Chapter 6

Conclusion and Future Work

This chapter concludes the thesis and presents the research possibilities in the direction of the proposed research work.

6.1 Conclusion

Numerical simulation of the the physical model is necessary to get the approximate solution when the corresponding different equation is complex. The complexity increases with the newly proposed fractional derivatives. This thesis presented collocation approximations for fractional integro-differential equations. Further, collocation approximations were also extended to solve fractional sub-diffusion equations defined in terms of generalized Caputo derivatives containing scale and weight functions. The stability and convergence of numerical scheme are derived analytically and verified numerically through various examples.

Chapter 1 introduced the thesis with basic definitions. The past and recent works on fractional integro-differential and fractional sub-diffusion equations were presented.

The motivation behind this research work and problem statement of the thesis was explained.

In Chapter 2, a convergent collocation method is developed for solving GFIDEs in terms of the B-operator. Jacobi poly-fractonomials are used as a basis in the proposed collocation method. The choice of the Jacobi poly-fractonomials helps to increase the accuracy in the approximated solution. The presented method works well on linear and nonlinear types of the GFIDEs and produces accurate solutions.

Chapter 3 presented a generalized model of PIDEFO and its numerical solution. This model results the solution of several standard fractional models such as 1) linear space-time fractional reaction-diffusion equation (FRDE), 2) space-time fraction diffusion equations (STFDE) and 3) time fractional telegraph equations (TFTE), in special case. And thus, could be considered as the generalized PIDEFO model. The proposed collocation method is used to find the numerical solution of the generalized PIDEFO. This could be analyzed with the two examples discussed in the thesis, one with unknown solution (Example (3.7.1)) and second with known solutions (Example (3.7.2)). The approximated solution is also compared with a special case of Example (3.7.2) in which the solution is known. It was observed that the proposed method obtains good accuracy. We discussed only three special cases of the proposed model. Some other models may also be obtained as a case of the presented model and such studies will be presented in future.

Chapter 4 approximated a numerical scheme for a new class of fractional diffusion equation in which the time derivative is considered as the generalized fractional derivative. The scheme uses the finite difference and collocation methods to find the numerical solution. The theoretical error and convergence analysis were validated numerically. Numerical examples showed that the proposed method achieves high accuracy in comparison to other methods [130, 132, 133, 134] presented recently.

Chapter 5 discussed a new collocation method based on the Legendre polynomials for solving GFADE with initial and boundary conditions. Due to the fact that GFD considered in this thesis involves scale and weight functions, we further study the effects of such functions on the dynamical behaviors of numerical solutions. Surprisingly, we find that an increasing scale function shifted the solution in upwards and a decreasing choice of scale function shifted the solution towards the origin. Whereas, increasing weight function contracts and decreasing weight function stretches the numerical solution. The error estimate and convergence results of the proposed collocation method are established theoretically. For checking the validity of numerical simulation, we consider three different typical examples containing both one and two-dimensional GFADE. It is concluded that our proposed method achieved good accuracy.

6.2 Future Work

Researcher have been working on the field of fractional calculus, especially on its operators, to provide clear definitions likewise in the classical calculus. This thesis, presented the collocation and finite difference based approximations of $A(B)$ operators and its application in solving fractional integro-differential and fractional sub-diffusion equations. However, there are other methods such as wavelet approach, finite element method etc. to be explored for the discussed problem in near future.

Most of the examples in this thesis were taken from literature with smooth data to provide a comparison with other existing methods in the literature. In upcoming research, it is expected to work on some real-discrete data and some more complicated model of fractional integro-differential and sub-diffusion equations in higher dimensions.

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