Thompson (MGT) theory. Firstly, the frequency approach method is adopted to derive the size-dependent expression of the inverse quality factor (QF) for evaluating TED in rectangular microbeam resonator. Furthermore, the impact of the material length scale parameter on TED associated with MCST for silicon micro-beam resonator is investigated in detail by comparing the present results with those predicted by classical continuum theory (CCT). The obtained findings reveal that the non-classical continuum theory (MCST) along with the MGT theory predicts a higher value of quality factor than the CCT for larger values of the material length scale parameter. Under the MCST, normalized frequencies become smaller as the microscale beam thickness is fixed. Also, it is found that the result of quality factor under the GN-III model is considerably larger than those estimated by the MGT and LS thermoleasticity theories. The second part of this chapter develops the variational formulation of the micro structural deflection and thermal moment of Timoshenko beam model considering Hamilton's principle and MCST which involves the material length scale parameter. The valid analytical formula for dimensionless deflection and the normalized thermal moment is derived using Fourier series and the Laplace transform approach with the appropriate boundary conditions. It has been noted that the maximum and minimum amplitudes of deflection varies significantly and the peaks are attained earlier in case of MCST as compared to CCT. It is further observed that the amplitude of deflection via all three models (LS, GN-III, and MGT) exactly matches with each other. It means that all three generalized thermoelastic theories provide steady-state deflection in this time range. Also, the peak values of thermal moment increase progressively as dimensionless time intensifies.

Next, in Chapter 5 a detailed analysis on MGT theory is presented by finding the analytical expression of the inverse quality factor for a Kirchhoff's microplate resonator by taking into consideration the MCST as non-classical continuum theory for the study of size effects and coupled MGT heat conduction equation. The Hamilton's principle is used in conjunction with the modified couple stress and Kirchhoff plate theories to derive governing equations based on the MGT thermoelasticity theory. This chapter therefore comprehensively evaluates the size effects of a micro plate resonator with a single material length-scale parameter in the context of the frequency shift and normalized attenuation. It has been found out that as the frequency shift diminishes, the attenuation reduces as well, causing an increase in the quality factor. It is observed that the impact of internal material length parameter l is prominent in the study of TED. Furthermore, this is discovered that as the material length-scale parameter's ratio l/h

to plate thickness is increased, the normalized frequency shift reduces dramatically. It is highlighted that the quality factor decreases as the relaxation parameter in the MGT model is increased. It is further investigated that the TED delineates non-monotonic behavior as a function of plate thickness. It implies that it will be an arduous task to predict the thermoelastic nature of any rectangular structure due to its non-monotone behavior when dealing with the different plate thicknesses.

In **Chapter 6**, Moore-Gibson-Thompson thermoelasticity theory is combined with nonlocal strain gradient theory (NSGT) and the Legendre wavelet method to predict the bending characteristics of the Euler-Bernoulli beam. The Laplace and Wavelet approximation methods are used to formulate and solve dimensionless deflection and temperature change equations with ramp-type heating boundary conditions. The graphical results showed the stiffness-softening effect by increasing the small-scale parameter due to nonlocal theory. It can be concluded that as the value of the nonlocal parameter increases, the deflection peak rises and the oscillation frequency decreases. It is noted that when the strain gradient parameter is considered, the stiffness-hardening nature of the beam resonator is illustrated. It means that stiffness hardening produces lower amplitude deflection and increases vibration frequency with intensifying strain gradient length scale parameter. The nonlocal beam theory (NBT) exhibited the highest deflection amplitude, while the strain gradient theory depicts a less range of dimensionless deflection. It implies that the declination of vibration frequency is maximum under nonlocal beam theory. A decrease in the non-dimensional deflection and temperature changes is also observed with increasing the ramp time parameter.

7.2 Future Scope

Recent advances in nanotechnology have led to the development of nano-electro-mechanical systems (NEMS), such as nanomechanical resonators, which have recently received significant attention from the scientific community. For the design and optimization of micro/nano-resonators, it is imperative to accurately calculate quality factors based on thermoelastic damping (TED). Certainly, it is apparent that theoretical investigations of TED can save money on research and development by optimizing resonators during preliminary design. TED is generated by the irreversible thermal flux in the coupling of the thermal and mechanical fields and the amount of dissipated energy during the vibration. However, there appear to be two challenging points due to the size-dependent effect. Firstly, for the devices at micro/nano-scales, the mechanical properties and behaviors cannot be accurately estimated by the classical elastic theories due to the size-dependent effect. Secondly, when the characteristic dimension of device structures becomes comparable to the intrinsic lengths of the material, the conventional Fourier theory of heat conduction, omitting the size-dependent effect, fails to give accurate thermal transport estimations. Therefore, it is crucial to carefully consider and properly analyze how damping processes affect oscillatory structures. Hence, there is scope to assess the TED modeling of micro/nano-resonators by adopting the improved theories of non-classical continuum mechanical theories within the framework of extended areas such as electro-thermoelasticity, visco-thermoelasticity, magneto-thermoelasticiy, and piezothermoelasticity. Further modification in the formulation of governing equations for TED modeling in the frame of these extended areas will be required. Limited research has been performed in this direction. Moreover, it is not always possible in general to find the analytical solution to coupled equations related to vibration analysis of microstructures, even numerically. So, there is scope to study these thermo-mechanical problems using numerical techniques like Finite Element, Wavelet Transform, and Finite difference methods. Also, Future research may explore advanced control strategies and optimization techniques to exploit thermoelastic damping for active vibration control applications, allowing for more efficient and precise vibration suppression in various engineering systems. The present thesis is therefore ended by keeping above points in mind for future investigation.