

**MATHEMATICAL STUDY OF SOME CRACK
PROPAGATION PROBLEMS IN COMPOSITE
ORTHOTROPIC MEDIA**



*Thesis submitted in partial fulfillment
for the award of the degree of*

Doctor of Philosophy

by

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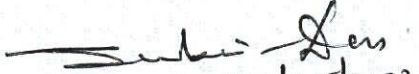
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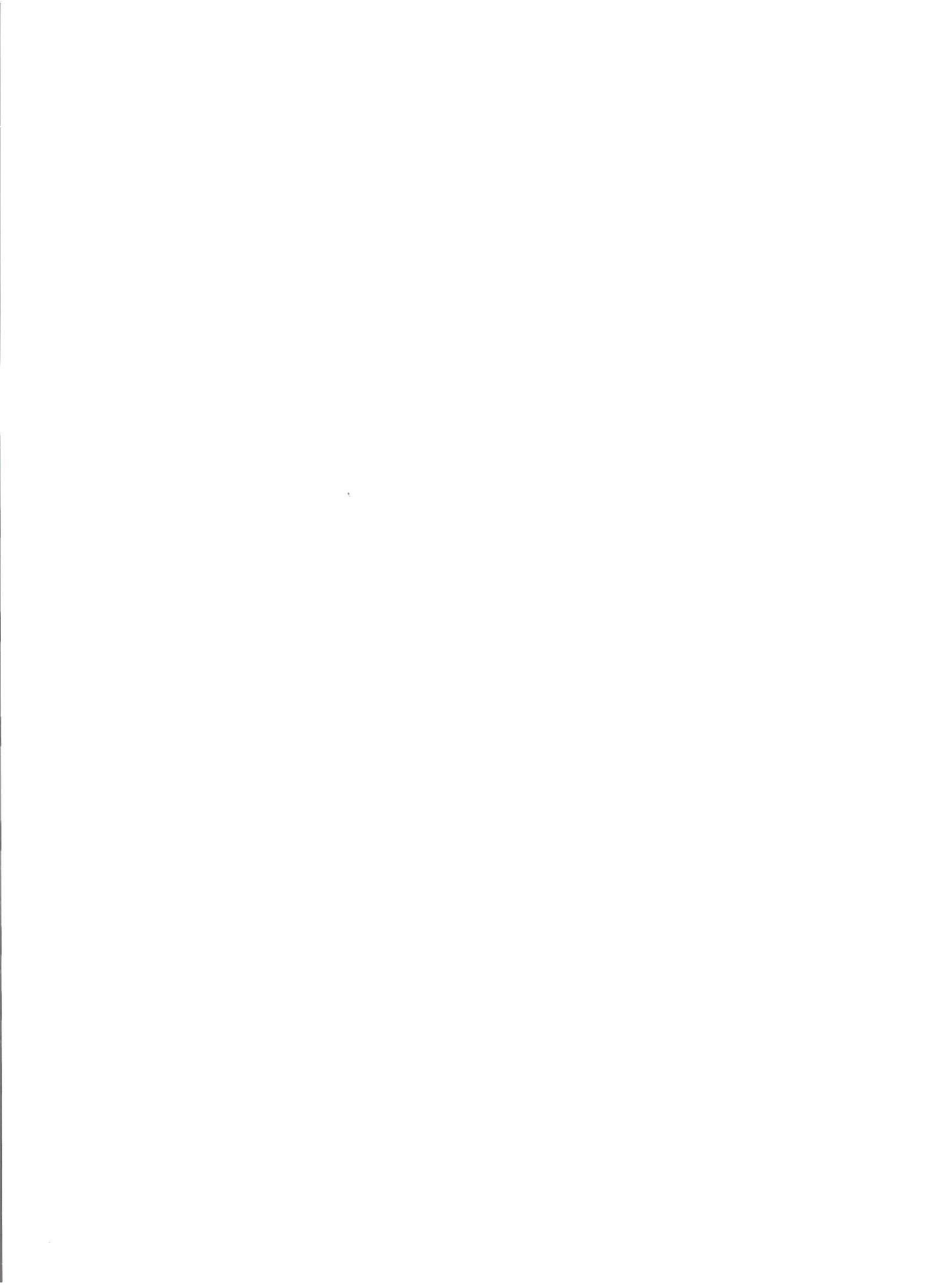
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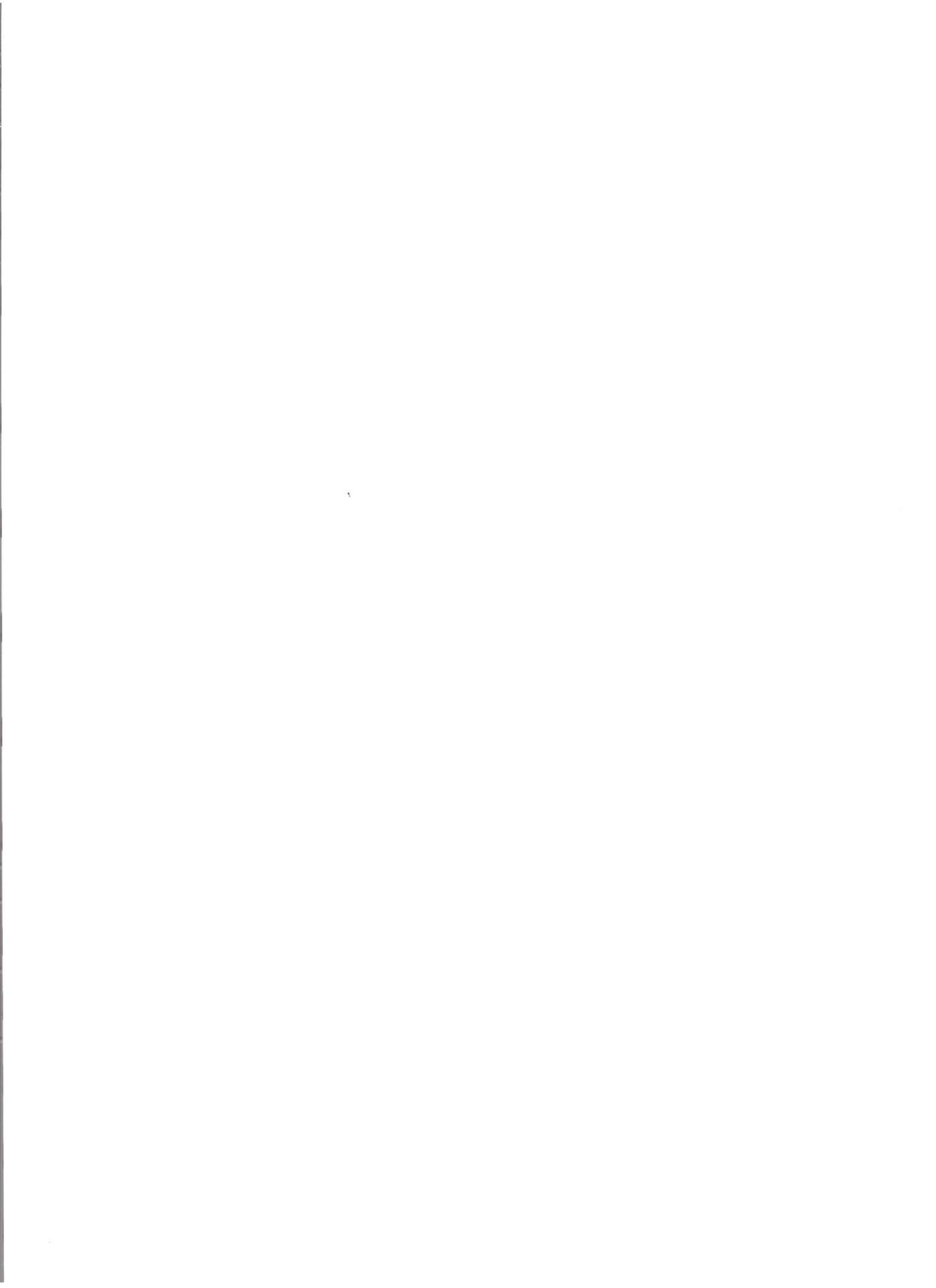

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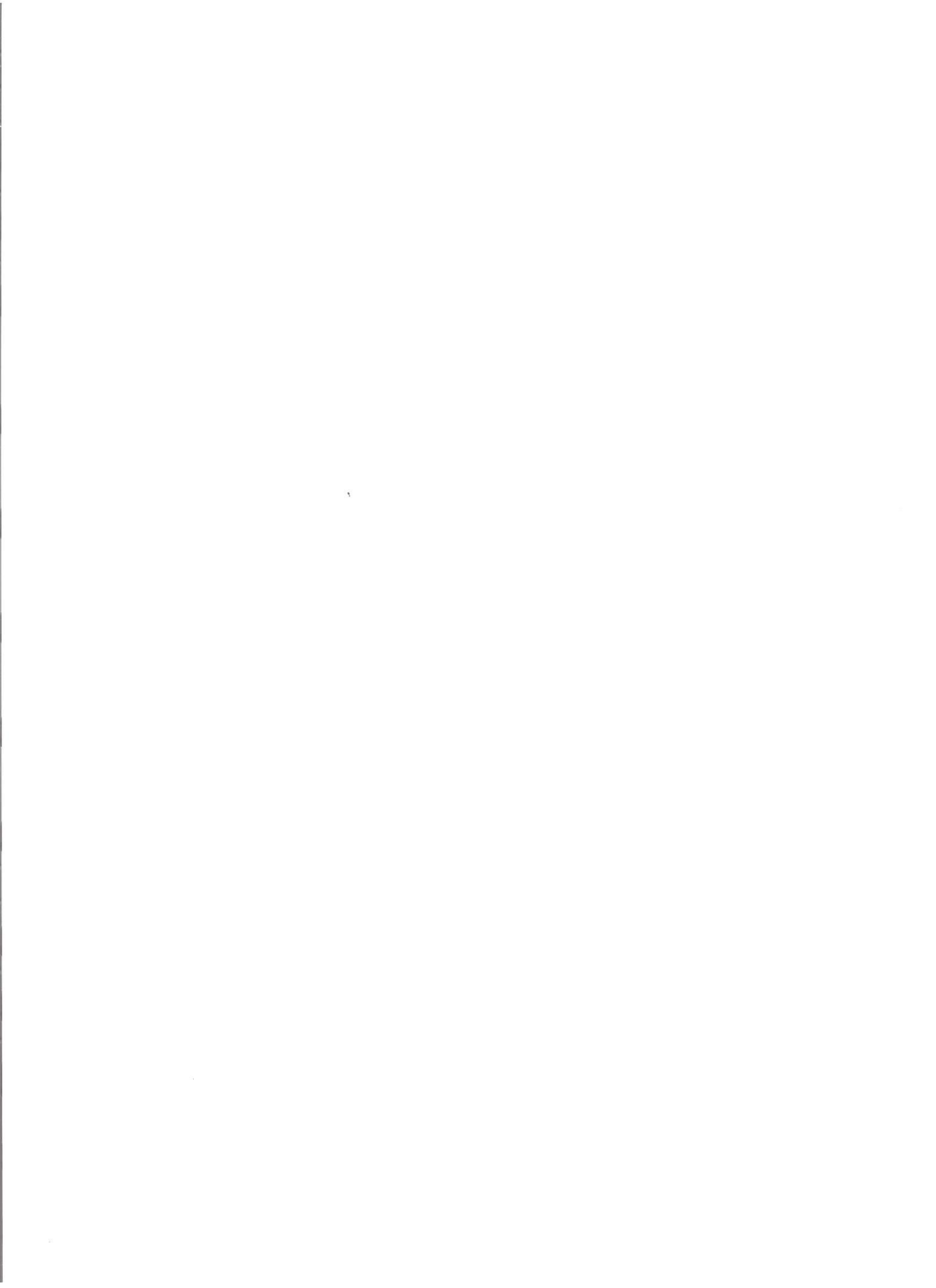
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Varanasi-221005

Neha Trivedi
Neha Trivedi

Dedicated to my family

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Preface

Almost all materials are susceptible to crack and wave propagation. Materials begin to weaken when they develop finite or semi-infinite cracks. This thesis aims to create numerous mathematical models of semi-infinite/finite cracks in infinite or bounded structures to investigate how crack tips propagate. The problems studied here can be applied in real life to improve safety standards in various industries and manufacturing processes. The handling of fracture parameters such as stress intensity factor and crack opening displacement at the crack tip has been achieved.

This thesis deals with the problem of various types of finite and semi-infinite cracks in different composite orthotropic media. There are five crack problems studied here, in which the first problem deals with three collinear cracks of finite length embedded at the semi-infinite orthotropic strip bonded by two orthotropic half-planes. The second problem consists of an edge crack in a vertical strip bonded by half plane, whereas the third type of crack is simply an edge crack situated in a vertical orthotropic strip. The problems from these models have been solved using the Schmith method along with Fourier transformation.

The fourth crack problem is an interfacial semi-infinite crack situated between two dissimilar orthotropic semi-infinite strips, and the fifth crack problem consists of two interfacial semi-infinite cracks situated at two different interfaces in the system of four orthotropic composite semi-infinite strips; these two problems have been solved using the Wiener-Hopf technique.

The crack problems studied in this thesis are a mathematical approach to save the material from failure by studying the physical factors which are responsible for crack propagation and material failure, viz. SIF, SMF, and COD. The thesis has been constructed in such a manner that the applications of the Fourier transform, as well as the methods for solving the models under consideration, including the Schmidt method, Asymptotic approach, and Weiner-Hopf technique, have been covered. It also discusses potential future works.

