Chapter 1

Introduction

1.1 Queueing theory and Queueing system

Queueing theory is a branch of operations research that examines how the line performs in various areas, such as transportation, telecommunication, manufacturing, blood sampling, etc. Studying queueing theory is primarily intended to reduce customers' waiting times. Accordingly, in the study of queueing theory, primarily six characteristics are discussed, *viz.*, arrival process (A), service process (B), number of servers (C), system capacity (N), service discipline (D), and population size (F). The notion of queueing theory was developed by the Danish mathematician A. K. Erlang (1878–1929). He contemplated the issue of figuring out how many phone circuits are needed to provide phone service so that customers do not have to wait too long for an open circuit. Erlang [1] developed a model of telephone calls and studied the holding times of conversation in telephone exchanges. Numerous researchers began to focus on the study of queueing theory after this novel literary contribution.

1.2 Preliminaries

1.2.1 Kendall-Lee notation

The queueing models are represented using the Kendall-Lee notation, which uses six abbreviations. The notation $A/B/C$, known as the Kendall notation, was first used in 1953 by Kendall [2]. The Kendall notation $A/B/C$ was later expanded by Lee [3] to the notation $A/B/C/N/D/F$, known as Kendall-Lee notation. The distribution of arrivals and service time are represented by the first two attributes, A and B, respectively. C stands for the number of servers in the system, N stands for the system capacity, D stands for the service discipline (*viz.,* first come first serve (FCFS), last come first serve (LCFS), etc.), and the population size is indicated by the letter F. For instance, $M/M/4/\infty/FCFS/\infty$ illustrates a queueing system with four servers and infinite system capacity, exponential inter arrival time and service time distribution, FCFS service discipline, and infinite population. It is presumed that $N = \infty$, service discipline is FCFS and $F = \infty$ if the last three characteristics (for instance, the $M/M/1$ queue) are not specified. The queueing model $M/E_2/1/K$ represents the system with a single server, Poisson arrival, Erlang service time distribution with two phases, and a finite system capacity of size K. The queueing model with three servers where the inter arrival and service time follow the general distribution is represented by the notation $GI/G/3$.

1.2.2 Markovian arrival Process (MAP)

Markovian arrival process with state space $\{1, 2, ..., m+1\}$ in which the transient states are $\{1, 2, ..., m\}$, and the absorbing state is $\{m+1\}$, is taken into consideration. Absorption is certain starting from any state. Assume that the process is in a transient state i $(1 \leq i \leq m)$. The sojourn time is exponentially distributed at each state i $(1 \leq i \leq m)$ with parameter λ_i. Once the sojourn time at state i $(1 \leq i \leq m)$ is completed, one of the following two possibilities may occur:

- The process enters the absorbing state, with probability $p_{i,j}$ $(1 \leq j \leq m)$ and it immediately restarts with state j.
- The process enters in the transient state j with probability $q_{i,j}$ $(1 \leq j \leq m,\ j \neq i)$.

Clearly,

$$\sum_{\substack{j=1 \\ j\neq i}}^{m} q_{i,j} + \sum_{j=1}^{m} p_{i,j} = 1,\ 1 \leq i \leq m.$$

Further, define, $d_{i,j} = \lambda_i p_{i,j}$, $1 \leq i,j \leq m$, $c_{i,j} = \lambda_i q_{i,j}$, $1 \leq i,j \leq m, i \neq j$, and $c_{i,i} = -\lambda_i$, $1 \leq i \leq m$. Then the elementary probability with an arrival (without an arrival) in an infinitesimal interval of length dt which leaves the process in state j, given that the Markov process is in state i, is $d_{i,j}dt$ $(c_{i,j}dt,\ i \neq j)$. Define the matrices $C = (c_{i,j})$ and $D = (d_{i,j})$, then $C+D$ is the infinitesimal generator of the underlying Markov process.

The matrix $C+D$ is irreducible and the stationary vector π of this Markov process satisfies the conditions,

$$\pi(C+D)=0,\ \pi\mathbf{e}=1,$$

where $\mathbf{e}$ is a column vector having all entry 1. The fundamental arrival rate of this Markov process is given by $\pi D\mathbf{e}$. Detail discussion on MAP can be found in Lucantoni et al. [4]. Let $N(t)$ be the number of arrivals in the time interval $(0,t]$ and let $J(t)$ be the state of the Markov process at time t. Define $P_{i,j}(n,t)=Pr\{N(t)=n,J(t)=j|N(0)=0,J(0)=i\}$ be the (i,j)th entry of the matrix $P(n,t)$, further, at time t and $t+dt$ the following balance (Chapman-Kolmogorov) equations are obtained

$$\begin{aligned}\frac{d}{dt}P(0,t) &= P(0,t)C,\\ \frac{d}{dt}P(n,t) &= P(n,t)C+P(n-1,t)D,\quad n\geq 1,\ t\geq 0,\end{aligned}$$

and $P(0,0)=I$. Here, I refers to an identity matrix with an appropriate dimension. Further, the matrix generating function of $P(n,t)$, i.e., $\hat{P}(z,t)=\sum\limits_{n=0}^{\infty}P(n,t)z^n$, satisfies the following matrix differential equation

$$\begin{aligned}\frac{d}{dt}\hat{P}(z,t) &= \hat{P}(z,t)(C+Dz),\quad |z|\leq 1,\\ \mathit{with}\quad \hat{P}(z,0) &= I.\end{aligned}$$

The following expression of $\hat{P}(z,t)$ is produced by solving the aforementioned matrix differential equation

$$\hat{P}(z,t) = e^{(C+Dz)t},\quad |z|\leq 1. \tag{1.1}$$

To deduce the Poisson arrival process from MAP, consider $m=1$, $C=-\lambda$, and $D=\lambda$, then the matrix generating function of $P(n,t)$ of MAP takes the form

$$\hat{P}(z,t) = e^{-(\lambda-\lambda z)t},\quad |z|\leq 1. \tag{1.2}$$

The above equation (1.2) is the probability generating function (PGF) of the Poisson arrival process. The proof is given below.

Theorem 1.1. *Let $\{N(t):t\geq 0\}$ be the Poisson arrival process and $P(n,t)=Pr\{N(t)=n\}$, n =0, 1, 2,...Then probability generationg function (PGF) of $P(n,t)$ is given by $\hat{P}(z,t)=e^{\lambda(z-1)t},\ |z|\leq 1$.*

Proof. The balance equations of the Poisson arrival process (or pure birth process) are given as follows (See, Medhi [5], page 25).

$$\frac{\partial}{\partial t}P(0,t) = -\lambda P(0,t), \tag{1.3}$$

$$\frac{\partial}{\partial t}P(n,t) = -\lambda P(n,t) + \lambda P(n-1,t), \quad n \geq 1, \tag{1.4}$$

with initial condition; $P(0,0) = 1$ and $P(n,0) = 0$, $n \geq 1$.

Define the PGF of $P(n,t)$, $n = 0, 1, 2,...$ as

$$\hat{P}(z,t) = \sum_{n=0}^{\infty} P(n,t)z^n, \quad |z| \leq 1. \tag{1.5}$$

Setting $t = 0$ in (1.5) the following equation is obtained

$$\hat{P}(z,0) = \sum_{n=0}^{\infty} P(n,0)z^n = 1, \quad |z| \leq 1. \tag{1.6}$$

Now multiplying z^n in (1.4) and summing over the range n =1, 2, 3,... and using (1.3), after some algebraic manipulation, the following equation is obtained

$$\frac{\partial}{\partial t}\hat{P}(z,t) = \hat{P}(z,t)\{\lambda(z-1)\}. \tag{1.7}$$

Solving (1.7), one can have

$$\hat{P}(z,t) = Ae^{\lambda(z-1)t}, \quad |z| \leq 1. \tag{1.8}$$

Using (1.6) in (1.8), $A = 1$ is obtained. Hence, the PGF of $P(n,t)$ is given by

$$\hat{P}(z,t) = e^{\lambda(z-1)t}, \quad |z| \leq 1. \tag{1.9}$$

This completes the proof. □

1.3 Bulk queues

Several queueing models are described in the literature, where the consumers enter the system either one by one or in bulk, and the server offers service either one by one or in bulk. Queues in which the customers arrive in bulk or are served in bulk are known as bulk queues. The study presented in this thesis is mainly focused on bulk queues with different vacation policies.

1.3.1 Bulk arrival queue

The term "bulk arrival queueing model" refers to a queueing model where customers enter to the system in groups or batches. The arriving customers group may be predetermined or random in such type of queueing models.

1.3.2 Bulk service queue

The term"bulk service queueing model" refers to a queueing model where the customers are served by the single server in batches. Several batch service rules are available in the literature, which includes fixed batch size bulk service rule, general bulk service rule, versatile bulk service rule, bulk service queue with variable capacity, etc.

Fixed batch size bulk service rule: Fixed batch size bulk service rule is introduced by Bailey [6] for scheduling the medical appointments. It states that a single server serves the waiting costumers in batches of fixed size, say b. After a service, if less than b customers are available in the waiting line, the server remains idle until the waiting line (queue size) hits the threshold value b, and if the queue length is more than b then the server serves a batch of size b.

General bulk service rule: The fixed batch size bulk service rule was proposed by Bailey [6] and was generalized into the general bulk service (GBS) rule by Neuts [7]. In this rule, the server specifies two thresholds, say a and b ($a \leq b$), for providing the service in batches. The minimum customers' requirement to start a service is a, and the maximum serving capacity of the server is b. After service, if c ($a \leq c \leq b$) customers are in the waiting line, then it takes all c customers for the service, and if more than b customers are in the waiting line, then it takes first b customers for the service, otherwise, the server remains idle in the system until the queue size reaches the minimum threshold.

Bulk service rule with variable capacity: In this rule, the server renders the service in batches, and b is the upper threshold of the serving capacity of the server. After one service, if the queue size is empty, then the server remains in the idle state in the system until the queue size becomes positive. At the beginning of the service, the server takes $Y(=i)$, $i = 1, 2, ..., b$ customers for the service, where Y is the random variable having finite support (Y is also called variable service capacity) with probability mass function (PMF) $\Pr(Y = i)$=y_i, $i = 1, 2, ..., b$ and $y_b > 0$. At the beginning of the service, if the queue size ≥ 1 but less than the chosen service capacity i then it does not wait for the queue length to reach i, but takes all customers for service, i.e., it serves min(i, hole queue length) customers with probability y_i.

Versatile bulk service rule: The versatile bulk service (VBS) rule, which is an extension of the general bulk service (GBS) rule, was first presented by Powell and Humblet [8], and later Kim et al. [9] gave the name VBS rule. According to the VBS rule, the server provides the service in batches, and a is the minimum threshold to serve. After one service, if the queue length is less than a, the server remains in the idle state in the system and waits for the queue length to reach a or more for starting service. At the beginning of the service, the server takes $Y(=i)$, $i = a, a+1, ..., B$ customers for the service, where B is the maximum serving capacity of the server, and Y is the random variable having finite support (Y is also called variable service capacity) with probability mass function (PMF) $\Pr(Y = i)$=y_i, $i = a, a+1, ..., B$ and $y_B > 0$. At the beginning of the service, if the number of customers is greater than or equal a but less than the chosen service capacity i then it does not wait for the queue length to reach i, but takes all customers for the service, i.e., it serves min(i, hole queue length) customers with probability y_i. In the case of $a = 1$, the VBS rule is reduced to a bulk service rule with variable capacity.

1.4 Second Optional Service (SOS)

In several practical situations, some customers may often require the subsidiary service facility offered by the same server, also called the second optional service (SOS), just after getting the first essential service (FES). For example, at a ticket counter every arriving customers book a ticket for a destination, however, some of them may also book a return ticket. At a barber's shop, customers enter for a haircut, but some of them may require a shave after their haircut. The concept of SOS was proposed in $M/G/1$ queueing model by Madan [10]. For every customer, the first service is essential and is called the first essential service (FES) some of them request the subsidiary service called the second optional service (SOS).

1.5 Vacation queue

In a classical queueing model server becomes completely inert during his idle period. Levy and Yechiali [11] pondered the concept of vacation policy for the first time to use the idle period of the server in some secondary jobs (*viz.*, machine maintenance) to increase the efficiency of the system. They analyzed the $M/G/1$ queueing system and proposed two different kind of vacation rule (I) Single vacation (II) Multiple vacation, which are defined here below.

Single vacation (SV): According to the SV rule, at the end of a service if server finds an empty queue or a queue with queue size less than the minimum threshold limit to start the service, then the server goes for a vacation and at the end of the vacation, if the server finds the queue with positive queue size (or greater than or equal to the minimum threshold limit), then he starts the service immediately, otherwise, he stays in the system in a dormant (inactive) state till he finds the queue with positive size (or greater than or equal to the minimum threshold limit to start the service).

Multiple vacation (MV): According to the MV rule, at the end of a service if server finds an empty queue or a queue with queue size less than the minimum threshold limit to start the service then the server goes for a vacation, and at the end of the vacation, if the server finds the queue with positive queue size (or greater than or equal to the minimum threshold limit), then he starts the service immediately, otherwise, goes for another vacation, and hence, the server takes repeated number of vacation until he finds the required number of customers in the queue for service at the end of the vacation.

Working vacation (WV): In a classical vacation of a server, primary service stops completely during the vacation period. However, in real-life situations, it has been seen that the server renders the service with different service rates during the vacation period instead of being completely inactive. Such queues are known as the working vacation (WV) queues, and it was proposed for the first time in 2002 by Servi and Finn [12]. In [12], authors generalized the classical vacation queue into WV queue. In WV queues, after a service, if there is any customer present in the queue, it serves the customers with service rate, say μ, otherwise, it goes for the vacation. During the vacation, if there is any customer present in the queue, then the server serves the customer with a lower service rate, say ν, than the actual service rate μ, i.e., $\nu \leq \mu$.

A queueing model fulfilled with some vacation rules is known as the vacation queueing model (or vacation queue). In such models, during the vacation period, the server either performs some supplementary work (*viz.*, machine maintenance, promoting companies policy, etc.) or remains inactive in the system.

1.6 Literature survey

The study of infinite buffer batch size dependent bulk service queuing models with different vacation rules has been carried out throughout the thesis. The mathematical findings of the Markovian and non-Markovian queues are analyzed using the supplementary variable technique (SVT) and the bivariate generating function method. In light of this, the literature survey is presented here below.

Literature survey on bulk queues: In the queueing theory, batch service queue has great applications in telecommunication (e.g., call center), computer communication (e.g., internet service), manufacturing industry (e.g., making the parts of machines), transportation (e.g., bus service, cargo loading), health care system (e.g., group or pool testing), etc. The bulk service queue was initially introduced by Bailey [6] in which author considered the fixed batch size bulk service queue for scheduling medical appointments. Later the fixed batch size service rule was expanded by Neuts [7] in which the author introduced general bulk service (GBS) rule, which was subsequently become famous among the researchers for analyzing the model in both continuous and discrete time setups. Powell and Humblet [8] introduced versatile bulk service (VBS) rule, which is an extension of GBS rule, later Kim et al. [9] named this rule as VBS rule. In GBS rule server has no choice to choose the next group size for serving the group, however, in VBS rule the server can opt the group size for the next service, this key difference makes the VBS queueing model more efficient and more realistic than the GBS queueing model. For a substantial application of the batch service queue in group/pool testing readers are prompted to see Bar-Lev et al. [13], Abolnikov and Dukhovny [14], Abolnikov and Dukhovny [15], Bar-Lev et al. [16] and Claeys et al. [17]. Abolnikov and Dukhovny [15] justified the pool testing method rather than individual testing by the optimization problem under some conditions. For a significant literature on bulk service queue in continuous time setup, see, e.g., Borthakur [18], Medhi [19], Curry and Feldman [20], Jacob and Madhusoodanan [21], Neuts [22], Jayaraman et al. [23], Lee et al. [24], Reddy and Anitha [25], Abolnikov and Dukhovny [15], Ho et al. [26], Banik [27], Banerjee and Gupta [28], Banerjee et al. [29], Banik [30], Gupta and Pradhan [31], Pradhan et al. [32], Pradhan et al. [33], Pradhan and Gupta [34], Pradhan and Gupta [35], Gupta and Banrejee [36], Pradhan [37], Bank and Samanta [38], Samanta and Bank [39], Goswami et al. [40], etc. For discrete time setup, readers can go through, Gupta and Goswami [41], Chaudhry and Chang [42], Janssen and Van Leeuwaarden [43], Goswami et al. [44], Claeys et al. [45], Yu and Alfa [46], Lee [47], Gupta and Banerjee [48], etc. For a key understanding of bulk service queues, readers are invited to see the books by Chaudhry and Templeton [49] and Medhi [50].

In most of the literature on the bulk service queue, the service time does not depends on the batch size under service. The bulk service queues in which the service time (or service rate) depends on the batch size under service are known as batch size dependent bulk service queues. Such queues are more appropriate in modeling crowd control in real-life scenarios because of the additional information about the distribution of the batch size with the server is obtained in this case. Direct application of the batch size dependent service queue reflects in the literatures Bar-Lev et al. [13] and Abolnikov and Dukhovny [15] as an example of group/pool screening for particular diseases (*viz.*, HIV). The concept of batch size dependent models can be understood adeptly from the model presented by Neuts [7]. Later few notable research have been published in literature on finite or infinite buffer batch size dependent bulk service queues, see, Curry and Feldman [20], Nuets [22], Chaudhry and Gai [51], Germs and van Foreest [52], Bar-Lev et al. [16], Claeys et al. [45], Claeys et al. [53], Banerjee et al. [54], Banerjee and Gupta [28], Banerjee et al. [29], etc. A recent coverage in batch size dependent bulk service queue, readers are refered to see, Gupta and Banerjee [55], Pradhan and Gupta [35], Pradhan [56], Yu and Alfa [46], Gupta et al. [57] and the references therein.

In several bulk queues, traffic is found to be highly irregular (bursty and correlated) where the Poisson/renewal arrival process does not fit. To analyze such bursty and correlated traffic, the nonrenewal arrival process, i.e., the Markovian arrival process (MAP) has been studied. The concept of MAP was introduced by Lucantoni et al. [4]. For notable research on continuous time bulk queues with correlated arrival readers can go through the literatures [34, 37, 38, 39, 58, 59, 60, 61]. Pradhan and Gupta [34], Pradhan and Gupta [37], and Banerjee et al. [61] considered $MAP/G_r^{(a,b)}/1$, $BMAP/G_r^{(a,b)}/1$, and $MAP/G_r^{(a,b)}/1/N$ queue, respectively, and obtained the joint distribution of queue and server content at various epochs. Bank and Samanta [38], Samanta and Bank [39], and Singh et al. [60] analyzed $BMAP/G^{(a,Y)}/1$, $BMAP/R^{(a,b)}/1$, and $MAP/R^{(a,b)}/1$ queue, respectively, for queue length distribution at different epochs. The authors of the references [34, 37, 38, 39, 60, 61] conducted their analysis using the supplementary variable technique (SVT), embedded Markov chain technique (EMCT), and roots method. However, in discrete time setup, readers can go through, Yu and Alfa [46], Gupta et al. [57], Chaudhry and Gupta [62] and the references therein.

Several queueing model are presented in the literature in which the customers enter in the system in groups or batches (see, e.g., Briere and Chaudhry [63] and Madan and Malalla [64]) such queues are known as bulk arrival queues. Literatures on bulk arrival queues in which customers are served in batches, readers are refered to see the references [29, 35, 38, 39] and the references therein.

Literature survey on vacation queue: Since the origin of queueing theory, it has been the objective of the researchers to increase the efficiency of the system by reducing congestion in terms of queue length, waiting time, rejection probability, etc. Towards this end, researchers are always keen to introduce new ideas and methodologies in queueing theory from time to time. In queueing system, vacation theory plays a strong character in increasing the efficiency of the system.

Various type of vacation rules are studied in the literature, *viz.*, single vacation (SV), multiple vacation (MV), working vacation (WV), multiple adaptive vacation, etc. Theory of vacation was introduced for the first time in 1975 by Levy and Yechiali [11]. In subsequent year researchers have taken the great interest to analyze the vacation queueing model because of its applicability in real life queues, *viz.*, manufacturing, computer communication, telecommunication, transportation, etc.

Literature on the vacation queueing model (either SV or MV) in which the single server serves the single customer, can be found in the references [4, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83]. Schouten [66], Scholl and Kleinrock [67], Li and Zhu [70] and Frey and Takahashi [73] analyzed $M/G/1$ queue with MV, whereas $M/M/1$ queue with MV has been considered by Altman and Nain [71], Mao et al. [78] and Kalidass et al. [79]. Karaesmen and Gupta [72] presented $GI/M/1$ queue with MV and obtained queue length distribution at arrival as well as random epoch. Yang and Ke [80] and Wu et al. [81] analyzed $M/G/1$ queue with SV. The queueing model with both SV and MV has been considered by Takagi [69], Cong [75], Gupta et al. [76], Gupta and Sikdar [77] and Kempa [82], etc.

For remarkable studies on bulk service queues with SV where the authors have used the supplementary variable technique (SVT) to analysis the model and obtain queue length (or probability generation function of the queue length) distribution at different epoch, readers are invoked to see the references [84, 85, 86, 87, 88, 89] and the references therein. In several queueing systems, the server takes the repeated vacation, such queues are known as MV queues. Literatures on bulk service queues with MV, where authors analyzed different queueing models using the SVT, can be found in the references [90, 91, 92, 93, 94, 95, 96, 97]. Bulk service queueing models in which SV and MV both have been analyzed, can be found in the references [24, 98, 99, 100, 101] and the references therein. Lee et al. [24] analyzed $M/G^b/1$ queue and obtained the PGF of the queue length distribution at random epoch. Sikdar and Gupta [99] considered $M^X/G^Y/1/N$ queue and obtained the stationary queue length distribution at various epoch by using SVT. Recently, Gupta et al. [101] worked for obtaining the joint distributions at various epoch by using the SVT and considering transition probability matrix for the $M/G_r^{(a,b)}/1/N$ queue with SV and MV. Few researchers have considered vacation queueing models in discrete time setup as well, see, e.g., Samanta et al. [98], Chang and Choi [102], and Nandy and Pradhan [103]. Samanta et al. [98] and Chang and Choi [102] considered finite buffer $Geo^X/G^{(a,b)}/1/N$ and $Geo^X/G^Y/1/K+B$ queue with SV (MV), respectively, and obtained the queue length distribution at various epoch. Nandy and Pradhan [103] worked for the joint distribution of the queue and server content for $Geo/G^{(a,b)}/1$ with SV (MV). On the vacation queueing model, the excellent bibliography can be found in the survey paper of Doshi [104] and Ke et al. [105]. For the quality literature on vacation theory, readers are referred to see Takagi [106] and Tian and Zhang [107].

Most of the works on vacation queueing systems assume random vacation time of the server, however, only a few authors analyzed the vacation queues in which the vacation time depends on the queue length. Such a vacation policy helps in improving the efficiency of practical problem. In this connection, a notable contribution has been added by Harris and Marchal [68] where they considered $M/G/1$ queue and obtained the PGF of the stationary queue length distribution using the stochastic decomposition method. Shin and Pearce [74] considered queue length dependent vacation in $BMAP/G/1$ queueing model and obtained the stationary distribution of queue length at departure epoch. Thangaraj and Rajendran [108] analyzed group arrival group service and single service queueing model where the server takes a fast vacation and slow vacation depending on the queue length. $BMAP/G/1/N$ queue with E-limited service and queue length dependent vacation considered by Banik [109] and numerically shown that the queue length dependent vacation policy helps in reducing the congestion. Recently, Gupta et al. [101] obtained the joint distribution of queue and server content as well the joint distribution of the queue content and type of vacation (i.e., queue length at vacation initiation epoch) for $M/G_r^{(a,b)}/1/N$ with SV (MV). Their graphical results reflects that the queue length dependent vacation policy makes the system more efficient than the queue length independent vacation policy. Lavanya et al. [110] analyzed $M^X/G^{(a,b)}/1$ queue with two types of vacations and setup time using SVT.

Literature survey on Second Optional Service (SOS): The terminology of second optional service in queueing systems was proposed by Madan [10], where he analyzed the $M/G/1$ queueing model using SVT. For every customer, the first service is essential and is called first essential service (FES), some of them request the subsidiary service called second optional service (SOS). Later, many researchers have analyzed different queueing models with a second optional service, see, Medhi [111], Al-Jararha and Madan [112], Wang [113], and Choudhury and Tadj [114].

For the bulk queues with SOS, few papers are available in literature. Choudhury and Paul [115] considered $M^X/G/1$ queue with SOS and N policy, and obtained the queue length distribution at various epochs. Maraghi et al. [116] considered $M^X/G/1$ queue with vacation, SOS and random system breakdowns, they obtained steady state PGF of queue size at arbitrary epoch. Ayyappan and Shyamala [117] considered $M^X/G/1$ queue with SOS, Bernoulli Schedule server vacation and random break downs and obtained the time dependent probability generating functions in terms of the Laplace transforms and the corresponding steady state results explicitly. Ayyappan et al. [118] considered fixed batch size bulk service queue with multiple vacation and SOS. They obtained various steady state probabilities of interest using the PGF technique. Madan and Malalla [64] analyzed batch arrival queue with the second optional service, breakdown at random time, and delayed repair. They derived the queue length distribution and some performance measures of interest. Ayyappan and Supraja [119] analyzed $M^X/G^{(a,b)}/1$ queue with unreliable server, SOS, two different vacations, and restricted admissibility policy and obtained the queue length distribution at random and departure epoch using the SVT. Singh et al. [120] analyzed bulk arrival queue with different m-SOS, vacation, and unreliable server using SVT. Ayyappan and Deepa [97] considered $M^X/G^{(a,b)}/1$ queue with SOS, MV, and setup time. They obtained the PGF of the queue size at different epochs using SVT. For the current work on the bulk queues with SOS, readers are invoked to see, Laxmi and George [121], Deepa and Azhagappan [122] and the references therein.

1.7 Motivation and Objective

The application of batch service queue in real-life congestion draws the attention of the researchers to discover more about such queuing model to apply it in various fields, *viz.*, computer network and telecommunications, transportation, manufacturing, etc. In literature, several research articles are available for batch service queues with batch size independent service, however, in most practical situations, the batch size under service affects the service time. That's why considering batch size dependent service queues plays a key role in improving system efficiency instead of batch size independent service queues. Group service queues with vacation has made a good impact on researchers in the past few decades, as in such models the idle time of the server can be used for some other task (*viz.*, machine maintenance). In most of the vacation queueing models, the server goes for the vacation, and the vacation time is independent of the queue length at vacation initiation epoch. However, going for vacation by observing the queue length at vacation initiation epoch reduces the waiting time, reduces congestion of the system, and increases system efficiency which is observed in Gupta et al. [101] and Banik [109]. For example, if the server observes a shorter queue length at vacation initiation epoch, it takes a longer vacation and uses that time for supplementary work, however, if the server observes a longer queue length at vacation initiation epoch, it takes a shorter vacation that reduces the waiting time of customer and reduces congestion of the system. As a result, the information of the queue length dependent vacation increases the efficiency of the system instead of queue length independent vacation. The literature on finite buffer batch size dependent bulk service queue with queue length dependent vacation and different batch size bulk service rule can be found in the thesis by Gupta [123]. However, in the practical situation (*viz.*, in computer network, digital data or signals transmits continuously through an Ethernet link in packets), several queueing systems have been observed in which considerabily large (infinite) waiting space is provided. This motivated the author to study infinite buffer batch size dependent bulk service queueing models with queue length dependent vacation (SV and MV) policy and with various batch service rules. The main objective of the thesis is to see whether the queue length dependent vacation also increases the efficiency of infinite buffer batch size dependent bulk service queue or not as infinite buffer queues are more realistic than finite buffer queueing system. Towards this end, our objective is to consider different infinite buffer batch size dependent bulk service queueing models with queue length dependent vacation (SV and MV) and obtain the joint probabilities of queue size and server content as well as the joint probabilities of the queue size and type of vacation at different epochs.

1.8 Organization of the thesis

Queueing models or queueing systems are studied to increase the efficiency of the queueing system. Throughout this thesis, infinite buffer batch service queueing models with queue length dependent single (multiple) vacations have been studied. In light of this, this thesis contains six chapters which briefed as follows.

Chapter 1 presents a brief literature survey which is mainly focused on bulk queues, vacation (SV and MV) queues, and SOS queues. The motivation and objective of the thesis are placed after the literature survey for the readers. Finally, Chapter 1 ends with the organization of the thesis.

Chapter 2 analyzes the infinite capacity Markovian bulk service queueing system with SV and MV. A single server serves the customers in batches following the GBS rule. The inter arrival time, the service time, and the vacation time follow the exponential distribution. Using the bivariate generating function method, the joint probabilities of the queue size and server content and the joint probabilities of the queue size and type of vacation are obtained. Numerical results are displayed to see the behavior of the performance measures.

Chapter 3 analyzes the infinite capacity batch size dependent bulk service non-Markovian queueing system with queue length dependent SV and MV. Customers' services are performed in batches as per the GBS rule with general service time distribution and the service time depends on the batch size under service. The (SV and MV) model has been studied in an unified way in the model. The vacation time is generally distributed and depends on the queue length at the vacation initiation epoch. Arrivals follow the Poisson process. Mathematical analysis has been done using the SVT and the bivariate generating function approach. The joint probabilities of the queue size and server content and the joint probabilities of the queue size and type of vacation at various epochs have been obtained. Numerical results have been presented to validate the system efficiency.

Chapter 4 analyzes the batch size dependent bulk service queue with SV and MV where the arrivals follow the MAP. The customers are served in batches by a single server according to the GBS rule with general service time distribution and the service time depends on the size of the batch under service. The server goes for the vacation with generally distributed vacation time which depends on the size of the queue at the vacation initiation epoch. Using the well known SVT and the bivariate generating function approach, the joint probabilities of the queue size and server content and the joint probabilities of queue size and type of vacation have been obtained. Finally, various performance measures have been discussed to observe the efficiency of the system.

In **Chapter 5**, the bulk arrival bulk service queue with SV (MV) and second optional service (SOS) has been analyzed. Customers arrive in the group following Poisson manner. Customers are served by a single server following the VBS rule with general service time distribution, and batch size dependent service rate. Queue length dependent vacation policy has been assumed, and the vacation time of the server also generally distributed and depends on the queue length at the vacation initiation epoch. The service (FES and SOS) time and the vacation time follow the general distribution. Mathematical analysis has been done using the SVT and the bivariate generating function approach. The joint probabilities of the queue size and server content and the joint probabilities of the queue size and type of vacation at various epochs have been obtained. Numerical results have been presented to validate the model system efficiency.

The concluding remarks of the author's work depicted in Chapter 2 to Chapter 5 are presented in **Chapter 6**. The scope of future study are also presented in Chapter 6.