MATHEMATICAL STUDY OF SOME CRACK PROPAGATION PROBLEMS IN COMPOSITE ORTHOTROPIC MEDIA

Thesis submitted in partial fulfillment

for the award of the degree of

Doctor of Philosophy

by

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It is certified that the work contained in the thesis titled "**Mathematical study of Some Crack Propagation problems in Composite Orthotropic Media**" by **Neha Trivedi** has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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I, **Neha Trivedi**, certify that the work embodied in this thesis is my own bona fide work and carried out by me under the supervision of **Prof. Subir Das** from **July 2018** to **May 2023** at the **Department of Mathematical Sciences**, Indian Institute of Technology (Banaras Hindu University), Varanasi. The matter embodied in this thesis has not been submitted for the award of any other degree/diploma. I declare that I have faithfully acknowledged and given credits to the research workers wherever their works have been cited in my work in this thesis. I further declare that I have not willfully copied any other's work, paragraphs, text, data, results, *etc.*, reported in journals, books, magazines, reports, dissertations, theses, *etc.*, or available at websites and have not included them in this thesis and have not cited as my own work.

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Neha Trivedi

Dedicated to my family

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Preface

Almost all materials are susceptible to crack and wave propagation. Materials begin to weaken when they develop finite or semi-infinite cracks. This thesis aims to create numerous mathematical models of semi-infinite/finite cracks in infinite or bounded structures to investigate how crack tips propagate. The problems studied here can be applied in real life to improve safety standards in various industries and manufacturing processes. The handling of fracture parameters such as stress intensity factor and crack opening displacement at the crack tip has been achieved.

This thesis deals with the problem of various types of finite and semi-infinite cracks in different composite orthotropic media. There are five crack problems studied here, in which the first problem deals with three collinear cracks of finite length embedded at the semi-infinite orthotropic strip bonded by two orthotropic half-planes. The second problem consists of an edge crack in a vertical strip bonded by half plane, whereas the third type of crack is simply an edge crack situated in a vertical orthotropic strip. The problems from these models have been solved using the Schmith method along with Fourier transformation.

The fourth crack problem is an interfacial semi-infinite crack situated between two dissimilar orthotropic semi-infinite strips, and the fifth crack problem consists of two interfacial semi-infinite cracks situated at two different interfaces in the system of four orthotropic composite semi-infinite strips; these two problems have been solved using the Wiener-Hopf technique.

The crack problems studied in this thesis are a mathematical approach to save the material from failure by studying the physical factors which are responsible for crack propagation and material failure, viz. SIF, SMF, and COD. The thesis has been constructed in such a manner that the applications of the Fourier transform, as well as the methods for solving the models under consideration, including the Schmidt method, Asymptotic approach, and Weiner-Hopf technique, have been covered. It also discusses potential future works.