Conclusion and Future Scope

The work of this thesis has close connections with *dual-Baer* modules, *dual-Rickart* modules, which are dual concepts of *Baer* modules and *Rickart* modules, respectively. Baer modules and Rickart modules are module theoretical notions of Baer rings and Rickart rings. Motivated by these dual-like properties of modules, we have introduced principally quasi-dual-Baer module, Σ-dual-Rickart module and finite Σ-dual-Rickart module.

In Chapter 2, we studied principally quasi-dual-Baer (PQ-dual-Baer) modules, which is a generalization of the quasi-dual-Baer modules and the dual concept of principally quasi-Baer modules. In general, every dual-Baer and quasi-dual-Baer modules are PQ-dual-Baer module, but the converse need not be true, counterexamples have been constructed to illustrate it. To fulfill the gap among these notions, we found some conditions under which these notions are equivalent to each other. We characterized semisimple Artinian rings and von Neumann regular rings in terms of PQ-dual-Baer modules. We also studied the endomorphism ring of PQ-dual-Baer modules.

In chapter 3, we introduced the class of Σ-dual-Rickart modules, which is properly contained in the class of dual-Rickart modules. We constructed some examples of Σ-dual-Rickart modules that are not Σ-Rickart modules and vice-versa. We find conditions for which Σ-dual-Rickart modules are Σ-Rickart modules and vice-versa.

We also characterized semisimple Artinian rings, hereditary rings, semi-hereditary rings and von Neumann regular rings in terms of Σ-dual-Rickart modules.

In Chapter 4, we have defined the class of finite Σ-dual-Rickart modules, which generalizes the class of Σ-dual-Rickart modules and dual of finite Σ-Rickart modules. We proved that every cohereditary module is a finite Σ-dual-Rickart module. Finite Σ-dual-Rickart modules have some nice properties rather than Σ-dual-Rickart modules, like; "A ring R is hereditary if and only if every injective R-module is finite Σ-dual-Rickart R-module", "Endomorphism ring of a finite Σ-dual-Rickart module is left Coherent" etc.

In Chapter 5, We studied some properties of purely extending modules, which were not studied earlier. Also, we generalized the notion of purely Baer modules as well as purely extending modules. We have constructed some examples and counterexamples which delimit our results.

Throughout the thesis, we characterized von Neumann regular, semisimple Artinian rings in terms of our notions introduced in the last three chapters.

Future Scope

During the study, we found a number of research problems that are unsolved and unavailable in the literature on which work can be done in the future. We would like to enlist a few of them as follows:

1. We can generalize the class of projective modules in terms of cogenerated modules and then characterize Σ-dual-Rickart modules in terms of this class.

2. Also, we can find some theoretical applications of finite Σ-dual-Rickart modules in terms of f-projective modules, regular rings, semisimple Artinian rings, intrinsically injective modules, etc.

3. We can also extend the theory of Σ-dual-Rickart modules and finite Σ-dual-Rickart modules with the help of pure theory of modules.

4. We characterized Σ-dual-Rickart modules and finite Σ-dual-Rickart modules in terms of $Add(M)$. $Add(M)$ plays a fundamental role in the theory of tilting modules. So, we can relate the theory of Σ-dual-Rickart modules in terms of tilting modules.