

# STUDIES ON SOME INFINITE BUFFER BATCH SIZE DEPENDENT BULK SERVICE QUEUES WITH QUEUE SIZE DEPENDENT VACATION



Thesis submitted in partial fulfillment  
for the Award of Degree

*Doctor of Philosophy*

*by*

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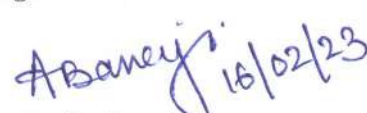
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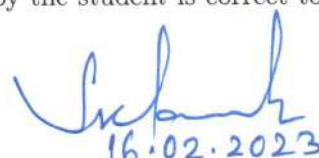
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# Preface

Queueing theory provides the study of controlling or managing the congestion in various areas (*viz.*, computer network, telecommunication, transportation, etc.). In literature, the information of the server content and queue size has gain remarkable attention to reduce the congestion in the batch size dependent bulk service queue. This motivates the author to analyze the infinite buffer batch size dependent bulk service queue and queue size dependent single and multiple vacation for obtaining the joint probabilities of the queue content and server content and joint probabilities of the queue content and type of vacation. Towards this end, in Chapter 2, author first develop the mathematical procedure for obtaining required joint probabilities by considering  $M/M^{(a,b)}/1$  queue with single and multiple vacation. In chapters 3, 4, and 5, queue length dependent vacation policy is studied along with batch size dependent service in infinite buffer queues.

Chapter 1 is introductory that contains some basic definitions and a literature survey related to batch service queueing models with several vacation policy. The chapter begins with some basic notations that represent the queueing systems. Some basic definitions are provided to understand the considered batch service queueing models with server's vacation in forthcoming chapters. After that, a literature review related to batch service queueing models with server's vacation is presented, then the motivation and objective of the thesis is presented. The chapter ends with the organization of the thesis.

Chapter 2 analyzes  $M/M^{(a,b)}/1$  queueing model with single vacation (SV) and multiple vacation (MV). The customers are served in batches according to the general bulk service (GBS) rule. The inter arrival time, the service time, and the vacation time follow the exponential distribution. SV and MV both cases have been studied in an unified way for the considered model. Using the bivariate generating function method, the joint probabilities of the queue size and server content and the joint probabilities of the queue size and type of vacation are obtained. Numerical results are provided to see the behavior of the performance measures.

Chapter 3 describes the analysis of  $M/G_r^{(a,b)}/1$  queueing system with queue length dependent SV and MV. Customers' services are performed in batches as per the GBS rule with general service time distribution and the service time depends on the batch size under service. The server takes either SV or MV. The SV and MV both cases have been analyzed in an unified way in the model. The vacation time is generally distributed and depends on the queue length at the vacation initiation epoch. Arrivals follow the Poisson

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process. SVT and the bivariate generating function approach have been used for mathematical analysis. The joint probabilities of the queue size and server content and the joint probabilities of the queue size and type of vacation at various epochs are obtained. Numerical results are presented, in which a comparison study has been presented to show the effect of state dependent vacation. A cost model is also presented to show the effect of our model.

Chapter 4 analyzes the  $MAP/G_r^{(a,b)}/1$  queueing model. The customers are served in batches according to the GBS rule with general service time distribution and the service time depends on the size of the batch under service. The SV and MV both the cases have been studied in an unified way in the same model. The vacation time is generally distributed and depends on the size of the queue at the vacation initiation epoch. Using the SVT and the bivariate generating function approach, the joint probabilities of the queue size and server content and the joint probabilities of queue size and type of vacation are obtained. Finally, various performance measures are discussed to observe the efficiency of the system. Numerical results are also presented.

In chapter 5,  $M^X/G_r^{(a,Y)}/1$  queueing model with SV (MV) and second optional service (SOS) has been discussed. Customers arrive in the group following Poisson manner. Customers are served according to the versatile bulk service (VBS) rule. The first service is essential called the first essential service (FES). The service (FES) time depends on the batch size under service. The server takes either SV or MV which have been studied in an unified way in the same model. The vacation time depends on the queue length at the vacation initiation epoch. The service (FES and SOS) time and the vacation time follow the general distribution. The joint probabilities of the queue size and server content and the joint probabilities of the queue size and type of vacation at various epochs have been obtained using SVT and the bivariate generating function approach. Numerical results have been presented to validate the system efficiency.

In Chapter 6, the overall work has been concluded and also the future scope is presented.

**Keywords:** Infinite buffer, Batch size dependent bulk service, General bulk service (GBS) rule, Versatile bulk service (VBS) rule, Batch arrival, Joint probabilities, Markovian arrival process, Bivariate generating function, Poisson queue, Non-Poisson queue, Queue length dependent vacation, Single vacation, Multiple vacation, Single server, Supplementary variable technique, Second optional service.