Appendix A

The values of all the unknown coefficients used from Eq.(5.3.1)-(5.3.15) are given below

$$a_1 = \frac{-(\alpha_1^{(1)} + \alpha_2^{(1)})}{2\alpha_2^{(1)}} e^{2(\gamma_1^{(1)} - \gamma_2^{(1)})\xi h}, \tag{A.1}$$

$$b_1 = \frac{(\alpha_1^{(1)} - \alpha_2^{(1)})}{2\alpha_2^{(1)}} e^{-2(\gamma_1^{(1)} + \gamma_2^{(1)})\xi h}, \tag{A.2}$$

$$a_2 = \frac{(\alpha_1^{(1)} + \alpha_2^{(1)})}{2\alpha_2^{(1)}} e^{2(\gamma_1^{(1)} + \gamma_2^{(1)})\xi h}, \tag{A.3}$$

$$b_2 = \frac{-(\alpha_1^{(1)} + \alpha_2^{(1)})}{2\alpha_2^{(1)}} e^{-2(\gamma_1^{(1)} - \gamma_2^{(1)})\xi h}, \tag{A.4}$$

$$a_3 = \frac{-(\alpha_1^{(4)} + \alpha_2^{(4)})}{2\alpha_2^{(4)}} e^{-2(\gamma_1^{(4)} - \gamma_2^{(4)})\xi h}, \tag{A.5}$$

$$b_3 = \frac{(\alpha_1^{(4)} - \alpha_2^{(4)})}{2\alpha_2^{(4)}} e^{2(\gamma_1^{(4)} + \gamma_2^{(4)})\xi h}, \tag{A.6}$$

$$a_4 = \frac{(\alpha_1^{(4)} - \alpha_2^{(4)})}{2\alpha_2^{(4)}} e^{-2(\gamma_1^{(4)} + \gamma_2^{(4)})\xi h}, \tag{A.7}$$

$$b_4 = \frac{-(\alpha_1^{(4)} + \alpha_2^{(4)})}{2\alpha_2^{(4)}} e^{2(\gamma_1^{(4)} - \gamma_2^{(4)})\xi h}, \tag{A.8}$$

$$a_5 = (e^{\gamma_1^{(1)}\xi h} + a_1 e^{\gamma_2^{(1)}\xi h} + a_2 e^{-\gamma_2^{(1)}\xi h}),$$

$$b_5 = (e^{-\gamma_1^{(1)}\xi h} + b_1 e^{\gamma_2^{(1)}\xi h} + b_2 e^{-\gamma_2^{(1)}\xi h}), \tag{A.9}$$

$$a_6 = (\alpha_1^{(1)} e^{\gamma_1^{(1)}\xi h} + \alpha_2^{(1)}(a_1 e^{\gamma_2^{(1)}\xi h} - a_2 e^{-\gamma_2^{(1)}\xi h})), \tag{A.10}$$

$$b_6 = (\alpha_1^{(1)} e^{-\gamma_1^{(1)}\xi h} - \alpha_2^{(1)}(b_1 e^{\gamma_2^{(1)}\xi h} - b_2 e^{-\gamma_2^{(1)}\xi h})), \tag{A.11}$$

$$a_7 = \frac{(a_5\alpha_2^{(2)} + a_6)}{2\alpha_2^{(2)}} e^{\gamma_2^{(2)}\xi h}, b_7 = \frac{(b_5\alpha_2^{(2)} + b_6)}{2\alpha_2^{(2)}} e^{\gamma_2^{(2)}\xi h}, \tag{A.12}$$

$$a_8 = \frac{-(\alpha_1^{(2)} + \alpha_2^{(2)})}{2\alpha_2^{(2)}} e^{(\gamma_1^{(2)} - \gamma_2^{(2)})\xi h}, b_8 = \frac{(\alpha_1^{(2)} - \alpha_2^{(2)})}{2\alpha_2^{(2)}} e^{-(\gamma_1^{(2)} + \gamma_2^{(2)})\xi h}, \tag{A.13}$$

$$a_9 = \frac{(a_5\alpha_2^{(2)} - a_6)}{2\alpha_2^{(2)}} e^{\gamma_2^{(2)}\xi h},\ b_9 = \frac{(b_5\alpha_2^{(2)} - b_6)}{2\alpha_2^{(2)}} e^{\gamma_2^{(2)}\xi h}, \tag{A.14}$$

$$a_{10} = \frac{(\alpha_1^{(2)} - \alpha_2^{(2)})}{2\alpha_2^{(2)}} e^{(\gamma_1^{(2)} + \gamma_2^{(2)})\xi h}, b_{10} = \frac{-(\alpha_1^{(2)} + \alpha_2^{(2)})}{2\alpha_2^{(2)}} e^{-(\gamma_1^{(2)} - \gamma_2^{(2)})\xi h}, \tag{A.15}$$

$$a_{11} = (a_7 + a_9), \qquad b_{11} = (b_7 + b_9), \qquad a_{12} = (a_8 + a_{10}), \tag{A.16}$$

$$b_{12} = (b_8 + b_{10}), \qquad a_{13} = \alpha_2^{(2)}(a_7 - a_9), \qquad b_{13} = \alpha_2^{(2)}(b_7 - b_9), \tag{A.17}$$

$$a_{14} = (\alpha_1^{(2)} + \alpha_2^{(2)}(a_8 - a_{10})), \quad b_{14} = -(\alpha_1^{(2)} - \alpha_2^{(2)}(b_8 - b_{10})), \tag{A.18}$$

$$a_{15} = (a_{11}\alpha_2^{(3)} + a_{13}), b_{15} = (b_{11}\alpha_2^{(3)} + b_{13}), \tag{A.19}$$

$$a_{16} = (a_{12}\alpha_2^{(3)} + a_{14}), b_{16} = (b_{12}\alpha_2^{(3)} + b_{14}), a_{17} = -(\alpha_1^{(3)} + \alpha_2^{(3)}), \tag{A.20}$$

$$b_{17} = (\alpha_1^{(3)} - \alpha_2^{(3)}), a_{18} = (a_{11}\alpha_2^{(3)} - a_{13}), b_{18} = (b_{11}\alpha_2^{(3)} - b_{13}), \tag{A.21}$$

$$a_{19} = (a_{12}\alpha_2^{(3)} - a_{14}), b_{19} = (b_{12}\alpha_2^{(3)} - b_{14}), a_{20} = -(\alpha_1^{(3)} - \alpha_2^{(3)}), \tag{A.22}$$

$$b_{20} = -(\alpha_1^{(3)} + \alpha_2^{(3)}), a_{21} = (a_{15}e^{-\gamma_2^{(3)}\xi h} + a_{18}e^{\gamma_2^{(3)}\xi h}), \tag{A.23}$$

$$b_{21} = (b_{15}e^{-\gamma_2^{(3)}\xi h} + b_{18}e^{\gamma_2^{(3)}\xi h}), a_{22} = (a_{16}e^{-\gamma_2^{(3)}\xi h} + a_{19}e^{\gamma_2^{(3)}\xi h}), \tag{A.24}$$

$$b_{22} = (b_{16}e^{-\gamma_2^{(3)}\xi h} + b_{19}e^{\gamma_2^{(3)}\xi h}), \quad a_{23} = (e^{-\gamma_1^{(3)}\xi h} + a_{17}e^{-\gamma_2^{(3)}\xi h} + a_{20}e^{\gamma_2^{(3)}\xi h}), \tag{A.25}$$

$$b_{23} = (e^{\gamma_1^{(3)}\xi h} + b_{17}e^{-\gamma_2^{(3)}\xi h} + b_{20}e^{\gamma_2^{(3)}\xi h}), \tag{A.26}$$

$$a_{24} = -(e^{-\gamma_1^{(4)}\xi h} + a_3e^{-\gamma_2^{(4)}\xi h} + a_4e^{\gamma_2^{(4)}\xi h}), \tag{A.27}$$

$$b_{24} = -(e^{\gamma_1^{(4)}\xi h} + b_3e^{-\gamma_2^{(4)}\xi h} + b_4e^{\gamma_2^{(4)}\xi h}), \quad a_{25} = \alpha_2^{(3)}(a_{15}e^{-\gamma_2^{(3)}\xi h} - a_{18}e^{\gamma_2^{(3)}\xi h}), \tag{A.28}$$

$$b_{25} = \alpha_2^{(3)}(b_{15}e^{-\gamma_2^{(3)}\xi h} - b_{18}e^{\gamma_2^{(3)}\xi h}), a_{26} = \alpha_2^{(3)}(a_{16}e^{-\gamma_2^{(3)}\xi h} - a_{19}e^{\gamma_2^{(3)}\xi h}), \tag{A.29}$$

$$b_{26} = \alpha_2^{(3)}(b_{16}e^{-\gamma_2^{(3)}\xi h} - b_{19}e^{\gamma_2^{(3)}\xi h}), \tag{A.30}$$

$$a_{27} = (\alpha_1^{(3)}e^{-\gamma_1^{(3)}\xi h} + \alpha_2^{(3)}(a_{17}e^{-\gamma_2^{(3)}\xi h} - a_{20}e^{\gamma_2^{(3)}\xi h})), \tag{A.31}$$

$$b_{27} = (-\alpha_1^{(3)}e^{\gamma_1^{(3)}\xi h} + \alpha_2^{(3)}(b_{17}e^{-\gamma_2^{(3)}\xi h} - b_{20}e^{\gamma_2^{(3)}\xi h})), \tag{A.32}$$

$$a_{28} = -(\alpha_1^{(4)}e^{-\gamma_1^{(4)}\xi h} - \alpha_2^{(4)}(a_3e^{-\gamma_2^{(4)}\xi h} - a_4e^{\gamma_2^{(4)}\xi h})), \tag{A.33}$$

$$b_{28} = (\alpha_1^{(4)}e^{\gamma_1^{(4)}\xi h} - \alpha_2^{(4)}(b_3e^{-\gamma_2^{(4)}\xi h} + b_4e^{\gamma_2^{(4)}\xi h})), \tag{A.34}$$

$$\begin{aligned} a_{29} = {} & ((C_{22}^{(1)}\,\alpha_1^{(1)}\gamma_1^{(1)} - C_{12}^{(1)})e^{\gamma_1^{(1)}\xi h} + (C_{22}^{(1)}\,\alpha_2^{(1)}\gamma_2^{(1)} - C_{12}^{(1)})(a_1e^{\gamma_2^{(1)}\xi h} \\ & + a_2e^{-\gamma_2^{(1)}\xi h}) - (C_{22}^{(2)} - C_{12}^{(2)})(a_7e^{\gamma_2^{(2)}\xi h}\,\alpha_2^{(2)}\gamma_2^{(2)} + a_9e^{-\gamma_2^{(2)}\xi h})), \end{aligned} \tag{A.35}$$

$$\begin{aligned} b_{29} = {} & ((C_{22}^{(1)}\,\alpha_1^{(1)}\gamma_1^{(1)} - C_{12}^{(1)})e^{-\gamma_1^{(1)}\xi h} + (C_{22}^{(1)}\,\alpha_2^{(1)}\gamma_2^{(1)} - C_{12}^{(1)})(b_1e^{\gamma_2^{(1)}\xi h} \\ & + b_2e^{-\gamma_2^{(1)}\xi h}) - (C_{22}^{(2)}\,\alpha_2^{(2)}\gamma_2^{(2)} - C_{12}^{(2)})(b_7e^{\gamma_2^{(2)}\xi h} + b_9e^{-\gamma_2^{(2)}\xi h})), \end{aligned} \tag{A.36}$$

$$\begin{aligned} a_{30} = {} & -((C_{22}^{(2)}\,\alpha_1^{(2)}\gamma_1^{(2)} - C_{12}^{(2)})e^{\gamma_1^{(2)}\xi h} - (C_{22}^{(2)}\,\alpha_2^{(2)}\gamma_2^{(2)} - C_{12}^{(2)})(a_8e^{\gamma_2^{(2)}\xi h} \\ & + a_{10}e^{-\gamma_2^{(2)}\xi h}), \end{aligned} \tag{A.37}$$

$$\begin{aligned} b_{30} = {} & -((C_{22}^{(2)}\,\alpha_1^{(2)}\gamma_1^{(2)} - C_{12}^{(2)})e^{-\gamma_1^{(2)}\xi h} - (C_{22}^{(2)}\,\alpha_2^{(2)}\gamma_2^{(2)} - C_{12}^{(2)})(b_8e^{\gamma_2^{(2)}\xi h} \\ & + b_{10}e^{-\gamma_2^{(2)}\xi h}), \end{aligned} \tag{A.38}$$

$$\begin{aligned} a_{31} = {} & (\mu_{12}^{(1)}(\alpha_1^{(1)} + \gamma_1^{(1)})e^{\gamma_1^{(1)}\xi h} + \mu_{12}^{(1)}(\alpha_2^{(1)} + \gamma_2^{(1)})(a_1e^{\gamma_2^{(1)}\xi h} - a_2e^{-\gamma_2^{(1)}\xi h}) \\ & - \mu_{12}^{(2)}(\alpha_2^{(2)} + \gamma_2^{(2)})(a_7e^{\gamma_2^{(2)}\xi h} - a_9e^{-\gamma_2^{(2)}\xi h})), \end{aligned} \tag{A.39}$$

$$\begin{aligned} b_{31} = {} & (-\mu_{12}^{(1)}(\alpha_1^{(1)} + \gamma_1^{(1)})e^{-\gamma_1^{(1)}\xi h} + \mu_{12}^{(1)}(\alpha_2^{(1)} + \gamma_2^{(1)})(b_1e^{\gamma_2^{(1)}\xi h} - b_2e^{-\gamma_2^{(1)}\xi h}) \\ & - \mu_{12}^{(2)}(\alpha_2^{(2)} + \gamma_2^{(2)})(b_7e^{\gamma_2^{(2)}\xi h} - b_9e^{-\gamma_2^{(2)}\xi h})), \end{aligned} \tag{A.40}$$

$$a_{32} = -(\mu_{12}^{(2)}(\alpha_1^{(2)} + \gamma_1^{(2)})e^{\gamma_1^{(2)}\xi h} + \mu_{12}^{(2)}(\alpha_2^{(2)} + \gamma_2^{(2)})(a_8e^{\gamma_2^{(2)}\xi h} - a_{10}e^{-\gamma_2^{(2)}\xi h}), \tag{A.41}$$

$$b_{32} = (\mu_{12}^{(2)}(\alpha_1^{(2)} + \gamma_1^{(2)})e^{-\gamma_1^{(2)}\xi h} - \mu_{12}^{(2)}(\alpha_2^{(2)} + \gamma_2^{(2)})(b_8e^{\gamma_2^{(2)}\xi h} - b_{10}e^{-\gamma_2^{(2)}\xi h}), \tag{A.42}$$

$$a_{33} = \frac{(a_{29}b_{32} - a_{31}b_{30})}{(a_{32}b_{30} - a_{30}b_{32})}, b_{33} = \frac{(b_{29}b_{32} - b_{31}b_{30})}{(a_{32}b_{30} - a_{30}b_{32})}, \tag{A.43}$$

$$a_{34} = \frac{(a_{29}a_{32} - a_{31}a_{30})}{(b_{32}a_{30} - b_{30}a_{32})}, b_{34} = \frac{(b_{29}a_{32} - b_{31}a_{30})}{(b_{32}a_{30} - b_{30}a_{32})}, \tag{A.44}$$

$$a_{35} = ((C_{22}^{(2)}\,\alpha_1^{(2)}\gamma_1^{(2)} - C_{12}^{(2)})(a_{33} + a_{34}) + (C_{22}^{(2)}\,\alpha_2^{(2)}\gamma_2^{(2)} - C_{12}^{(2)})(a_7 + a_9 + a_{33}a_{10} + b_{10}a_{34})$$
$$- (C_{22}^{(3)}\,\alpha_2^{(3)}\gamma_2^{(3)} - C_{12}^{(3)})(a_{15} + a_{16}a_{33} + b_{16}a_{34} + a_{18} + a_{19}a_{33} + b_{19}a_{34})), \qquad \text{(A.45)}$$
$$b_{35} = ((C_{22}^{(2)}\,\alpha_1^{(2)}\gamma_1^{(2)} - C_{12}^{(2)})(b_{33} + b_{34}) + (C_{22}^{(2)}\,\alpha_2^{(2)}\gamma_2^{(2)} - C_{12}^{(2)})(b_7 + b_9 + b_{33}a_{10} + b_{10}b_{34})$$
$$- (C_{22}^{(3)}\,\alpha_2^{(3)}\gamma_2^{(3)} - C_{12}^{(3)})(b_{15} + a_{16}b_{33} + b_{16}a_{34} + b_{18} + a_{19}a_{33} + b_{19}a_{34})), \qquad \text{(A.46)}$$
$$a_{36} = -((C_{22}^{(3)}\,\alpha_1^{(3)}\gamma_1^{(3)} - C_{12}^{(3)}) + (C_{22}^{(3)}\,\alpha_2^{(3)}\gamma_2^{(3)} - C_{12}^{(3)})(a_{17} + a_{20})), \qquad \text{(A.47)}$$
$$b_{36} = -((C_{22}^{(3)}\,\alpha_1^{(3)}\gamma_1^{(3)} - C_{12}^{(3)}) + (C_{22}^{(3)}\,\alpha_2^{(3)}\gamma_2^{(3)} - C_{12}^{(3)})(b_{17} + b_{20})), \qquad \text{(A.48)}$$
$$a_{37} = (\mu_{12}^{(2)}(\alpha_1^{(2)} + \gamma_1^{(2)})(a_{33} - a_{34}) + \mu_{12}^{(2)}(\alpha_2^{(2)} + \gamma_2^{(2)})(a_7 + a_8a_{33} + b_8a_{34} - a_9 - a_{10}a_{33}$$
$$- b_{10}a_{34}) - \mu_{12}^{(3)}(\alpha_2^{(3)} + \gamma_2^{(3)})(a_{15} + a_{16}a_{33} + b_{16}a_{34} - a_{18} - a_{19}a_{33} - b_{19}a_{34})), \qquad \text{(A.49)}$$
$$b_{37} = (\mu_{12}^{(2)}(\alpha_1^{(2)} + \gamma_1^{(2)})(b_{33} - b_{34}) + \mu_{12}^{(2)}(\alpha_2^{(2)} + \gamma_2^{(2)})(b_7 + a_8b_{33} + b_8b_{34} - b_9 - a_{10}b_{33}$$
$$- b_{10}b_{34}) - \mu_{12}^{(3)}(\alpha_2^{(3)} + \gamma_2^{(3)})(b_{15} + a_{16}b_{33} + b_{16}a_{34} - b_{18} - a_{19}b_{33} - b_{19}b_{34})), \qquad \text{(A.50)}$$
$$a_{38} = -(\mu_{12}^{(3)}(\alpha_1^{(3)} + \gamma_1^{(3)}) - \mu_{12}^{(3)}(\alpha_2^{(3)} + \gamma_2^{(3)})(a_{17} - a_{20})), \qquad \text{(A.51)}$$
$$b_{38} = (\mu_{12}^{(3)}(\alpha_1^{(3)} + \gamma_1^{(3)}) - \mu_{12}^{(3)}(\alpha_2^{(3)} + \gamma_2^{(3)})(b_{17} - b_{20})), \qquad \text{(A.52)}$$
$$a_{39} = \frac{(a_{35}b_{38} - a_{37}b_{36})}{(a_{38}b_{36} - a_{36}b_{38})}, b_{39} = \frac{(b_{35}b_{38} - b_{37}b_{36})}{(a_{38}b_{36} - a_{36}b_{38})}, \qquad \text{(A.53)}$$
$$a_{40} = \frac{(a_{35}a_{38} - a_{37}a_{36})}{(b_{38}a_{36} - b_{36}a_{38})}, b_{40} = \frac{(b_{35}a_{38} - b_{37}a_{36})}{(b_{38}a_{36} - b_{36}a_{38})}, \qquad \text{(A.54)}$$
$$a_{41} = ((C_{22}^{(3)}\,\alpha_1^{(3)}\gamma_1^{(3)} - C_{12}^{(3)})(a_{39}e^{-\gamma_1^{(3)}\xi h} + a_{40}e^{\gamma_1^{(3)}\xi h}) + (C_{22}^{(3)}\,\alpha_2^{(3)}\gamma_2^{(3)}$$
$$- C_{12}^{(3)})(e^{-\gamma_2^{(2)}\xi h}(a_{15} + a_{16}a_{33} + b_{16}a_{34} + a_{17}a_{39} + b_{17}a_{40}) + e^{-\gamma_2^{(2)}\xi h}(a_{18}$$
$$+ a_{19}a_{33} + b_{19}a_{34} + a_{20}a_{39} + b_{20}a_{40}))), \qquad \text{(A.55)}$$
$$b_{41} = ((C_{22}^{(3)}\,\alpha_1^{(3)}\gamma_1^{(3)} - C_{12}^{(3)})(b_{39}e^{-\gamma_1^{(3)}\xi h} + b_{40}e^{\gamma_1^{(3)}\xi h}) + (C_{22}^{(3)}\,\alpha_2^{(3)}\gamma_2^{(3)}$$
$$- C_{12}^{(3)})(e^{-\gamma_2^{(2)}\xi h}(b_{15} + a_{16}b_{33} + b_{16}b_{34} + a_{17}b_{39} + b_{17}b_{40}) + e^{\gamma_2^{(2)}\xi h}(b_{18} + a_{19}b_{33}$$
$$+ b_{19}b_{34} + a_{20}b_{39} + b_{20}b_{40}))), \qquad \text{(A.56)}$$
$$a_{42} = -((C_{22}^{(4)}\,\alpha_1^{(4)}\gamma_1^{(4)} - C_{12}^{(4)})e^{-\gamma_1^{(4)}\xi h} + (C_{22}^{(4)}\,\alpha_2^{(4)}\gamma_2^{(4)} - C_{12}^{(4)})(a_3e^{-\gamma_2^{(4)}\xi h} + a_4e^{\gamma_2^{(4)}\xi h})), \qquad \text{(A.57)}$$
$$b_{42} = -((C_{22}^{(4)}\,\alpha_1^{(4)}\gamma_1^{(4)} - C_{12}^{(4)})e^{\gamma_1^{(4)}\xi h} + (C_{22}^{(4)}\,\alpha_2^{(4)}\gamma_2^{(4)} - C_{12}^{(4)})(b_3e^{-\gamma_2^{(4)}\xi h} + b_4e^{\gamma_2^{(4)}\xi h})), \qquad \text{(A.58)}$$
$$a_{43} = (\mu_{12}^{(3)}(\alpha_1^{(3)} + \gamma_1^{(3)})(e^{-\gamma_1^{(3)}\xi h}a_{39} - e^{\gamma_1^{(3)}\xi h}a_{40}) + \mu_{12}^{(3)}(\alpha_2^{(3)} + \gamma_2^{(3)})(e^{-\gamma_2^{(3)}\xi h}(a_{15}$$
$$+ a_{16}a_{33} + b_{16}a_{34} + a_{17}a_{39} + b_{17}a_{40}) - e^{\gamma_2^{(3)}\xi h}(a_{18} + a_{19}a_{33} + b_{19}a_{34}$$
$$+ a_{20}a_{39} + b_{20}a_{40}))), \qquad \text{(A.59)}$$
$$b_{43} = (\mu_{12}^{(3)}(\alpha_1^{(3)} + \gamma_1^{(3)})(e^{-\gamma_1^{(3)}\xi h}b_{39} - e^{\gamma_1^{(3)}\xi h}b_{40}) + \mu_{12}^{(3)}(\alpha_2^{(3)} + \gamma_2^{(3)})(e^{-\gamma_2^{(3)}\xi h}(b_{15}$$
$$+ a_{16}b_{33} + b_{16}b_{34} + a_{17}b_{39} + b_{17}b_{40}) - e^{\gamma_2^{(3)}\xi h}(b_{18} + a_{19}b_{33} + b_{19}b_{34}$$
$$+ a_{20}b_{39} + b_{20}b_{40}))), \qquad \text{(A.60)}$$

$$a_{44} = (\mu_{12}^{(4)}(\alpha_1^{(4)} + \gamma_1^{(4)})e^{-\gamma_1^{(4)}\xi h} + \mu_{12}^{(4)}(\alpha_2^{(4)} + \gamma_2^{(4)})(e^{-\gamma_2^{(4)}\xi h}a_3 - e^{\gamma_2^{(4)}\xi h}a_4), \tag{A.61}$$

$$b_{44} = (\mu_{12}^{(4)}(\alpha_1^{(4)} + \gamma_1^{(4)})e^{\gamma_1^{(4)}\xi h} - \mu_{12}^{(4)}(\alpha_2^{(4)} + \gamma_2^{(4)})(e^{-\gamma_2^{(4)}\xi h}b_3 - e^{\gamma_2^{(4)}\xi h}b_4), \tag{A.62}$$

$$a_{45} = \frac{(a_{41}b_{44} - a_{43}b_{42})}{(a_{44}b_{42} - a_{42}b_{44})}, b_{45} = \frac{(b_{41}b_{44} - b_{43}b_{42})}{(a_{44}b_{42} - a_{42}b_{44})}, \tag{A.63}$$

$$a_{46} = \frac{(a_{41}a_{44} - a_{43}a_{42})}{(b_{44}a_{42} - b_{42}a_{44})}, b_{46} = \frac{(b_{41}a_{44} - b_{43}a_{42})}{(b_{44}a_{42} - b_{42}a_{44})}, \tag{A.64}$$

$$a_{47} = \frac{-(a_{21} + a_{22}a_{33} + b_{22}a_{34} + a_{23}a_{39} + b_{23}a_{40} + a_{24}a_{45} + b_{24}a_{46})}{(b_{21} + a_{22}b_{33} + b_{22}b_{34} + a_{23}b_{39} + b_{23}b_{40} + a_{24}b_{45} + b_{24}b_{46})}, \tag{A.65}$$

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Research Publications

International Journal Publications

[**1**] Neha Trivedi, Subir Das and Holm Altenbach, Study of collinear cracks in a composite medium subjected to time-harmonic wave disturbance, ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik, volume 101, page e202000307, Wiley Online Library, 2021.

[**2**] Neha Trivedi, Subir Das and E-M Craciun, The mathematical study of an edge crack in two different specified models under time-harmonic wave disturbance, Mechanics of Composite Materials, volume 58, pages 1–14. Springer, 2022.

[**3**] Neha Trivedi and Subir Das, Semi-infinite Moving Crack under Antiplane Shear Loading, ZAMPJournal of Applied Mathematics and Physics/Zeitschrift für angewandte Mathematik und Physik, volume 73(6), page 229.Springer, 2022.

[**4**] Neha Trivedi and Subir Das, Two semi-infinite moving cracks situated at two different interfaces of four semi-infinite orthotropic strips in a composite medium, Applied Mathematical Modelling (under review), 2023.